THE RELATIVE FRACTION OF RAYLEIGH AND LOVE WAVES IN AMBIENT VIBRATION WAVEFIELDS AT DIFFERENT EUROPEAN SITES

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Abstract - The way of interpreting results of ambient vibration based techniques in the context of near surface site structure investigations very often depends on the assumptions about the nature of the wavefield. Taking into account the relative fraction of Rayleigh and Love waves on the horizontal components is important in order to improve the evaluation of H/V spectral ratios by realistic forward modeling. Within this study we employed the modified 3c-SPAC method to determine the Rayleigh to Love wave partitioning at eight different European sites. Site structures, frequency dependence and temporal stationarity were analyzed in order to find characteristic features in the results. We found a dominance of Love waves between 70 and 90 % over a broad frequency range within the reliable band for analysis. In conclusion, there seems to be no systematical effect of the site on the mean level of Love waves. However, we observed deviations from the mean level inside the reliable frequency band. These effects are characterized by a higher variance of the partitioning in the vicinity of the fundamental frequency of the site. This can be attributed to the Rayleigh wave ellipticity, the Airy phase of Love waves and, for some sites, also to deviations from the assumption that surface waves dominate the wavefield.

1. Introduction

Although the nature of the ambient vibration wavefield is not known in much detail, we have some basic ideas about its general behavior (see for a review Bonnefoy-Claudet et al., 2006a). In order to let common analysis methods like f-k, SPAC and H/V spectral ratios work properly, stationarity and mainly surface wave character of the wavefield have to be assumed. Supported by many studies in the past, it has turned out that the dominance of surface waves can indeed be assumed. For the analysis of three-component records (H/V, 3c-f-k, 3c-SPAC) it is furthermore of considerable interest to have information about the relative partitioning between Rayleigh and Love waves. In particular, techniques that use forward modeling of Rayleigh wave ellipticities to fit the shape of H/V spectra are depending on realistic estimates for the fraction of Love waves (see Arai and Tokimatsu, 2000). Moreover, the dispersion analysis of the horizontal ground motion requires knowledge about the presence of Rayleigh and Love waves in the analyzed data window (Tokimatsu, 1997). Presently there is no consensus about the relative fraction of Rayleigh and Love waves since only a few studies were carried out that provide some estimates (see Bonnefoy-Claudet et al., 2006a). The results cannot be generalized and not all can be compared due to different methods used and due to the restriction to individual sites.
Some of these studies were carried out in Japan using the 3c-SPAC method and reporting a dominance of Love waves. Only one equivalent study about volcanic tremors is known in Europe (Chouet et al., 1998). In fact, at least in Europe, no studies are known that have investigated the partitioning between Rayleigh and Love waves systematically using the same technique at different sites.

The spatial autocorrelation method (SPAC) introduced by Aki (1957) became very popular in the context of ambient vibration analysis in the last years. Many modification and extensions of SPAC (e.g. recently Cho et al., 2006) were developed since 1957 one being the modified SPAC method (MSPAC: Bettig et al., 2001). MSPAC enables the use of arbitrary array layouts which is an important advantage for array measurements in urban areas compared to the traditional method. Although SPAC was introduced by Aki as a three-component method, only a minority of the studies employed all three component recordings to investigate additionally the propagation properties of Love waves. The advantage of SPAC compared to f-k analysis is that assumptions concerning propagating directions of Rayleigh and Love waves on the horizontal components are unnecessary. Furthermore, being the issue of this study, the Rayleigh to Love wave partitioning on the horizontal components (in the following called $\alpha$) can be estimated in addition to the single mode phase velocities. In particular, we employed the extension of MSPAC on three components (Köhler et al., 2006) in order to compare $\alpha$ systematically at different European sites where ambient vibration array measurements were carried out in the last years.

