PROJECT DESCRIPTION

Operator algebras has in the last 40 years developed from a rather special discipline within functional analysis to become a central field in mathematics often described as “non-commutative geometry”, see for example the book of Field medalist Alain Connes with this title. In Norway there is a group of 11 mathematicians who are active in the field, Professor emeritus Erik Alfsen, Oslo, Professor Erik Bedos, Oslo, Professor Ola Bratteli, Oslo, Professor Trond Digernes, Trondheim, Professor Magnus B. Landstad, Trondheim, Postdoctoral Fellow Nadia S. Larsen, Postdoctoral Fellow Sergey Neshveyev, Oslo, Professor Christian Skau, Trondheim, Professor Erling Størmer, Oslo, Associate Professor Lars Tuset, Oslo, together with the postdoc Toke Meier Karlsen, Trondheim (from 01/07/04). As noted in the NRF evaluation ”Research in Mathematics in Norwegian Universities and Colleges” from 2002, the group is among the strong groups in operator algebras internationally, and has a wide network of international collaborators. This can also be seen from the publication lists of the individual participants. This collaboration entails both a lot of travel and also invitations of mathematicians from abroad to Norway. The group has been partly financed through projects in NFR (and NAVF) in four consecutive periods since 1994, the last expiring in 2003, and this has been extremely beneficial for the research activity.

The following researchers have visited Trondheim since 1999, largely (but not exclusively) funded through the NFR grant: Vaughan Jones 15 - 20/05/00 as Onsager lecturer, T. Giordano, June 2000 and October 2001, I. Putnam, June 2000 and October 2001, J. Quigg, December 98 and June 2003, J. Renault, November 99, A. van Daele, May 99. April 00 and May 2003, S. Vaes, May 99, A. Maes, May 99, J. Kustermanns, May 99 and April 00, G. Murphy, April 00, S.Sakai, June 2000, S.Bezyglui, January 1999, J. Hunton, April 1999, J. Lutzen,University of Copenhagen, May 2000, Søren Eilers, April 2003, Arnoud Jacobs May 2003, Lydia Delvaux 10-17.5.2003, Chris Phillips Aug. 2003, Marc Rieffel,Oct. 2003, Toke Carlsen,Dec.2003, Gwion Evans,Dec.2003, Ian Putnam,March and April 2004, Thierry Giordano, March 2004, Isaac Kornfeld,May 2004.

Similarly, the following researchers have visited Oslo since 1999: Alain Connes, 01/09/00 to receive an honorary doctor degree at UiO , D.E.Evans, 16/12/00 - 8/01/01, 15-23/02/02, 14-23/02/03, R. Conti, 27/11 - 04/12/99,

29/4 - 27/5/00, 26/05 - 19/06/01, October 01, June 02, S.L Woronowicz , 11 - 13/5/99, 29/09 - 2/10/99, 24 - 28/08/00, Fred Schulz , 23/05 - 19/06/99, 28/05 - 17/06/00, 23/05 - 11/06/01, 26/05-23/06/02, P.E.T.Jorgensen, 31/5 - 26/6/98, 28/12/98 - 8/1/99, 27/12/99 - 21/01/00, 7/07-16/07/00, 13/12/00 - 9/01/01, August 2001, A.Paolucci, 3/12/98 - 11/01/99, V.Golodets, 13 - 22/6/99, October 01, Y. Nakagami, 15-16/9/99, W.Bergmann, 20 - 27/9/99, J.Hjelmborg, 14 - 17/11/99, 1/1 - 1/6/00, 13 - 20/9/00, 15/10 - 4/11/00, S.Neshveev, 6 - 12/12/99, 15/3 - 16/4/00, M. Rørdam, 16 - 20/03/00, S.Sakai, 15-19/6/00, E.Effros, 15 - 19/6/00, L.Kadison, March 01 and October 01, W.Arveson, October 01, J. Renault, October 01, Kornel Szlachanyi, October 01, A. van Daele, November 01, A.Jacobs, June 9-17 03 and , Etienne Blanchard, November 01, Ilan Hirshberg December 01, J. Quigg, December 01, M. Laca, December 01, Nadia S. Larsen, December 01, Michael Müger December 01, Steve Kaliszewski, Dec 01, Nate Brown, August 2002, S.Albeverio, September 2002, Lyudmila Turowska, October 2002, Vasyl Ostrovsky, 24/02-01/03/03, N.Chris Phillips August 03, Masaki Izumi August 03, Alfons vanDaele September 03, Jacqui Ramagge 6.-17. oktober 03, Marc Rieffel, 13 - 14 October 03, Dafydd Gwion Evans 27 – 29 Nov. 03, Dorin Ervin Dutkay 11 - 20 Nov. 03, Palle Jorgensen 1-18 June 04, Iain Raeburn 12-20 June 04, Louis Labuchagne 13-17 June 04, Tuong Tong Tat 13 - 14 June 04, Marcelo Laca 19 -25 June 04, Mary Beth Ruskai 21 - 29 June 04, Akitaka Kishimoto approximately 22 August - 19 September.

The Oslo branch of the group is part of the SUP program ” SUPREMA , New contexts for geometry and arithmetic” started 2003 jointly with the algebraic geometers and topologists in Oslo (see http://www.math.uio.no/~rognes/suprema/).
The group earlier participated in a large network called “Non-commutative geometry” in the EU program “Training and Mobility of Researchers”, and has in that connection had several postdoctoral researchers in Norway: Alan Forrest (1/4/97 - 28/2/99), Jesper Villadsen (1/8/97-30/6/98) and Fabien Durand(1/10/97 - 30/11/97) in Trondheim, and Alexis Alevras (1/8/97 - 31/7/99) and Roberto Conti (1/10/98 - 31/9/99) in Oslo. The group now participates In the large EU Research Training Network in ”Quantum Spaces - Noncommutative Geometry” since June 2002 (see http://www.cf.ac.uk/maths/opalg/qsng.html) This network financed Svendsen´s stay in Oslo 01.09.2002 - 30.11.2003 and Toke Meier Carlsens stay in Trondheim 01.07.2004 - 30.09.2005.

