Introduction

Flow in the surf zone is a major area of the hydrodynamics research of concern to coastal engineers. The flow has complicated features due to the wave breaking as well as other periodic motions. Non-intrusive techniques such as laser Doppler velocimetry (LDV) and particle image velocimetry (PIV) have been used for the measurement of surf zone flow fields in recent years. Nadaoka et al (1989) used fibre-optic 2D LDV and measured the turbulent velocity and Reynolds stress field under a spilling breaker. By using a flow visualization technique, they found obliquely descending eddies. The vorticity and deformation rate analysis using LDV data revealed the role of these eddies in momentum transport. Ting and Kirby (1994), (1995), (1996) carried out a series of experiments on turbulence transport due to spilling and plunging breakers. They found the turbulent kinetic energy transports landward under a spilling breaker by the undertow and seaward under a strong plunging breaker. During the experiments, the laser source could not provide enough power for the blue line. Therefore, only the green line was used and, in that situation, the longitudinal and vertical components of velocity have to be measured separately by repeating the measurement at the same point in the flow. Reynolds stress was thus not directly measured. Petti and Longo (2001) performed turbulence and water surface elevation measurements in the swash zone introduced by plunging and collapsing breakers on a 1/10 smooth slope breach. Velocity measurements were made using a 1D LDV system. Vertical spatial steps of 1mm from the bottom were taken and allowing them to measure the turbulence near the bed. By using LDV data, they derived the length and velocity macro and micro scales and Eulerian frequency spectra.

The particle image velocimetry (PIV) technique has been employed by several researchers, e.g. Skyner(1996), Perlin et al(1996), Lin & Rockwell(1994). The advantage of the PIV method is its capability of measuring the spatial distribution of instantaneous velocity field. From the velocity measurements, the ensemble average over the turbulent velocity field can be made. However the speed of a PIV camera, 15-25 frames/second, is not fast enough for turbulence measurements since the frequency of turbulence in hydraulic flows is in the range of 10-100 Hz. Therefore, in order to measure turbulence and Reynolds stress, one will have to use LDV with the disadvantage of only point measurements. This means that velocity measurement has to be repeated at different locations over the field to get the whole field measured. This may however be achieved for periodic flow where phase-averaging may be applied.

In the present study a three-component, fibre-optic LDV has been used to measure the velocity field in the surf zone. The measurements covered a length along the beach
covering pre-plunging, plunging to well-developed spilling breakers. The data are used for the evaluation of each term appearing in RANS depth-averaged shallow-water equations. The project is still underway.

Apparatus

An inclined beach of slope 1:20 has been built in an existing wave flume (11 meters long, 0.3 meters wide and 0.45m deep). The side-walls of the flume and the bottom plate of the beach were made with Perspex to allow the laser beam or laser sheet to shine through for LDV or PIV measurement.

Regular waves with period, T=2.4 s, were generated by a piston wave paddle. The still water depth was set up to 0.342 m. A maximum wave height of 0.40 m was obtained before the beach. In these conditions, the waves break at 5.8m from the wave paddle. The breaking locations are only approximately repetitive with a variation of ±0.02m been found in the data analysis.

The fibre-optic, three-component LDV system has a 4-Watt Argon-Ion laser source (a TSI product, from ESPRC equipment loan pool). Surface elevation was measured with wave probes made from stainless steel wires. By running the FIND programme (a TSI product) on a PC, the simultaneous measurements of three components of velocity and surface elevation can be made in a multi-channel manner (TSI DataLink).

The details of the wave flume, measurement locations may be found in the sketch in Fig. 1.

Experimental Results

LDV measurements were conducted at 12 horizontal locations. The measurements were along vertical lines through the depth, at 15-60 vertical locations depending on the depth variation.

The green line of laser (514nm wavelength) was used to measure longitudinal velocity and the blue line (488nm) vertical velocity. A sampling time of 15 minutes was used and with the wave period, T=2.4 seconds, 375 wave cycles were measured in a coincident data rate up to 400 Hz. Hence an average size of data set of 6.0 millions (4 channels) was obtained for evaluating the turbulent velocity at each point. Typical time series for longitudinal and vertical velocity components and surface elevation (u, w, η) are shown in Fig.2: (a) in the shallow surf zone and (b) in the wave crest region.

The phase average for N samples within a period is defined, for u for example, as

\[ \hat{u} = \frac{1}{N} \int_{\tau}^{\tau+T} u \, dt' \]

where \( \tau = T / N \) (N=128 here). Assuming the transform, \( \Delta x = -c \Delta t \), is valid with celerity c locally uniform, velocity vector maps (\( \hat{\mathbf{v}} - c, \hat{\mathbf{u}} \)) are presented in Figs.3(a-f) with surface elevation (\( \eta \)) to demonstrate the complete process from pre-plunging to spilling breaker. At location 1 (Lx=5540mm) the wave is steepening. At location 2 (Lx=5737mm) plunging has occurred and the roller is growing. At the next two locations 3 and 4, (Lx=5887, 5962mm) the breaker becomes spilling and is bore-
like at location 5 (Lx=6412mm). At location 6 (Lx=7200mm) the strength of the bore has clearly decreased. Since wave motion after breaking is rather complicated, the boundaries of this classification are not too distinct.