2. Method and data

For the SPAC method circular or at least regular arrays are required for the azimuthal averaging of the autocorrelations (coherency) between two stations. For MSPAC the averaging process is extended to all the station-combinations (co-array) that lay within a ring. The division of the co-array into rings of radii $r_{\text{min}}=r_1$ and $r_{\text{max}}=r_2$ depends on the distribution of stations pairs and is done manually before processing. The three component spatial averaged autocorrelation coefficients ($z,r,t$) for a particular ring $k$ are related to the Rayleigh and Love wave phase velocities and $\alpha$ (fraction of Rayleigh waves) as follows (Köhler et al., 2006),

\[
\rho_{z,k}(\omega) = \frac{2}{r_2^2-r_1^2} \frac{c_R(\omega)}{\omega} \left[ r \cdot J_1|w_R| \right]_{r_i}^2 \tag{1}
\]

\[
\rho_{r,k}(\omega) = \frac{4}{r_2^2-r_1^2} \frac{\alpha c_R(\omega)}{\omega} \left[ \frac{c_R(\omega)}{\omega} \left[ J_0|w_R| \right]_{r_i}^2 + r \cdot J_1|w_R| \right]_{r_i}^2 \tag{2}
\]

\[
\rho_{t,k}(\omega) = \frac{4}{r_2^2-r_1^2} \frac{\left(-\alpha\right)c_L(\omega)}{\omega^2} \left[ J_0|w_L| \right]_{r_i}^2 + \frac{1-\alpha}{\omega^2} \left[ J_0|w_L| \right]_{r_i}^2 \]

\[
+ \frac{c_L(\omega)}{\omega} \left[ J_0|w_L| \right]_{r_i}^2 + r \cdot J_1|w_L| \right]_{r_i}^2 \tag{3}
\]
where the arguments of the Bessel functions \( w_{R/L} \) correspond to \( \frac{\omega \cdot r}{c_R(\omega)} \) and \( \frac{\omega \cdot r}{c_L(\omega)} \).

The left hand sides of equations 1-3 were calculated from the data in the time domain for each frequency \( \omega = 2\pi f_c \). For this purpose the traces were bandpass filtered between \( 0.9 f_c \) and \( 1.1 f_c \) and correlated over a time segment of frequency dependent length. After the ring averaging for each time segment, the mean was calculated by averaging over all time segments inside the data window chosen for processing. This data window had a length of about 15 to 20 min for all computations. By means of a simple grid search we derived the model parameters \( c_R \), \( c_L \) and \( \alpha \) (right hand sides) from the data (step width for \( \alpha \) was 0.05). For the processing see also Köhler et al. (2006). Note that \( \alpha = 0 \) does not mean necessarily the absence of Rayleigh waves. There could be still power on the vertical component in the case of a vertical orientated polarization. This is implying that \( \alpha \) is also affected by the frequency dependent Rayleigh wave ellipticity. Obviously, the equations 1-3 do not include multiple modes. However, as shown in Köhler et al. (2006), the effect of higher modes on \( \alpha \) can be neglected.

![Figure 1. Locations of the ambient vibration measurement sites used within this study: 1: Colfiorito, 2: Hamburg, 3: Weil a. Rhein. 4: Grenoble, 5: Liege, 6: Uccle (Brussels), 7: Pulheim, 8: Thessaloniki](image)

The ambient vibration data sets used in this study were acquired within the SESAME (Bard, 2004) and the recent HADU project. They include eight European sites (Figure 1) with different subsurface structures from deep sedimentary basin (e.g. Grenoble, France) towards very shallow sediment cover over basement rocks (e.g. Liege). The majority of the
studies were carried out within urban areas, whereas Colfiorito was a more rural site. There is data available recorded at daytime as well as at night. More than one array was installed within the measurement area at several of the eight locations. See table I for an overview of these sub-sites, the array configurations and the simplified one layer over halfspace Vs models for each site.