The group has arranged several conferences. To keep up the close connection with our Danish colleagues we have almost yearly had a workshop supported by the Foundation for Danish-Norwegian Collaboration either in Schæffergaarden in Copenhagen or in Lysebu in Oslo. The 8´th such conference was held in Lysebu 17-19/8/01 and the 9´th in Schæffergaarden 29/11-01/12/02 . The 10´th will be arranged in Lysebu in December 2004. In 3 – 5 September 2004 the group will arrange the first Abel Symposium in Voksenåsen, see http://www.matematikkforeningen.no/abelsymposiene/symp2004.php. The Oslo part of the group also participated in 2 day workshops arranged by SUP at the end of May 2003 and 2004. In June 2000 we arranged a one day international mini-conference in Oslo.

In the fall of 2001 the group participated in a program at The Centre for Advanced Study at the Norwegian Academy of Science and Letters entitled "Non-Commutative Phenomena in Mathematics and Theoretical Physics" where approximately 20 mathematicians visited in addition to the Norwegian participants. A 3 day workshop which also included many foreign participants was a part of the programme. The longer visits included Palle Jorgensen ,Iowa , David Evans, Cardiff , William Arveson, Berkeley , Jean Renault, Orleans, Michael Mueger, Amsterdam, Steve Kalizewski and John Quigg, Arizona.More details about the programme and visitors can be found in http://www.math.uio.no/~bratteli/SHS/info.html

Professor S.L. Woronowicz, Warsaw, was a Professor II at NTNU 1996-2000. Professor A. Van Daele, Univ. Leuven, Belgium has been part of the Math. Department at NTNU during the academic year 2002-3, and he has had a position as visiting professor in the spring term 2003. This has been a source of inspiration both for the permanent members and the students in the research group. Professor Masamichi Takesaki, UCLA, will similarly visit Trondheim in the fall of 2004.

Bratteli, Skau and Størmer participated until the end of 2001 in a NATO project on noncommutative dynamical systems together with Giordano in Ottawa, Vershik in St.Petersburg, and Golodets in Kharkov.
The group in Oslo has also been part of an European mobility program for young researchers in mathematics called OMATS
which financed visits by Arnoud Jakobs in the fall of 01 and John Holland in the spring of 02. A survey of the various programs we participate in can be found at http://www.math.uio.no/~bratteli/#Activities

Bratteli was awarded the Fridtjof Nansen´s price for excellence in research by The Norwegian Academy of Science and Letters 3 May 2004.

The scientific activity of the group is presently within the following branches of the theory of operator algebras:

Erik Alfsen: Geometry of state spaces of operator algebras.

Erik Bedos: Discrete magnetic Laplacians, amenability aspects for quantum groups, projective unitary representations and twisted Fourier analysis of discrete groups.

Ola Bratteli: Wavelets and operator theory, noncommutative dynamical systems.

Toke Meier Carlsen: C*-algebras associated to shift spaces

Trond Digernes: p-adic quantum systems.

Magnus Landstad: Hecke algebras and groupoid actions on C*-algebras.

Nadia S. Larsen: Hecke algebras and C*-completions, crossed products of C*-algebras by semigroups of endomorphisms.

Sergey Neshveyev : Noncommutative ergodic theory, boundary theory of quantum discrete groups, noncommutative differential calculus, rigidity properties of group actions.

Christian Skau: Induced equivalence relations of discrete group actions on Cantor sets, K-theoretic data defined by dynamics.

Erling Størmer: Contractions on von Neumann algebras, noncommutative information theory and entropy.

Lars Tuset: Cyclic cohomology for Hopf algebras, tensor categories, Martin boundary for compact quantum groups

The details of done and planned research, as well as all publications since 1999 can be read off the descriptions of the individual participants. One can see from these descriptions that the participants work within disciplines with great potential both for further research and applications.

There have been rather few cand.scient students majoring in operator algebras. The reason is that it is rather difficult to enter the field because of large requirements of initial knowledge. The students who nevertheless have taken a degree in the field have therefore often been quite good and have continued towards a dr.scient degree. E.Bedos and T.Digernes have given some students more computational tasks, for example to use IT technology to compute the spectra of operators coming from quantum lattice systems. In this area there is a potential for educating engineers and computer experts with special knowledge of non-commutative theory.
The details on master and doctoral students associated to the group can be read off the description of the individual participants in the group

We will now describe the research of the individual participants the last 5 years and their research plans for the future.

ERIK ALFSEN

E. Alfsen has now completed his joint work with Fred Shultz (Wellesley, USA) on the geometry of state spaces of operator algebras. The first part of this material was included in their book [1].

[1] Alfsen, Erik M.; Shultz, Frederic W., State Spaces of Operator Algebras (subtitle: Basic Theory, Orientations and C*-products), xii + 350 pp, Birkhauser (2001)

[2] E. Alfsen & F.W. Shultz, Geometry of State Spaces of Operator Algebras (subtitle: Jordan algebras and axiomatic characterization) , , xiii + 467 pages, Birkhäuser (2003).

ERIK BEDOS

Some years ago, E. Bédos studied the problem of computing spectra of bounded self-adjoint operators using C*-algebraic methods and was advisor for two cand. scient. (master) theses on this theme. He was especially interested in

the special case of 3D discrete magnetic Laplacians and what may be said about their spectral properties.

Together with R. Conti (Erlangen), who was EU-post.-doc in Oslo in 1998 -1999, he also initiated a study of infinite tensor products of projective unitary representations.

More recently, together with G. J. Murphy (Cork), L. Tuset (Oslo) and R. Conti (Erlangen) he has been working on the concepts of amenability and co-amenability for quantum groups.

In the future, Erik Bédos intends to :

1) Write up his results on numerical approximations of the integrated density of states of discrete 3D magnetic Laplacians.

2) Investigate aspects of amenability and actions of quantum groups (with L. Tuset).

3) Present a unified approach to twisted Fourier analysis on discrete groups.

4) Study the relationship between non-commutative 3-tori and some 6-dimensional discrete Heisenberg- groups.

[1] Bédos, Erik: “An introduction to 3D discrete magnetic Laplacians and noncommutative 3-tori”. J. of Geom. and Phys. 30 (1999), no. 3,204--232.

[2] Bédos, Erik, Murphy, Gerard J. and Tuset, Lars : “ On co-amenability of compact quantum groups”. J. of Geom. and Phys. 40 (2001), 130-153.

