NFR project in Operator Algebras 2005-2007. Report of results

EB and LT (with R. Conti) introduce and study several amenability properties for unitary corepresentations and *-representations of algebraic quantum groups, which may be used to characterize amenability or co-amenability of such groups. As a background for this study, they also investigate the involved tensor C*-categories.

EB (with R. Conti) study norm convergence and summability of Fourier series in the setting of reduced twisted group C*-algebras of discrete groups. For amenable groups, Følner nets give the key to Fejèr summation. They show that Abel-Poisson summation holds for a large class of groups, including e.g. all Coxeter groups and all Gromov hyperbolic groups. As a tool in the presentation, they introduce notions of polynomial and subexponential H-growth for countable groups w.r.t. proper scale functions, usually chosen as length functions. These coincide with the classical notions of growth in the case of amenable groups.

OB has joinly with Akitaka Kishimoto and Derek W Robinson proved that certain quasi-free flows on the Cuntz algebra of order infinity have the Rohlin property and therefore are cocycle-conjugate with each other. This, in particular, shows that any unital separable nuclear purely infinite simple C*-algebra has a Rohlin flow.

The latter three authors has also in a paper "Approximately inner derivations" studied the connection between approximately inner flows on C*-algebras and various notions of convergence of the associated generators. In particular they examine the relationship between the space of pointwise convergence of the generators and various cores related to spectral subspaces,

TMC In "C*-crossed Products and Shift Spaces" (with Sergei Silverstrov) it is shown how the C*-algebra of a shift space can be constructed as one of Exel's crossed products of an endomorphism and a transfer operator, and a survey of the properties of the C*-algebra of a shift space is given.

The paper "Ordered K-groups associated to substitutional dynamics" (with Søren Eilers) is a continuation of the paper "Augmenting dimension group invariants for substitution dynamics". For certain shift spaces, including Sturmian and primitive substitutional shifts, we show how to compute the ordered K_0 group (which is a flow invariant of the shift space) of the corresponding C*-algebra, and show by example that this can be used to distinguish shift spaces of substitutions which are not distinguishable by any other (to us) known flow invariant.

TD. A joint paper with V. S. Varadarajan and D. Weisbart, entitled *“Schrødinger Operators on Local Fields: Self-adjointness and Path Integral Representations for*

Propagators", was submitted for publication in December 2007. It established a Feynman-Kac formula for the propagator of a Hamiltonian type operator with inner symmetries over a configuration space of the form K^n, where K is a local, totally disconnected field. To allow for inner symmetries the Hilbert space was of the form L^2(K^n) tensored with C^m, and thus H was matrix-valued. That meant that the usual exponential under the integral sign in the Feynman-Kac formula had to be replaced by a time-ordered exponential.This was an extension of an earlier result by Varadarajan concerning scalar-valued Hamiltonians over K^n.

MBL and A. Van Daele have shown that a commutative or co-commutative multiplier Hopf *-algebra always comes from a group G with a compact open subgroup and that it is unique. For the special case with G abelian, this is equivalent to the existence of a non-zero continuous function f such that both f and its Fourier transform have compact support.

They continue their work on compact and discrete subgroups of algebraic quantum groups.

S. Kaliszewski, MBL and John Quigg have introduced a new approach to the Hecke C*-algebra of a pair (G,H) by looking at a Morita equivalent ideal in the group C*-algebra of G. In particular they gave conditions on when this ideal is the whole algebra. The case where G and H are semi-direct products are treated in detail.

NSL in collaboration with N. Brownlowe, I. F. Putnam and I. Raeburn, analyse the Bost-Connes Hecke C^*-algebra and its analogues determined by finite sets of prime numbers by means of identifying ideals organised in composition series, which result in subquotients that are C*-algebras fitting into Elliott's classification program.

NSL in collaboration with I. Raeburn give a new proof, based on direct limits of Hilbert spaces, of a theorem of Mallat which produces a wavelet from a filter function.

NSL in collaboration with I. Raeburn give a general construction of projective multi-resolution analyses in Hilbert modules over C*-algebras. With the resulting theory a specific example of modules over the algebra of continuous functions on the n-torus due to Packer and Rieffel is revisited. New examples involving Hilbert modules over the algebra of continous functions on the infinite path space of a graph are explored.

NL and SN in collaboration with M. Laca establish a phase-transition result and a structural description for the C*-algebra of the Hecke pair arising from the orientation preserving affine groups of the ring of 2x2 matrices over the integers and over the rationals. This C*-algebra arises in the context of the GL_2-system of Connes and Marcolli. The main theorem classifies a large class of KMS-states. The authors continue the investigation of this C*-algebra with the goal of unravelling the behaviour of all possible KMS-states.

MBL and NSL undertake an analysis and comparison of C*-algebras arising as completions of algebras associated to Hecke pairs of a group-subgroup and a finite-dimensional representation of the subgroup with finite range. The main tool used in this analysis is the construction of a locally compact completion of the given group and a compact open completion of the subgroup. Several classes of examples are explored. The article "Generalised Hecke algebras and C^*-completions" will appear in Internat. J. Math.

SN and LT study the Dirac operator on the quantum sphere, and obtain a local index formula for it. At the time of publishing the paper was one of a few examples of a successful combination of the techniques of q-differential geometry and of Connes' non-commutative geometry. The non-classical behaviour of the geodesic flow that was observed, still awaits a satisfactory explanation and may be a key for the analysis of higher dimensional examples.