Table I. Description of measurement sites

<table>
<thead>
<tr>
<th>site (index in figure 1, name, country)</th>
<th>sub-site</th>
<th>number of array stations</th>
<th>time of day</th>
<th>depth [m]</th>
<th>Vs ratio at depth</th>
<th>model*</th>
<th>number of time windows</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Colfiorito, (I)</td>
<td>A</td>
<td>12</td>
<td>day</td>
<td>70</td>
<td>5.0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>B &amp; D</td>
<td>12</td>
<td>day &amp; night</td>
<td>48</td>
<td>12.0</td>
<td>1</td>
<td>1 &amp; 2</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>12</td>
<td>night</td>
<td>65</td>
<td>7.5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2 Hamburg (D)</td>
<td></td>
<td>3 x 7</td>
<td>day</td>
<td>55</td>
<td>4.5</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3 Weil a. Rhein (D)</td>
<td></td>
<td>2 x 25</td>
<td>day</td>
<td>280</td>
<td>4.0</td>
<td>3</td>
<td>2</td>
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<tr>
<td>4 Grenoble (F)</td>
<td>E</td>
<td>13</td>
<td>day</td>
<td>800</td>
<td>4.3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>B.Pasteur</td>
<td>13</td>
<td>day &amp; night</td>
<td>600</td>
<td>4.3</td>
<td>4</td>
<td>2 &amp; 1</td>
</tr>
<tr>
<td></td>
<td>Campus</td>
<td>13</td>
<td>day</td>
<td>700</td>
<td>4.3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5 Liege Botanique (B)</td>
<td></td>
<td>10</td>
<td>day</td>
<td>10</td>
<td>5.0</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6 Uccle (B)</td>
<td></td>
<td>10</td>
<td>day</td>
<td>120</td>
<td>3.5</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>7 Pulheim (D)</td>
<td></td>
<td>12</td>
<td>day</td>
<td>230</td>
<td>6.3</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>8 Thessaloniki (GR)</td>
<td>ago</td>
<td>13</td>
<td>early morn.</td>
<td>80</td>
<td>-</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>villa</td>
<td>13</td>
<td>day</td>
<td>260</td>
<td>-</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>kal</td>
<td>13</td>
<td>day</td>
<td>420</td>
<td>-</td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>


3. Results and discussion

The determined \( \alpha \) had to be interpreted very carefully due to the necessity of correct estimates for the Rayleigh and Love wave phase velocities. We chose to trust only \( \alpha \) at frequencies that were showing reasonable velocities. In particular, we considered both a lower and an upper frequency limit. At higher frequencies aliasing can cause steps or increasing velocity branches in the dispersion curve leading to wrong \( \alpha \) estimates. A restricted grid search to exclude aliasing minima was able to broaden the frequency band in these cases. Towards lower frequencies the resolution of the method and consequently the resolution of \( \alpha \) decreases. Moreover, the lack of energy can cause unstable results. By comparing the estimated velocities with forward computed dispersion curves in the case of already existing site models, the reliability of the determined Rayleigh to Love wave partitioning was evaluated indirectly.
Figure 2. Contour plots for each model parameter representing the results of the grid search for the Grenoble Bon Pasteur (left) and the Pulheim site (right). Black stands for low misfit. Grey scale and contour lines are logarithmic. White triangles are the model parameters corresponding to the lowest misfit and white error bars are the standard deviations. The white curves show the theoretical dispersion curves (fundamental and first higher mode). The red/dark dashed lines correspond to the bounds of the frequency band of stable $\alpha$ level. The white/light dashed line represents the lower limit of the reliable frequency range chosen for interpretation. The corresponding upper limit is identical to the rightmost red line.

In Figure 2 the results of two grid searches were plotted for each model parameter $c_R$, $c_L$ and $\alpha$ exemplary for the Pulheim and Grenoble Bon Pasteur site. The error bars of all model parameters represent one standard deviation calculated by means of the curvature of the error surface and the uncertainties of the autocorrelation coefficients (see Köhler et al., 2006). Considering the best fitting models as well as the whole error surface for each model parameter, aliasing (velocity steps), decreasing resolution (see error bars) and missing energy (variation in Rayleigh wave dispersion curve) can be observed in Figure 2. Based on such grid search plots, a reliable frequency range was chosen visually for each array layout at all sites on which further visualization and interpretation was restricted (lower bound: white dashed line; upper bound: rightmost red line).
Figure 3 shows the results for $\alpha$ as a function of frequency for each site. The thick solid curves represent the mean value for each time window and the thin curves one standard deviation given from the grid search error bars. Except the sites Uccle and Liege Botanique, one or more other time windows, indicated by the different gray scale, were analyzed to prove temporal stationarity in the available data. The frequency band between the two solid vertical lines represents the position and approximately width of the maximum in the H/V spectral ratio (fundamental frequency).