[3] Bédos, Erik, Murphy, Gerard J. and Tuset, Lars : “Amenability and co-amenability of algebraic quantum groups”. Int. J. of Math. and Math. Sciences 31 (2002), 577-601.

[4] Bédos, Erik, Murphy, Gerard J. and Tuset, Lars : “Amenability and co-amenability of algebraic quantum groups, II “. Journ. of Funct. Anal. 201 (2003), 303--340.

[5] Bédos, Erik and Tuset, Lars : “Amenability and co-amenability for locally compact quantum groups”. Int. J. of Math. 14 (2003), 865--884.

[6] Bédos, Erik and Conti, Roberto : “On infinite tensor products of projective unitary representations”. Rocky Mount. J. of Math. 34 (2004), 467--494.

[7] Bédos, Erik, Conti, Roberto and Tuset, Lars : “On amenability and co-amenability of algebraic quantum groups and their co-representations”. Revised May 2003. To appear in the Can. Journ. Math. (2005).

Cand. Scient. / Master students 1999-2004.

1. Roy A. Syversrud : Numerical approximations of spectra of 3-dimensional discrete magnetic Laplace operators (fall 2000).
2. Wasif Arif: On the summation of Fourier series in reduced group C*-algebras (started spring 2003- expected to be finished during fall 2004).

OLA BRATTELI

Ola Bratteli has worked in the interface of wavelet analysis and operator theory in collaboration with P.E.T.Jorgensen , see [3], [10], [11], [16], and these investigations have resulted in a research monograph/textbook on wavelet analysis and operator theory, see [1]. This work leads to an analysis of certain representations of certain operator relations called the Cuntz relations, and as a spinoff he has written several papers on representations and states of these algebras, see [6], [12], [14]. The motivation for this work is explained below. Another spinoff is a collaboration with Jørgensen, K.H.Kim and F.Roush on a decision procedure for isomorphism of two AF algebras defined by single incidence matrices [4], [5], [7]. He has written papers with A. Kishimoto on C*-dynamical systems in [2], [8]

[1] Bratteli, O. and Jorgensen, P.E.D., Wavelets Through A Looking Glass. The World of the Spectrum, xxiv + 398 pp, 147 illustrations, Birkauser, Boston (2002)

[2] Bratteli, Ola; Kishimoto, Akitaka, AF flows and continuous symmetries, Reviews of Mathematical Physics 13 (2001), 1505-1528

[3] Bratteli, Ola; Jørgensen, Palle E. T., Wavelet filters and infinite dimensional unitary groups, Proceedings of the International Conference on Wavelet Analysis and Applications, Donggao Deng, Daren Huang, Rong-Qing Jia, Wei Lin, Jianzhong Wang (eds.), International Press, Boston (2001), pp. 35--64.

[4] Bratteli, Ola; Jørgensen, Palle E. T.; Kim, Ki Hang; Roush, Fred, Decidability of the isomorphism problem for stationary AF-algebras and the associated ordered simple dimension groups, Ergod.Theory and Dyn.Sys. 21 (2001), 1625-1655 with Corrigendum, Ergod.Theory and Dyn.Sys. 22 (2002), 633.

[5] Bratteli, Ola; Jørgensen, Palle E. T.; Kim, Ki Hang; Roush, Fred, Computation of isomorphism invariants for stationary dimension groups, Ergod.Theory and Dyn.Sys. 22 (2002), 99 - 127.

[6] Bratteli, O., P.E.T.Jorgensen and V.Ostrovsky, Representation Theory and Numerical AF invariants. The representations and centralizers of certain states on Od, Mem. Amer. Math. Soc. 168 (2004), no. 797, 178 pp.

[7] Bratteli, Ola; Jørgensen, Palle E. T.; Kim, Ki Hang; Roush, Fred, Non-stationarity of isomorphism between AF algebras defined by stationary Bratteli diagrams. Ergodic Theory Dynam. Systems 20 (2000), no. 6, 1639--1656.

[8] Bratteli, Ola; Kishimoto, Akitaka, Trace scaling automorphisms of certain stable AF algebras. II. Q. J. Math. 51 (2000), no. 2, 131--154.

[9] Bratteli, Ola; Kishimoto, Akitaka, Homogeneity of the pure state space of the Cuntz algebra. J. Funct. Anal. 171 (2000), no. 2, 331--345.

[10] Bratteli, Ola; Evans, David E.; Jorgensen, Palle E. T., Compactly supported wavelets and representations of the Cuntz relations. Appl. Comput. Harmon. Anal. 8 (2000), no. 2, 166--196.

[11] Bratteli, Ola; Jorgensen, Palle E. T., Convergence of the cascade algorithm at irregular scaling functions. The functional and harmonic analysis of wavelets and frames (San Antonio, TX, 1999), 93--130, Contemp. Math., 247, Amer. Math. Soc., Providence, RI, 1999.

[12] Bratteli, O.; Jorgensen, P. E. T.; Kishimoto, A.; Werner, R. F., Pure states on O_d. J. Operator Theory 43 (2000), no. 1, 97--143.

[13] Bratteli, Ola; Jørgensen, Palle E. T.; Robinson, Derek W., Spectral asymptotics of periodic elliptic operators. Math. Z. 232 (1999), no. 4, 621--650.

[14] Bratteli, Ola; Jorgensen, Palle E. T., Iterated function systems and permutation representations of the Cuntz algebra. Mem. Amer. Math. Soc. 139 (1999), no. 663, x+89 pp.