**RANS Depth-Averaged Shallow-Water Equations**

Consider 2DV flow at a point with periodic (or mean) and turbulent components:

\[ u = \bar{u} + u' \]
\[ w = \bar{w} + w' \]  
(1)

\[ \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial z} = 0 \]  
(2)

\[ \frac{\partial (\bar{u}^2)}{\partial t} + \frac{\partial (\bar{u} \bar{v})}{\partial x} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} - \frac{\partial (u'^2)}{\partial x} - \frac{\partial (u'w')}{\partial z} \]  
(3)

Kinematic free-surface boundary condition is given by:

\[ w_s = \frac{\partial \eta}{\partial t} + \bar{u} \frac{\partial \eta}{\partial x} \]
\[ \bar{w}_s + w' = \frac{\partial (\bar{r} + \eta')}{\partial t} + (\bar{u} + u') \frac{\partial (\bar{r} + \eta')}{\partial x} \]  
(4)

Time-averaging gives

\[ \bar{w}_s = \frac{\partial \bar{r}}{\partial t} + \bar{u} \frac{\partial \bar{r}}{\partial x} + u' \frac{\partial \eta'}{\partial x} \]  
(5)

The last term is usually ignored but has not been evaluated.

Integrating over depth \( \int_{b}^{s} ... dz \) where for example \( \bar{u}h = \int_{b}^{s} \bar{u} dz \), and applying the Leibnitz and the time-averaged kinematic free-surface boundary condition gives for continuity

\[ \frac{\partial \bar{r}}{\partial t} + \frac{\partial (\bar{r} \bar{h})}{\partial x} = 0 \]  
(6)

and force/momentum

\[ \frac{\partial \left( \int_{b}^{s} \bar{r} \bar{h} dz \right)}{\partial t} + \frac{\partial \left( \int_{b}^{s} \bar{r} \bar{h}^2 dz \right)}{\partial x} = -gh \frac{\partial \bar{r}}{\partial x} - \frac{1}{\rho} \int_{b}^{s} \frac{\partial p_D}{\partial x} dz - u'w' \bar{h} - u'w'_{b} - \int_{b}^{s} \frac{\partial u'^2}{\partial x} dz \]  
(7)

where \( p_D \) is the dynamic pressure (total-hydrostatic) and again applying the Leibitz theorem
\[
-\int_b^s \frac{\partial u'^2}{\partial x} \, dz = -\frac{\partial}{\partial x} \left( \int_b^s u'^2 \, dz \right) + u'^2 \frac{\partial \bar{h}}{\partial x} - u'^2 \frac{\partial z}{\partial x} \tag{8}
\]

Putting \( \bar{h} = \bar{h} + (u' - \bar{h}) \) and assuming \( u' \) at the bed is zero gives

\[
\frac{\partial (\bar{h} \bar{u})}{\partial t} + \frac{\partial (\bar{h} \bar{u})}{\partial x} + \frac{\partial (w^2)}{\partial z} = -gh \frac{\partial \bar{h}}{\partial x} - \int_b^s \frac{\partial p_D}{\partial x} \, dz - \bar{u}' \bar{w}_b - \frac{\partial}{\partial x} \left( \int_b^s (u' - \bar{h})^2 \, dz \right) - \frac{\partial}{\partial x} \left( \int_b^s u'^2 \, dz \right) + u'^2 \frac{\partial \bar{h}}{\partial x} \tag{9}
\]

(The Roman numerals are referred to below). The dynamic pressure term (IV), the surface and bed shear stress terms (V, VI) requires modelling; this also applies to the ‘dispersion’ term (VII). The normal turbulent stress terms, (VIII), (IX), are not usually included and will be evaluated. Note terms in \( \frac{\partial}{\partial x} \) may be transformed to \(-\frac{1}{c} \frac{\partial}{\partial t}\) in periodic conditions.