In order to compare the mean level of $\alpha$ with the subsurface structure of the particular site, we defined a frequency band in which the value for $\alpha$ is more or less stable. The lower and upper bounds of this band are indicated by the dashed vertical lines in both Figure 2 and 3. Note that for Thessaloniki there are more than one measurement sites within one diagram. In particular, three sites within the city of Thessaloniki were plotted together. They are all located over a strongly dipping velocity contrast. The gray scale goes from light gray (shallow) to black (deep) in this case. The five different $\alpha$ curves for the Hamburg site belong to three arrays of different apertures. Their array midpoints, however, were located at the same position. In that way a broader frequency band could be analyzed. A similar situation existed at the site Weil a. Rhein where two different arrays configurations were installed at the same location but at two different days. For each day a time window was chosen for processing.

Considering all sites we made a first order observation, confirming previous studies, that there is a dominance of Love waves on the horizontal components (> 50 %). Restricting our interpretation to the stable frequency bands (red lines), we observed the majority of $\alpha$ values between 0.1 and 0.4. Looking at the results in more detail, especially at the sites that show a clear plateau in the curve (Grenoble all sites, Weil a. Rhein, Hamburg, Colfiorito E, Pulheim), or ignoring some peaks (Colfiorito B and D), we even observed a mean level of Love waves between 70 – 90 %. Note that the frequency axes are plotted logarithmically.

The different behavior of $\alpha$ at lower frequencies is characterized by higher variance in the vicinity and within the frequency band of the H/V spectral maximum (Colfiorito B, D and E, Hamburg, Weil a. Rhein, Liege, Uccle, Thessaloniki, Pulheim). Some sites (Colfiorito B, D and E, Hamburg, Uccle, Pulheim) even show a clear trough in the $\alpha$ curve within the H/V maximum band, partially going down to zero. Furthermore, this trough is associated with unstable behavior of the Rayleigh wave dispersion curve while Love wave velocities seem to be unaffected (see Pulheim Figure 2). The last observation can be explained by lack of Rayleigh wave energy on the vertical component due to the Rayleigh wave ellipticity maximum (pure horizontal polarization). At the same frequencies the Love wave Airy phases corresponding to a minimum in the group velocity curve of Love waves lead to higher amplitudes on the horizontal components and consequently to a decreasing $\alpha$. In general, a mixture of both phenomena, additionally affected by the lower resolution limit of the method, are causing instabilities in the $\alpha$ curve at lower frequencies. Moreover, for low impedance contrasts Bonnefoy-Claudet et al. (2006b) proposed the existence of a significant portion of body waves.

The Grenoble sites are too deep or the apertures of the arrays are too small, respectively, to resolve $\alpha$ directly at the H/V peak frequency. However, above the H/V peak a different behavior of the $\alpha$ curves compared to all other sites can be observed. This feature will be discussed later.
Figure 3. The relative fractions of Rayleigh waves (Alpha) for all analyzed sites and time windows (different gray scale) together with uncertainties (thin curves). Vertical lines: H/V maximum (solid) and limits for stable frequency band (dashed).
Except the Colfiorito sites and Weil a. Rhein, for which the standard deviation is clearly higher than at other sites, the standard deviation as well as the variation between the different time windows is in the order of +/-10 % in the defined stable frequency band. While the standard deviation is standing for the uncertainty within the time window (lack of stationarity and resolution), the difference between $\alpha$ curves represents the stationarity at the site over the total measurement time. Depending on the available data, that period corresponds to several hours up to two days (Weil a. Rhein).

DiGiulio et al. (2006) reported for the Colfiorito A site difficulties to derive a stable dispersion curve over a wide frequency band employing f-k analysis. Therefore, it is likely that the assumptions for the analysis are not fulfilled here, e.g. due to 3D effects or non-surface wave signals. Considering the broad error bars in Figure 3, this site is indeed an example where $\alpha$ should not be interpreted at all. However, the sites B (D) and E within the Colfiorito basin show very stable results for $\alpha$ below 0.4. Interestingly, the time windows recorded at night (the two brighter curves) contain more Rayleigh waves then the day data recorded around noon. Furthermore, the variation with frequency is higher at night. Site E shows again slightly lower values although recorded at night, too. Grenoble Bon Pasteur is another example where day and night data were analyzed at the same site. However, no significant difference can be observed (compare the two darkest curves in Figure 3) in contrast to Colfiorito B (D). Therefore, differences between day and nighttime could not be generalized based on observations of this study.

The difference between day and night at Colfiorito and the absence of such a difference for the Grenoble data may exist due to the rural character of Colfiorito. One might think e.g. about absence of cultural noise at night.