Ola Bratteli intends to continue the collaboration with Palle Jorgensen on wavelets, and spectra of transfer operators. Wavelet theory stands on the interface between signal processing and harmonic analysis. It concerns the mathematical tools involved in digitizing continuous data with view to storage, and the synthesis process, recreating the desired picture (or time signal) from the stored data. The algorithms involved go under the name of filter banks, and their efficiency derives in part from the use of (hidden) self-similarity in the data which is analyzed. Observations or time signals are functions, and classes of functions make up spaces. Numerical correlations add structure to the spaces at hand, Hilbert spaces. There are operators in the spaces deriving from the discrete data and others from the spaces of continuous signals. The first ones are good for computations, while the second reflect the real world. The operators between the two are the focus of this research. Relations between operations in the discrete and continuous domains are considered. These relations are studied as symbols because symbols are programmable. The mathematics involved in assigning operators to the symbolic relations is called representation theory. The combination of these areas opens up exciting new opportunities at the interface of mathematics and engineering. A main point is the study of intertwining operators between, on one side, the “discrete world” of high-pass/low-pass filters of signal processing, and on the other side, the “continuous world” of wavelets. There are significant operator-algebraic and representation-theoretic issues on both sides of the divide, and the intertwining operators throw light on central issues for wavelets in higher dimensions. Tools from diverse areas of analysis, as well as from dynamical systems and operator algebras, and even, more speculatively, from the algebraic theory of adeles and solenoids, merge into the research on wavelet analysis.

More specifically, Bratteli is now concerned with the question of whether the representation of the Cuntz algebra associated to a multiresolution wavelet theory decomposes into a finite number of irreducible representations. This is known if the wavelets in question haqve compact supports l10] and there are indications that the result may carry over to more general situations. Bratteli has also given as thesis problems to his two Ph.D. students to analyze wavelets over more general spaces (Nilpotent Lie groups, adeles or fractals) admitting scaling and translation, and also to analyze how the crossed product by an endomorphism depends on an associated transfer operator, by using the analysis of transfer operators apposite to wavelet theory.

Students: Rune Kleveland had a NFR grant for a dr.scient study in 1997 - 2000, but has not delivered a thesis. Kjetil Røysland has started on his dr.scient studies from the summer of 2002. Sindre Duedahl took his cand.scient degree in May 2003, and has enrolled as a dr.scient student from 1 June 2004 with support from SUP.

TOKE MEIER CARLSEN

Toke Meier Carlsen is joining the group as an EU QSNG - postdoc in Trondheim 01.07.2004 - 30.09.2005.

Carlsen has worked on the interplay between C*-algebras and symbolic dynamical systems. The research has mainly been focused on C*-algebras associated with shift spaces ([3] and [7]). In a series of four papers with Søren Eilers ([4], [5], [8] and [9]) a thoroughly investigation of the C*-algebras associated with a special class of shift spaces which include substitution shift spaces has been conducted. In another direction, the C*-algebras of sofic shifts has been proved to be Cuntz-Krieger algebras ([2]).

Toke Meier Carlsen intends to continue the work on C*-algebras associated with shift spaces and hope to extend some of the results to bigger classes of symbolic dynamical systems such as higher dimensional symbolic dynamical systems and symbolic dynamical systems with an infinite alphabet. This work has been initiated with [6].

[1] Boel, Søren, Carlsen, Toke Meier and Hansen, Nielsen Richard: A Useful Strengthening of the Stone-Weierstrass Theorem, Amer. Math. Monthly 108 (2001), no.7, 642-643.

[2] Carlsen, Toke Meier: On C*-algebras Associated with Sofic Shifts, J. Operator Theory 49 (2003), no. 1, 203-212.

[3] Carlsen, Toke Meier and Matsumoto, Kengo: Some Remarks on the C*-algebras Associated with Subshifts, to appear in Math. Scand.

[4] Carlsen, Toke Meier and Eilers, Søren: Augmenting dimension group invariants for substitution dynamics, to appear in Ergodic Theory Dynam. Systems.

[5] Carlsen, Toke Meier and Eilers, Søren: A graph approach to computing nondeterminacy in substitutional dynamical systems, submitted for publication.

[6] Carlsen, Toke Meier: Symbolic dynamics, partial dynamical systems, Boolean algebras and C*-algebras generated by partial isometries, in preparation.

[7] Carlsen, Toke Meier: Cuntz-Pimsner C^*-algebras associated with subshifts, submitted for publication.

[8] Carlsen, Toke Meier and Eilers, Søren: Matsumoto K-groups associated to certain shift spaces, submitted for publication.

[9] Carlsen, Toke Meier and Eilers, Søren: Ordered K-groups associated to substitutional dynamics, preprint.
□

TROND DIGERNES

Trond Digernes has been working with V. S. Varadarajan in two distinct, but in some ways related, fields since the early 1990's: 1) Finite approximations of quantum systems. 2) Arithmetical physics.
Although it is fair to say that finite dimensional models have not played a major role in physics yet, they have been studied by many mathematicians and physicists, both for their own sake and as approximations to infinite models;

among the more prominent names here we mention H. Weyl, J. Schwinger, and Y. Nambu. From the point of view of approximations this opens new perspectives: finite models can be made to converge not only to conventional models built on R and C, but also to models built on any local field or ring, such as the p-adic numbers Q_p and the ring of adeles. This brings us to the realm of arithmetical physics which has been studied by, among others, I. Volovich, Yu. Manin and E. Witten. It can be argued that physics at and beyond the Planck scale requires new models for space-time, and that certain versions of arithmetical physics may be a possible framework for this.

In the spirit of finite approximations Digernes, Varadarajan and E. Husstad have proved that any Weyl system based on a (separable) locally compact group, can be obtained as a limit (in a natural sense) of Weyl systems based on finite groups (ref. [1] below). This includes, of course, quantum systems based on local fields (like R, C and Q_p, and also on local rings (like the ring of adeles).

It is also of interest to find out whether a Weyl system based on, say, a non-Archimedean field, exhibits phenomena that do not appear in the conventional setting. In a recent paper Digernes and Varadarajan address this question (ref. [A] below). They show that in certain models for the irreducible representation of a Heisenberg group, the vacuum sector exhibits a fermionic structure. This phenomenon occurs, for example, in a model based on the 2-adic numbers Q_2.

[A] Digernes, T.; Varadarajan, V.S., “MODELS FOR THE IRREDUCIBLE REPRESENTATION OF A HEISENBERG GROUP”. Submitted for publication December 2003, to the journal “Infinite Dimensional Analysis, Quantum Probability and Related Topics” (World Scientific).

[1] Digernes, T., “Finite approximations and physics over unconventional fields”, in the series “Advanced studies in pure mathematics”, vol. 38, p. 267-269, Mathematical Society of Japan.

[2] Digernes, T.; Husstad, E.; Varadarajan, V. S.: “Finite approximation of Weyl systems.” Math. Scand. 84 (1999), no. 2, 261--283.

