Analyse the sources of equifinality in hydrological model using GLUE methodology

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Abstract The equifinality problem has been universally found in hydrological models. The Generalized Likelihood Uncertainty Estimation methodology (GLUE) is widely used to quantify the parameter uncertainty in a variety of hydrological models. This study makes a comprehensive discussion about the sources of equifinality in a distributed conceptual hydrological model. The study is performed using the model DTVGM on Chao River basin in north China. It analyses the impacts of three aspects on the equifinality, including the number of parameters, the systematic errors of input data and the object function. The Monte Carlo scatter figures of parameters, the relative width of 95% confidence intervals and the percent of observations bracketed by the 95% confidence intervals are used as criteria to estimate uncertainties in parameters and the simulated stream flow. In addition, the study gives an example of reducing the equifinality in the model by using a baseflow separation method; this can be used in basins that only have discharge data. The results indicate that: (a) overparametrization, systematic errors of input data and too little constraint conditions are the sources of equifinality in model DTVGM; (b) systematic errors of input data are not only increasing the predictive uncertainty caused by equifinality, but also reduce accuracy of the model; (c) the uncertainty of parameters has been greatly reduced through the secondary model objective which uses baseflow as an additional condition to constrain the model parameters.

Key words GLUE; equifinality; hydrological model; DTVGM; multi-objective function

INTRODUCTION

The traditional calibration method for hydrological models aims to find an optimal set of parameter values, which makes a good fit to observed data. However, all model calibrations and subsequent predictions will be subject to uncertainty, which arises in that no rainfall–runoff model is a true reflection of the processes involved, and it is impossible to specify the initial and boundary conditions required by the model with complete accuracy. There are four important sources of uncertainties in hydrological modelling (Refsgaard & Storm, 1996; Engeland et al., 2005): (a) uncertainties in input data; (b) uncertainties in output data used for calibration; (c) uncertainties in model parameters; (d) uncertainties in model structure. The conceptual model is not necessarily deterministic. The ultimate target of simulation is likely to find a stochastic–conceptual model, which contains the random variables that reflect the unavoidable uncertainty of data. However, almost all the conceptual models are conceptual–deterministic models (Kirkby, 1978). Hydrologists have also been exploring stochastic–conceptual models. However, it will be very complicated to study the modelling uncertainty, considering all of the four error sources. The problem can be simplified by considering these uncertainties separately based on certain assumptions. Beven & Binley (1992) and Beven & Freer (2001) proposed the Generalized Likelihood Uncertainty Estimation (GLUE) method in order to quantify the parameters uncertainty. Multiple sets of parameters can make an equally good or bad simulation, which is named equifinality in the literature. There are many different model structures and many different parameter sets in a model that may be acceptable in reproducing the observed data (Freer et al., 1996; Zak & Beven, 1999).

In order to reduce the model uncertainty through reducing the equifinality, hydrologists carry out many studies on formulation of the objective function, parameters feasible scope, etc. Franks et al. (1997) used catchment saturated area extent, which is estimated by an approach based on the combination of the topographic index and the Saturation Potential Index from the ground truth data, as a secondary modelling objective for model evaluation. And through the specification of a
secondary modelling objective, the parametric and predictive uncertainty has been greatly reduced, especially in constraining the catchment effective saturated transmissivity parameter. Mo & Beven (2004) used two data sets, which are the observations of CO₂ and heat fluxes, before and after an irrigation event in a wheat field to constrain the model. It showed that some parameters are strongly conditioned by the observed fluxes. Besides, the GLUE method was used to compare the uncertainty and parameter differences between the two-source canopy model and the three-source canopy model. Gallart et al. (2007) used water table records and the distribution of parameters obtained from point observations to reduce the uncertainty of predictions for both streamflow and groundwater contribution. Then, increasing the additional constraint information, which are independent from the original data, can reduce the equifinality of the model. Furthermore, Multi-objective and multi-criteria approach can be used in model calibration to reduce the equifinality (Franks et al., 1998; Kuczera & Mroczkowski, 1998; Yapo et al., 1998; Choi & Beven, 2007). There were also some studies on the equifinality of the Xin’anjiang model by GLUE approach in China (Zeng, 2005; Huang & Xie, 2007; Shu et al., 2008). All these papers mostly focus on the parameter uncertainty, reducing the uncertainty of parameter and model, improving the accuracy of simulations, etc. There are only a few papers about the sources of equifinality. This paper seeks to provide a comprehensive discussion about the sources of equifinality in hydrological models and the impacts on parameters of hydrological models which are quantified using the GLUE methodology. A monthly distributed conceptual model called DTVGM was used.

**METHODOLOGY**

**The DTVGM model**

The monthly Distributed Time-Variant Gain Model (DTVGM), which was developed based on the Time-Variant Gain Model (TVGM) (Xia et al., 1997), has been applied in the Chaobai River basin and the Heihe River basin (Xia et al., 2003; Wang et al., 2005). The primary equations of the model are presented in Table 1.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_m = MF \cdot (T_m - T_{m0})$</td>
<td>snowmelt</td>
</tr>
<tr>
<td>$ET_p = [(1 - KAW) \cdot f(P / ET_p) + KAW \cdot AW / WM] \cdot ET_p$</td>
<td>actual evapotranspiration</td>
</tr>
<tr>
<td>$Rs = g1(\frac{AW}{WM})^{g2} \cdot P$</td>
<td>surface runoff</td>
</tr>
<tr>
<td>$Rs = g1(\frac{AWi}{WMI})^{g2} \cdot P$</td>
<td>underground runoff</td>
</tr>
<tr>
<td>$R = Rg + Rs$</td>
<td>total runoff</td>
</tr>
</tbody>
</table>

$MF$ is snowmelt rate (mm°C⁻¹month⁻¹), $T_{m0}$ is the Snowmelt temperature (°C), $T_m$ is mean monthly air temperature (°C), $ET_p$ is potential evapotranspiration (mm month⁻¹), $ThinkU$ is soil depth.