The non-hydrostatic pressure may be determined from the RANS vertical force/momentum equation:

\[
\frac{\partial \bar{h}}{\partial t} + \frac{\partial (\bar{h} \bar{u})}{\partial x} + \frac{\partial (w^2)}{\partial z} = -\frac{1}{\rho} \frac{\partial p_D}{\partial x} - \frac{\partial (u' \bar{w})}{\partial x} - \frac{\partial (w^2)}{\partial z} \tag{10}
\]

\[
- \left[ \frac{p_D}{\rho} \right]_z = \int_z^s \frac{\partial \bar{h}}{\partial t} \, dz + \int_z^s \frac{\partial (\bar{h} \bar{u})}{\partial x} \, dz + \left[ \bar{w}^2 \right]_z + \int_z^s \frac{\partial (u' \bar{w})}{\partial x} \, dz + \left[ w^2 \right]_z \tag{11}
\]

If the surface pressure is zero, applying the Leibnitz theorem and the kinematic free surface boundary condition gives

\[
\left. \frac{p_D}{\rho} \right|_z = \frac{\partial}{\partial t} \left[ \bar{h} \bar{w} \right]_z + \frac{\partial}{\partial x} \left[ \bar{h} \bar{w} \right]_z - \bar{h} \bar{w}^2 + \int_z^s \frac{\partial (u' \bar{w})}{\partial x} \, dz - \left[ w^2 \right]_z \tag{12}
\]

In hydraulic jumps at least influence of turbulence is negligible. At point \( z \)

\[
\left. \frac{p_D}{\rho} \right|_z = \frac{\partial}{\partial t} \left[ \bar{h} \bar{w} \right]_z + \frac{\partial}{\partial x} \left[ \bar{h} \bar{w} \right]_z - \bar{h} \bar{w}^2 \tag{13}
\]

Using \( \frac{\partial}{\partial x} \equiv -\frac{1}{c} \frac{\partial}{\partial t} \) gives

\[
\left. \frac{p_D}{\rho} \right|_z = \frac{\partial}{\partial t} \left[ \bar{h} \bar{w} \right]_z - \frac{1}{c} \frac{\partial}{\partial t} \left[ \bar{h} \bar{w} \right]_z - \bar{h} \bar{w}^2 \tag{14}
\]
For the depth-averaged horizontal equation (9) we require

\[
- \frac{1}{\rho} \frac{\partial}{\partial x} \int p_b dz = - \frac{1}{\rho} \frac{\partial}{\partial x} \int p_b dz + \frac{p_{Db}}{\rho} \frac{\partial \eta}{\partial x} - \frac{p_{Db}}{\rho} \frac{\partial z_h}{\partial x}
\]  

(15)

and \( p_{Db} = 0 \). Again using \( \frac{\partial}{\partial x} \equiv - \frac{1}{c} \frac{\partial}{\partial t} \) gives

\[
- \frac{1}{\rho} \frac{\partial}{\partial x} \int p_b dz = \frac{1}{\rho c} \frac{\partial}{\partial t} \int p_b dz - \frac{p_{Db}}{\rho} \frac{\partial z_h}{\partial x}
\]  

(17)

Note that \( z_h \) is not cyclic and transformation to the time derivative is not valid.

**Data Analysis**

Matlab® programmes have been written for the evaluation of the terms I to IX in Eq. 9. Time variations are shown in Fig.4(a) for location 4 (Lx=5962mm). This shows that all the turbulence terms, IV, V, VIII, IX are very small compared to the other terms. The profile of each term is quite complex although the overall balance is achieved to a close approximation. A 128 point spline interpolation was used and the profiles could possibly be improved by using other interpolation algorithm. The dynamic (non-hydrostatic pressure), \( p_d/\rho \), given by Eq.14, is shown in Fig. 4(b) to be particularly complex.

**Conclusions**

Turbulent velocity fields in the surf zone have been measured with laser Doppler velocimetry. The measurement area covers the region from plunging to spilling breaking. Using the LDV data, evaluation of the terms in the RANS shallow-water equation was undertaken and they were shown to balance to a close approximation. It is found that the turbulence stress terms are negligible in the RANS shallow-water equation but that non-hydrostatic (dynamic) pressure and dispersion terms are significant. Numerical and physical modelling work is still ongoing.

**Acknowledgement**

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**References**

Nataoka, K., Hino, M., Koyano, Y., Structure of the turbulent flow field under breaking wave in the surf zone. J. Fluid Mechanics. 204, 359-387.


Figure 1 Sketch of Measurement location
Fig. 2(a). Time series of surface elevation and velocity measured at 10mm above the bed (at $z = 192$mm) for $L_x=5540$mm.

Fig.2(b) Time series of surface elevation and velocity measured in wave crest region at 128mm above the bed (at $z=310$mm) for $L_x=5540$mm.
3(a) velocity vectors (U-C, W) at Lx=5540mm → 1.03m/s

3(b) velocity vectors (U-C, W) at Lx=5737mm → 1.03m/s

3(c) velocity vectors (U-C, W) at Lx=5887mm → 1.03m/s
Fig. 3. Velocity vector maps at (a) Lx=5540mm; (b) Lx=5737mm; (c) Lx=5887mm; (d) Lx=5962mm; (e) Lx=6412mm; Lx=7200mm. The ‘thick’ surface covers the periodic mean $\pm 3\sigma$. 
Figure 4(a) Comparison of terms in depth-averaged force/momentum equation

Figure 4b Non-hydrostatic pressure, $\frac{p_D}{\rho}$, at $L_x=5962$mm