Trond Digernes plans to continue his collaboration with Varadarajan.
Among the topics singled out for attention, are the following:

1) The problem of giving a rigorous formalism for the Feynman integral of the propagator for p-adic quantum systems. This is important for the analysis of the dynamical structure of a p-adic quantum system.
2) Adelic oscillator and other adelic quantum systems. Since there does not seem to be a preferred prime p in nature, one eventually has to work with all the primes at the same time. This leads to the adelic theories. Digernes and Varadarajan plan to undertake a study of adelic quantum systems, using A. Weil's work on unitary groups as mathematical foundation.

Students :

Martin Wold Lund (will submit master thesis July 2004).
Jorunn Garnes (siv.ing. 1999).

MAGNUS B. LANDSTAD

Landstad has together with John Quigg and Steve Kaliszewski of Arizona State Univ. (in a paper completed in October 2003) shown that the study of certain Hecke algebras first studied by Bost and Connes can be simplified from a direct use of Morita-Rieffel equivalence. This has given more direct proofs and better insight without using tools like groupoids and semi group actions. They have just started a program of further applications and results about Hecke pairs from semi-direct product groups turns out to have applications into areas like Galois theory.

Landstad and Nadia S. Larsen started research in another direction in November 2003. They look at more general Hecke algebras and it seems that the same techniques can be used in this case, however there are interesting differences.

A.van Daele and M. B. Landstad have started another project based on their discovery that the existence of a compact, open subgroup in a group G is equivalent to the existence of a non-trivial pair of a convolution operator and a multiplication operator which commute. Other properties of the group G are also equivalent to this property. They want to see how this can be generalized to

quantum groups; the first goal is to find the proper analogue of a compact, open quantum subgroup in this setting.

[1] Kaliszewski,S.; Landstad, Magnus B. and Quigg, John.
“Hecke algebras and groups” 36 pages (preprint math.OA/0311222).

[2] Landstad, Magnus B. and Van Daele, A.
„Multiplier Hopf *-algebras and groups with compact open subgroups‰ (appr 20 pages, in preparation)

[3] Landstad, Magnus B. and Van Daele, A.
„Compact open quantum subgroups in multiplier Hopf *-algebras (appr 55 pages, in preparation)

[4] Landstad, Magnus B.,Traces on noncommutative homogeneous spaces.
J. Funct. Anal. 191 (2002), no. 2, 211--223.

Students:

Landstad have 1 cand.scient. student Margrethe Stene who plans to finish in the fall of 2004.
□

NADIA S. LARSEN

Nadia S. Larsen joined the group in Oslo as a university postdoc from July 2003. Her research interests are centered around the following topics: the analysis of *-representations and of C*-completions of Hecke algebras (joint with Laca, Raeburn, Ramagge), the study of crossed products of C*-algebras by semigroups of endomorphisms in the sense of Stacey, Laca-Raeburn, Exel (joint with Raeburn), description of the structure of certain Hecke C*-algebras arising in number theory by using methods of crossed products by semigroups and topological dynamical systems (joint with Brownlowe, Putnam and Raeburn).

[1] N.S.Larsen, Non-unital semigroup crossed products, Math. Proc. Royal Irish Acad. (2) 100A (2000), 205-218.

[2] N. S. Larsen and I. Raeburn, Faithful representations of crossed products by actions of N^k, Math. Scand. 89 (2001), 283-296.

[3] N. S. Larsen, I. F. Putnam and I. Raeburn, The two-prime analogue of the Hecke C*-algebra of Bost and Connes, Indiana Univ. Math. J. 51 (2002), 171-186.

[4] N. S. Larsen and I. Raeburn, Representations of Hecke algebras and dilations of semigroup crossed products, J. London Math. Soc. 66(2002), 198-212.

[5] N. S. Larsen, Crossed products by semigroups of endomorphisms and groups of partial automorphisms, Can. Math. Bull. 46 (2003), 98-112.

[6] M. Laca and N. S. Larsen, Hecke algebras of semidirect products, Proc. Amer. Math. Soc. 131 (2003), 2189-2199.

[7] N. Brownlowe, N. S. Larsen, I. F. Putnam and I. Raeburn, Subquotients of Hecke C*-algebras, preprint, august 2003.

Her current research plans include:

1) Work to develop and extend to other semigroups than the positive integers the recent construction of crossed products by single endomorphisms and transfer operators due to Exel. In Stacey's theory of crossed products by single endomorphisms, the extension to other semigroups brought in a large number of new and interesting examples and applications. There are already indications that extending Exel's theory will also give rise to new applications. One possible direction is suggested by the operator algebraic approach of Bratteli and Jørgensen to wavelet theory. (joint with Raeburn).
2) Work with M. Landstad on the study of *-representations, C*-completions and structure of generalised Hecke algebras.
3) Work with Laca, Raeburn and Ramagge on Hecke algebras and their C*-completions.

□

SERGEY NESHVEYEV

Sergey Neshveyev has during the last year worked on the computation of boundaries of discrete quantum groups and on the local index formula for q-spaces. These topics were stated as his main research plans in the last year application. Though the theories of the geometry of q-deformed spaces and noncommutative geometry of Connes have developed considerably over the last years, the connections between these two areas remain obscure. It is enough to say that the Dirac operators, the starting point of Connes' geometry, have so far been constructed only on quantum flag manifolds (other known Dirac-type operators, e.g. on SUq(2), do not look like the right ones for the moment). In thie papers [14-15] Neshveyev and Tuset study the Dirac operator on the quantum sphere and obtain a local index formula for it. Their considerations give one of few examples of a successful combination of the techniques of q-differential geometry and of Connes' noncommutative geometry.

His joint work with Lars Tuset on the index formula [14-15] is described in the part devoted to Tuset below. Concerning boundaries of discrete quantum groups, in a joint work with Masaki Izumi and Lars Tuset [16] it has been shown that the Poisson boundary of the dual of SU_q(n) is isomorphic to the quantum flag manifold. The result can be considered as a dual counterpart of classical results of Furstenberg on boundaries of semisimple groups. The proof relies on a discovered connection between the Poisson integral and the Berezin transform. The main technical result says that a sequence of Berezin transforms defined by a random walk on the dominant weights of SU(n) converges to the identity on the quantum flag manifold. This is a random q-analogue of some known results on quantization of coadjoint orbits of Lie groups.