M. Izumi, SN and LT show that for any non-trivial product type action of SUq(n) $(0 < q < 1)$ on an ITPFI factor N, the relative commutant of the fixed point algebra in N is isomorphic to the algebra of bounded measurable functions on the quantum flag manifold. This is equivalent to computing the Poisson boundary of the dual discrete quantum group.

SN and ES wrote the book Dynamical Entropy in Operator Algebras. A Series of Modern Surveys in Mathematics, Vol. 50. Springer, 2006.

M. Laca, NSL and SN develop a general framework for analyzing KMS-states on C*-algebras arising from actions of Hecke pairs. They then specialize to the system recently introduced by Connes and Marcolli and classify its KMS-states for inverse temperatures beta different from 0 and 1.In particular, they show that for each beta in (1,2] there exists a unique KMS-state, which solves a problem left open by Connes and Marcolli.

M. Izumi, SN and, R. Okayasu completely determine the ratio set of the orbit equivalence relation on the boundary of a non-amenable hyperbolic group, considered with the harmonic measure defined by a non-degenerate finite range random walk on the group. In particular, it is shown that such equivalence relations are never of type III0.

KR. In the paper on symmetries in projective multiresolution analyses KR gave an equivariant version of Packer and Rieffel's theorem on sufficient conditions for the existence of orthonormal wavelets in projective multiresolution analyses. The scaling functions that generate a projective multiresolution analysis are supposed to be invariant with respect to actions of affiliated groups. The main result in the paper gave sufficient conditions for the existence of wavelets with similar invariance.

KR with Dorin Dutkay analyze matrix-valued transfer operators. We prove that the fixed points of transfer operators form a finite dimensional C*-algebra. For matrix weights satisfying a low-pass condition we identify the minimal projections in this algebra as correlations of scaling functions, i.e., limits of cascade algorithms.

CFS. The orbit structure of Cantor minimal systems (X,T),where X is the Cantor set and T is a minimal homeomorphism,was completely described by Giordano, Putnam and Skau in a paper published in Crelle in 1995. It was shown there that a complete invariant for orbit equivalence was a simple (acyclic) dimension group with trivial infinitesimal subgroup.Furthermore, all such dimension groups occur as invariants. It was also pointed out in the same paper that a so-called AF-equivalence relation which is minimal, is orbit equivalent to a Cantor minimal system, i.e. there exists a homeomorphism mapping the AF-equivalence classes to the orbits of the Cantor minimal system.
The question that CFS and coworkers and put themselves was to find a similar description of the orbit structure of minimal Cantor systems,where the action was by Z-n for n greater than 1. (The case n=1 is the Cantor minimal case.) They, i.e. Giordano,Matui,Putnam and CFS have been able to answer the most important question, namely: Any minimal Cantor Z-n system is orbit equivalent to a Cantor minimal system (i.e. a minimal Z system). The complete invariant is again a simple (acyclic) dimension group with trivial infinitesimal subgroup.However, they do not know yet the range of the invariant,i.e. does every such group occur as an invariant for n greater than 1 ? To prove the result it turned out to be crucial to describe minimal AF-able ("affable") equivalence relations. These are equivalence relations that can be given a topology making them AF-equivalence relations,and so by their structure theorem are isomorphic to the cofinal relation on a simple Bratteli diagram. The one main technical tool to prove their result for Z-n actions is the so-called absorption theorem,which roughly speaking says that one can extend a minimal AF-equivalence relation on a thin set such that the new equivalence relation is affable. In fact,it is orbit equivalent to the original AF-equivalence relation.

The result they were able to prove is a breakthrough in the study of topological dynamical systems. This is a common reaction among some of the leading researchers working in this field of mathematics.One of their papers have also been accepted for publication by the Journal of the AMS (JAMS), which is a highly prestigious journal.

ES has the last years worked on positive linear maps on operator algebras. The first paper is shows a reduction theorem for capacity of positive maps of finite dimensional algebras, which reduces the computation of capacity to the case when the image of a nonscalar projection is never a projection. In the second paper some of these ideas are extended to study multiplicative properties of positive maps, in particular the definite set of a map, which is the Jordan subalgebra where the map is a Jordan homomorphism. Special attention is given to a smaller Jordan algebra on which the map is a Jordan automorphism. In a joint paper with W. Arveson asymptotic lifts of positive maps on von Neumann algebras are studied, and they characterize the case when the domain of the asymptotic lift can embedded as a Jordan subalgebra in terms of multplicative properties of the map. The last work is divided into two papers and treat separable states and their duality with positive maps. Several results inspired by quantum information theory characterizing separable states are given.

LT (with Gerard Murphy aqnd Johan Kustermans) They study differential calculi and related notions for quantum groups. They are particularly interested in the Hodge-Dirac operator, and develope a method for computing its spectrum. This analysis is done in painstaking detail in the case of the left covariant differential calulus of Woronowicz. The entire spectrum is computed. We also study commutators with 'smooth functions' having in mind Connes non-commutative geometry. These commutators are no longer bounded. Based on our computation of the spectrum, a formula is established, which indicates how things ought to be modified.