The Grenoble alpine valley is the deepest of all our sites and is showing very high $\alpha$ values (0.4 – 0.9) at frequencies below 1 Hz. We did not observe such a phenomenon at the other sites. The increasing of $\alpha$ towards lower frequencies is associated with very low Love wave velocities that lay outside any realistic range, if we compare them with the expected theoretical velocities (Figure 2). Furthermore, there are some instabilities in the Rayleigh wave dispersion curve (Figure 2). Therefore, the interpretation is difficult and ambiguous. A total disappearance of Love waves may explain that arbitrary velocities are found for the best fitting model. But on the other hand the Love wave dispersion curve is showing this unrealistic behavior at frequencies where still a percentage of 40 – 50 % Love waves should exist. Therefore, we have to take into consideration that the assumptions may not be fulfilled that we have claimed for the method (surface waves and parallel wavefronts). This might result from 2D/3D effects within the basin, e.g. standing waves and basin resonance. The interpretation is in line with Chaljub et al. (2006). The authors observed SH and SV-wave resonance at frequencies lower than 1 Hz in the Grenoble basin using both ambient noise and earthquake recordings. Furthermore, one may ask whether a high $\alpha$ is a general feature at low frequencies or an effect of the local site. But since we found low $\alpha$ values at low frequencies (~0.5 Hz) for Weil a. Rhein and Pulheim, which are also relative deep sites, a local effect is more likely. For frequencies lower than 1Hz ambient vibrations are exited predominantly by non-cultural sources. Therefore, the seismic waves causing the observed effects at the Grenoble site are most likely oceanic microseismicity and meteorological effects within the alpine valley. However, it does not explain the observed apparent portion of transversal waves since it is difficult to generate them by such phenomena.

The Belgian sites (Uccle and Liege Botanique) show a prominent step in the shape of the $\alpha$ curve which is not directly related to the H/V maximum. The fraction of Rayleigh
waves below (Liege) or above (Uccle) this step, respectively, are relatively high (40 and 50 %). Again, the cause of this observation is unclear. As one possible explanation we can think of dominant local sources at these sites. The peak for the Thessaloniki results between 1 and 2 Hz far away from the H/V frequency is another example for such kind of phenomena. Although it is observed within the cultural noise frequency band (> 1 Hz), the higher portion of Rayleigh waves may be attributed to the near sea (oceanic microseismicity). But since a similar effect is missing at the Hamburg site, this interpretation remains ambiguous.

Figure 4. fraction of Rayleigh waves (Alpha) for all frequencies in the stable band versus depth of the dominant seismic contrast, the respective S-wave velocity ratio and travel time within layer (up and down). The gray scale illustrates the variation within this frequency band. Dark colors stand for more frequent values.

In Figure 4 we plotted the α values for all frequencies within the band visually defined as stable and for all sites versus depth of the dominant seismic contrast, ratio of S-wave velocities at this depth and the mean travel time within the upper layer (up and down). The simplified site structures and the underlying models are listed in table I. The distribution or variation of α over frequency for a fixed depth, Vs-contrast or travel time is illustrated by means of a histogram. There seems to be no significant trend showing a dependence of the fraction of Rayleigh and Love waves on interface depth and velocity structure. Thus, as discussed above, only the behavior of α curves in the vicinity of the fundamental frequency is a systematical effect which can be attributed to the site structure.

4. Conclusions

Employing the 3c-MSPAC method we have shown that Love waves are predominantly present in the horizontal ambient vibration components irrespective of the 1D-subsurface structure and the time of day. Within a broad frequency band the proportion of Love waves
was found to lay between 60 and 90 %. The more unstable character of the $\alpha$ curve around the fundamental frequency of the site due to Rayleigh wave ellipticity and Love wave Airy phases could be described as a second general feature. However, as e.g. the Grenoble site showed us, there were also phenomena that we were not able to assign to simple site characteristics. These effects might be controlled by 3D-structures and local sources, or exist due to the failure of the method in the case of deviations from the plane surface wave character of the wavefield.

We conclude that $\alpha$ could successfully obtained by employing the 3c-MSPAC method and can therefore help to improve interpretation of H/V spectra and dispersion curve analysis. In particular, it provides an useful tool to investigate the behavior of ambient vibration wavefields in the near of the fundamental frequency of the site.

5. References