[1] Neshveyev S., On the K-property of quantized Arnold cat maps, J. Math. Phys. 41 (2000), 1961-1965.

[2] Golodets V.Ya., Neshveyev S., Entropy for Bogoliubov actions of torsion-free abelian groups on the CAR-algebra, Ergodic Theory Dynam. Systems 20 (2000), 1111-1125.

[3] Neshveyev S., Størmer E., The variational principle for a class of asymptotically abelian C*-algebras, Comm. Math. Phys. 215 (2000), 177-196.

[4] Neshveyev S., Entropy of Bogoliubov automorphisms of CAR and CCR algebras with respect to quasi-free states, Rev. Math. Phys. 13 (2001), 29-50.

[5] Golodets V.Ya., Neshveyev S., Entropy of automorphisms of II_1-factors arising from the dynamical systems theory, J. Func. Anal. 181 (2001), 14-28.

[6] Neshveyev S., Størmer E., Entropy in type I algebras, Pacific J. Math. 201 (2001), 421-428.

[7] Neshveyev S., Ergodicity of the action of the positive rationals on the group of finite adeles and the Bost-Connes phase transition theorem, Proc. Amer. Math. Soc. 130 (2002), 2999-3003.

[8] Neshveyev S., Størmer E., The McMillan theorem for a class of asymptotically abelian C*-algebras, Ergodic Theory Dynam. Systems. 22 (2002), 889-897.

[9] Neshveyev S., Størmer E., Ergodic theory and maximal abelian subalgebras of the hyperfinite II_1-factor, J. Func. Anal. 195 (2002), 239-261.

[10]Neshveyev S., Tuset L., Quantum random walks and their boundaries, in: Proceedings of the Symposium „Analysis of (Quantum) Group Actions on Operator Algebras (Kyoto, 2003), Surikaisekikenkyusho Kokyuroku 1332 (2003), 57-70.

[11] Neshveyev S., Størmer E., Maximal abelian subalgebras of the hyperfinite factor, entropy and ergodic theory, in: Proceedings of International Workshop on Operator Algebras and Operator Theory (Shanxi, 2001), Acta Math. Sin. (Engl. Ser.) 19 (2003), 599-604.

[12] Neshveyev S., Tuset L., The Martin boundary of a discrete quantum group, J. reine angew. Math. 568 (2004), 23-70.

[13] Laca M., Neshveyev S., KMS-states of quasi-free dynamics on Pimsner algebras, J. Funct. Anal. 211 (2004), 457-482.

[14] Neshveyev S., Tuset L., Hopf algebra equivariant cyclic cohomology, K-theory and index formulas, preprint 2003.

[15] Neshveyev S., Tuset L., A local index formula for the quantum sphere, preprint 2003.

[16] Izumi M., Neshveyev S., Tuset L., Poisson boundary of the dual of SUq(n), preprint 2004.

Sergey Neshveyev has the following research plans.

1. To continue the work with Lars Tuset and Masaki Izumi on boundaries of the duals of q-deformations of semisimple Lie groups. Though the Poisson boundary is computed for SU_q(n), the proof does not apply immediately to other groups and requires case-by-case study. One of the goals is to find a unified proof. Another problem is to compute the Martin boundary, which is conjecturally a q-deformation of the sphere in the dual of the Lie algebra.

2. Neshveyev and Tuset hope that the new phenomena they observed in [14-15], the most significant among which is a non-classical behavior of the geodesic flow, is a key for the analysis of higher dimensional examples, and can also help to explain numerous failed attempts to construct the Dirac operators on quantum groups.

3. To write a book with Erling Størmer on dynamical entropy in operator algebras. The plan is to summarize essentially everything what is known about Connes-Størmer-Narnhofer-Thirring's approach to the Kolmogorov-Sinai entropy and Voiculescu's approach to topological entropy.

CHRISTIAN SKAU

Christian Skau has mainly worked on the interplay between C*-algebras and (topological) dynamical systems. Given a dynamical system (X,T) one forms the C*-crossed product C*(X,T). One of the goals is to establish a connection between the orbit structure of (X,T) and the isomorphism type of C*(X,T) in analogy to what Connes and Krieger did in the von Neumann/measurable setting. For (X,T) a Cantor minimal system, Skau and his coworkers Giordano and Putnam did obtain a very satisfactory result. It turned out the ordered (sic!) K-theory was the link between orbit structure of the dynamical systems on one hand, and isomorphism type of the C*-algebras on the other hand. This breakthrough was the impetus to a widespread study of the (ordered) K-theoretic invariant in a purely dynamical setting. Skau, with his coworkers Durand and Host, proved that for substitution minimal symbolic systems the K-theoretic invariant could be computed by an effective algorithm, giving rise to a completely new invariant for studying such systems—independent of the classically studied invariants of spectral and entropy type. Similar results are true for Toeplitz symbolic systems. A crucial tool in all this is a special infinite graph called a Bratteli diagram,which first appeared in the study of special C*-algebras, but which can be interpreted dynamically to give a powerful method to study symbolic dynamical systems. Skau spent 3 weeks in May 2002 in Mathematisches Forschungsinstitut Oberwolfach, Germany,and 2 weeks at the Banff International Research Station for Mathematical Innovation and Discovery(BIRS),Banff,Canada, working with Giordano and Putnam. The results obtained so far has resulted in one published paper ([2] below), and in one preprint([1] below)to be published. Several other papers are in preparation.

[1] T.Giordano,I.Putnam,C.Skau:"Cantor minimal Z-2 systems", preprint(2004).

[2] T.Giordano,I.Putnam,C.Skau:"Affable equivalence relations and orbit structure of Cantor minimal systems",26 pages., Erg. Th. and Dyn. Syst. 24(2004), 441-475

[3] T.Giordano,I.Putnam,C.Skau: "K-theory and asymptotic index for certain almost one-to-one factors", Math.Scand.89 (2001),297-319.

[4] Skau, Christian, Ordered K-theory and minimal symbolic dynamical systems. Dedicated to the memory of Anzelm Iwanik. Colloq. Math. 84/85 (2000), , part 1, 203--227.

[5] Skau, Christian, Three pearls from elementary mathematics. (Norwegian) Normat 48 (2000), no. 2, 56--74, 95. 00A05

[6] Durand, F.; Host, B.; Skau, C., Substitutional dynamical systems, Bratteli diagrams and dimension groups. Ergodic Theory Dynam. Systems 19 (1999), no. 4, 953--993.

[7] Giordano, Thierry; Putnam, Ian F.; Skau, Christian F., Full groups of Cantor minimal systems. Israel J. Math. 111 (1999), 285--320.

Christian Skau will do research in the following fields, in collaboration with Ian Putnam, University of Victoria, Canada, Thierry Giordano, University of Ottawa, Canada, Jean Renault, University of Orleans, France, Christopher Phillips, University of Oregon, USA:

(1) Let G be a countable amenable group acting freely and minimally on a Cantor set.The problem is to show that the induced equivalence relation is AF. This is again equivalent to showing that the action is orbit equivalent to a Z-action. (In the measure-theoretic setting the analogous result has been shown by Connes, Feldman, Weiss). Recently Skau and his coworkers made a

breakthrough, proving that for a large class(potentially for all!) of minimal and free Z-2 actions of homeomorphisms on a Cantor set,the associated etale equivalence relations are affable, hence the dynamical systems are orbit equivalent to a Z-action. This is a research project that Skau wants to pursue with his coworkers Giordano and Putnam the next few years, and it will require a lot of hard work. However, the tools that the three have constructed are very powerful and bode well for the handling more general amenable groups.

(2) Recently great progress has been done in the study of C*-crossed products associated to minimal homeomorphisms of compact differential manifolds by C.Phillips and Q.Lin. It turns out that K-theoretic data are complete isomorphism invariants for these (simple) C*-algebras, in accordance with the Elliott Conjecture. It remains to be explored what the K-theoretic data mean in terms of the dynamics. Since orbit equivalence in this case coincides with (flip) conjugacy, the situation here is different than in the Cantor minimal case. There are examples of non-conjugate systems yielding the same K-theoretic data. However, a reasonable question to ask is whether the K-theoretic data yield, up to orbit equivalence, the “smallest” (in a specific sense) Cantor minimal extension of the given dynamical system. This is a research field that will be pursued by Skau and Phillips the next three years, and there has been several meetings between the two over the last year.

Students:

Skau presently has two Ph.D. students: Mats Molberg (since Sept.2001) and Heidi Dahl(since Aug. 2003). Molberg has two preprints that will be sent to journals for publication:

G.Hjort,M.Molberg:”Free continuous group actions”, preprint(2004).

M.Molberg:”AF-equivalence relations”,preprint(2004).

K.de Silva and Yaw Bimpeh (June 2000), M.Molberg (Aug.2001), H.Dahl (Aug.2003) received their cand.scient. degrees under supervision of Skau.

ANNE LOUISE SVENDSEN

Anne Louise Svendsen used her tenure as a post.doc in Oslo 01/09/2002 – 30/11/2003 to write part of her Ph.D, thesis into the paper [1] and to work on examples of entropy of endomorphisms arising from subfactors. The results are in [2].

[1]"Automorphims of subfactors from commuting squares ", submitted to Trans.Amer. Math. Soc

[2]" Endomorphisms and automorphisms from subfactors illustrating non-commutative entropy", submitted to Math.Scand

ERLING STØRMER

Erling Størmer has worked almost exclusively on noncommutative entropy in the last 5 years, i.e. the theory of entropy of automorphisms of C*- and von Neumann algebras. Much of the work is described in his survey article [11]. There are essentially two different set of definitions - the first by Connes and Størmer and its extensions by Connes, Narnhofer and Thirring, which are based on relative entropy of states, and the approximation entropies of Voiculescu which are refinements of mean entropy and are closer to the classical abelian ones. The interplay between the definitions is very important in the theory and much research has been related to this aspect. For example when the entropies coincide, one finds that the entropy of a tensor product of two automorphisms with respect to a product state is the sum of the entropies. This happens in the nicer cases, e.g. in the case of asymptotical abelianness with locality, studied by Neshveyev and Størmer the last year and a half. For this case they could solve a problem stated in Connes' book "Noncommutative geometry" on the variational principle, see [9]. In particular it follows that in the setting of the paper Voiculescu's topological entropy is the sup over all dynamical entropies in the sense of Connes, Narnhofer and Thirring, just as in the classical case. As a follow up, ref.[10], Neshveyev and Størmer could also show a version of the classical Shannon-McMillan-Breiman theorem in ergodic theory. The classical result played an important role in Ornstein's classification of Bernoulli shifts, and it is hoped that the noncommutative result could play a similar role in the classification of noncommutative Bernoulli shifts. In a paper

written in the fall of 2001, ref. [10], Neshveyev and Størmer studied masas, i.e. maximal abelian subalgebras, in the crossed product defined by an ergodic action on a probability space. When the action is weakly mixing, the masa generated by the unitary implementing the action, is singular, and they proved partial results towards the conjecture that the masa determines the action up to isomorphism. They also proved partial results using noncommutative entropy towards the conjecture that when the action is just ergodic then the entropy of the action is completely determined by the masa.

In 2001 Størmer started a collaboration with T. Oikhberg and H. Rosenthal on imbeddings of preduals of von Neumann algebras. It was shown, see [12], that if M is a semi-finte (resp. injective) von Neumann algebra, and N is a von Neumann algebra whose predual is isometric to a subspace of that of M, then N is semi-finite (resp. injective).

The last year and a half Størmer has also worked on problems in quantum computing and in the theory of completely bounded linear maps of operator algebras. Størmer will continue this investigation. At the same time he and S. Neshveyev will write a book on noncommutative entropy.

[4] Golodets, V. Ya.; Størmer, Erling, Generators and comparison of entropies of automorphisms of finite von Neumann algebras. J. Funct. Anal. 164 (1999), no. 1, 110--133.

[5] Størmer, Erling, Entropy of endomorphisms and relative entropy in finite von Neumann algebras. J. Funct. Anal. 171 (2000), no. 1, 34--52.

[5] Neshveyev, Sergey; Størmer, Erling, Entropy in type I algebras, Pacific J. Math. 201 (2001), 421-428.

[7] Neshveyev, Sergey; Størmer, Erling, The variational principle for a class of asymptotically abelian C*-algebras. Comm. Math. Phys. 215 (2000), no.1, 177--196.

[8] Neshveyev, Sergey; Størmer, Erling, The McMillan theorem for a class of asymptotically abelian C*-algebras, Ergodic Theory Dynam.Systems. 22 (2002), 889-897.

[9] Størmer, Erling, A survey of noncommutative dynamical entropy, Encyclopedia Math. 126, Springer-Verlag 2002.

[10]Neshveyev, Sergey; Størmer,Erling. Ergodic theory and maximal abelian subalgebras of the hyperfinite factor, J.Funct.Analysis, 195 (2002), 239-261.

[11]Neshveyev, Sergey; Størmer,Erling. Maximal abelian subalgebras of the hyperfinite factor, entropy and ergodic theory, Acta Math. Sinica, English Series, 19 (2003), 599-604.

[12]Oikhberg,Timor; Rosenthal, Haskell.P.; Størmer,Erling, A predual characterization of semi-finite von Neumann algebras, Advances in Quantum Dynamics, Contemporary Math. 335 (2003), 244-246.

[13] Størmer, Erling, Posititve linear maps of von Neuman algebras and capacity, Preprint May 2004.

Students: Størmer has had 3 cand.scient students who have finished, Sigmund Vik (his thesis is published in Math.Scand. 88 (2001), 257-278), Sverre Briseid (in 2001) and Kjetil Røysland (in 2002). Presently Størmer has one cand.scient student, Bjarte Berntsen. Simen Gaure is still working on his thesis.

□

LARS TUSET

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L. Tuset has since 1999 been an associate professor at The Faculty of Engineering, Oslo University College. For details on recent research within non-commutative geometry, quantum random walks, quantum groups and amenability, see the descriptions by coauthors S. Neshveyev and E. Bedos. Research within tensor categories and cyclic cohomology include: Definition of twisted cyclic cohomology and its relation to differential calculi for quantum groups, construction of the modular square with recipes for computation of the modular characteristic homomorphism, relation between embedding functors for non-symmetric tensor categories and absorbing monoids.

[1] Roberts John E.; Tuset, Lars, On the equality of q-dimension and intrinsic dimension. J. Pure Appl. Algebra 156 (2001), no. 2-3, 329--343.

[2] G.J. Murphy and L. Tuset, Aspects of compact quantum groups, to appear in Proc.AMS, 2003.

[3] D. Guido and L. Tuset, Representations of the direct product

of matrix algebras, Fundamenta Matematicae, Vol 169, 2001, no 1-3, 145--160.

[4] Kustermans Johan; Tuset, Lars, A survey of C*-algebraic quantum groups. II. Irish Math. Soc. Bull. No. 44 (2000), 6--54.

[5] Kustermans, Johan; Tuset, Lars, A survey of C*-algebraic quantum groups. I. Irish Math. Soc. Bull. No. 43 (1999), 8--63.

[6] J.Kustermans, G.J. Murphy and L.Tuset, Differential calculi over quantum groups and twisted cyclic cocycles, J. Geom. Phys 44, 2003, 570-594.

[7] J. Kustermans, J. Rognes and L. Tuset, The Connes-Moscovici approach to cyclic cohomology for compact quantum groups, K-Theory. Vol 26, 2002, no. 2, 101-137.

[8] J. Kustermans, G.J. Murphy and L. Tuset, Quantum groups, differential calculi and the eigenvalues of the Laplacian, accepted by Trans. Amer. Math. Soc.

[9] J. Kustermans, J. Rognes and L. Tuset, The modular square for quantum groups, accepted for publication in Banach Centre Proc.

[10] Bedos, E., Murphy G.J. and Tuset L.: Co-amenability for compact quantum groups. J. Geom. Phys. 40, 2001, 130-153.

[11] Bedos, E., Murphy G.J. and Tuset L.: Amenability and co-amenability for algebraic quantum groups, Int. J. Math. Math. Science. Vol 31, 2002, no.10, 577-601.

[12] Bedos, E., Murphy G.J. and Tuset L.: Amenability and co-amenability for algebraic quantum groups II, J. Funct.Anal. 201 (2003), 303 - 340.

[13] Bedos, E. and Tuset L.: Amenability for locally compact quantum groups II, Int. J. of Math. 14, no.8 (2003) 865-884.

[14] Bedos, E., Conti R. and Tuset L.: Amenability and co-amenability for algebraic quantum groups and their corepresentations, revised May 2003, to appear in Canadian J. of Math.

[15] M. Mueger, J.E. Roberts and L. Tuset, Representations of algebraic quantum groups and reconstruction theorems for tensor categories, accepted for publication in Alg. and Reps., 2002.

[16] S. Neshveyev and L. Tuset: The Martin Boundary for Discrete Quantum Groups, J. reine angew. Math. 568 (2004), 23-70.

[17] Neshveyev S., Tuset L., Hopf algebra equivariant cyclic cohomology, K-theory and index formulas, accepted for publ. in K-theory.

[18] Neshveyev S., Tuset L., Quantum random walks and their boundaries, to appear in RIMS Kokyuroku, an issue with the Kyoto Symposium Proceedings.

[19] The Local Index Formula for the Quantum Sphere, S.Neshveyev and L.Tuset. Accepted for publ. in Comm. Math. Phys., 2003.

[20] Poisson Boundary of the Dual of SUq (n). M.Izumi, S.Neshveyev and L.Tuset. Subm. to Annals of Math., 2004.

Research plans for L. Tuset:

- with S. Neshveyev:

  (a) Convergence of quantum random walks and further computations of boundaries of discrete quantum groups.
  (b) Formal deformation methods were developed by Nest & Tsygan in the 90's to generalize the Atiyah-Singer index theorem. We want to apply these methods in order to extend our index formula (in the spirit of

Connes & Moscovici) for the quantum sphere to arbitrary quantum flag manifolds. This way we should avoid some of the inherent combinatorial problems affiliated with the required operator-valued pseudo-differential calculus.

- with E. Bedos:

  Amenability and actions of quantum groups.

- with M. Mueger:

  Regular representation for tensor categories and abstract embedding theorems.