**The GLUE methodology**

The parameter uncertainty was evaluated by using the GLUE (Generalized Likelihood Uncertainty Estimation) methodology (Beven & Binley, 1992). In this method a large number of model runs are made with many different randomly chosen parameter values selected from a priori probability distributions. The acceptability of each run is evaluated against observed values and, if the acceptability is below a certain subjective threshold, the run is considered to be “non-behavioural” and that parameter combination is removed from further analysis. In this method, likelihood values serve as relative weights of each parameter set or simulated value. It is noted that the likelihood function and the threshold are subjectively determined, and this was discussed by Freer et al.
In this study, the Nash-Sutcliffe efficiency was chosen as the likelihood function:

$$L(\theta | Y) = \left(1 - \frac{\sigma_i^2}{\sigma_{obs}^2}\right)^N \quad \sigma_i^2 < \sigma_{obs}^2$$

where $L(\theta | Y)$ is the likelihood measure, $\sigma_i^2$ is the variance of errors for given parameter set $\theta$, and the observed discharge data set $Y$, $\sigma_{obs}^2$ is the variance of the observed data set, and $N$ is a parameter.

**Criteria for the comparison**

In this paper, two indices are used to compare the derived 95% confidence interval (95CI), which are the relative width of 95% confidence interval ($R - 95\text{CI}$) and the percent of observations bracketed by the 95CI ($P - 95\text{CI}$):

$$R - 95\text{CI} = \left(\frac{Q_{ub} - Q_{lb}}{Q_{obs}}\right)$$

$Q_{ub}$ is the upper boundary value of 95CI, $Q_{lb}$ is the lower boundary value of 95CI, $Q_{obs}$ is the observed discharge.

$$P - 95\text{CI} = \frac{NQ_{obs}}{NQ_{all}} \times 100\%$$

$NQ_{obs}$ is the number of observations bracketed by the 95CI, $NQ_{all}$ is the total number of observations. The goodness of calibration uncertainty is judged on the basis of the closeness of the $R - 95\text{CI}$ to 0 and the $P - 95\text{CI}$ to 100%.

**RESULTS AND DISCUSSION**

**Sensitivity analysis of model parameters**

The study area was Chao River basin upstream of the Miyun reservoir with drainage area of 5300 km², accounting for 40% of the Miyun Reservoir catchment area, which is one of the most important surface water resources of Beijing water supply. The data from the 1973 to 1982 was used for simulation. The model was calibrated based on the observed discharges at the watershed outlet (Xiahui station). There is rarely snow in winter in Chao River basin because of the little precipitation in the winter season. So snowmelt is not considered in the simulation with monthly time step. The parameters ranges of the model are shown in Table 2. The prior distribution of parameters is supposed to be a uniform distribution. A total of 200,000 runs of DTVGM were made using Monte Carlo-based random sampling of the original parameter ranges. The threshold value of the Nash-Sutcliffe efficiency used to select the behavioural parameter set was chosen to be 0.6. The results in Figs 1 and 2 show the ranking of the parameters sensitivity is as follows: $g_1$, $WM$, $g_2 > Kaw$, $Kr > WMi$.

**The impact of the numbers of parameters on equifinality**

In this paper, the impact of the number of parameters on equifinality in DTVGM is studied by reducing the number of parameters gradually, starting from the least sensitive parameter based on

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Physical meaning</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1$</td>
<td>The coefficients of surface water</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$g_2$</td>
<td>The coefficients of surface water</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$Kr$</td>
<td>The outflow efficient of groundwater</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$WMI$</td>
<td>Minimum soil-moisture storage</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>$WM$</td>
<td>Maximum soil-moisture storage</td>
<td>0.4</td>
<td>1</td>
</tr>
<tr>
<td>$Kaw$</td>
<td>Weight coefficient for evapotranspiration</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Analyse the sources of equifinality in hydrological model using GLUE methodology

Fig. 1 Dotty plots for six DTVGM parameters against likelihood value.

Fig. 2 Intersection of the hypersurface of the likelihood function. ($\hat{\theta}_j = \text{optimised parameter value}, \tilde{\theta}_j = \text{parameter values in the neighbourhood of } \hat{\theta}_j$).

the sensitivity analysis of parameters. The parameters $g1$ and $Kaw$, which are important and sensitive, are retained and chosen as example to demonstrate the effect of fixing other parameters on them. The parameters are fixed to the optimal values one by one in turn, by the sensitivity from weak to strong. The dotty plots of $g1$ and $Kaw$ are shown in Fig. 3.

By comparison with Fig. 1, the dotty plots show nearly no change when the $WMi$ parameter in Fig. 3(a) is fixed, which proves that $WMi$ is a “non-sensitive” parameter. When considering hydrological model improvement, the non-sensitive parameters can be fixed by experience or experiment. So the hydrological model can be simplified and the time of model calibration
shortened. With the increasing number of fixed parameters, the dotty plots become denser, and the peaks of response surface are sharper. So it demonstrates that equifinality is reduced. Figure 3 also demonstrates that as more parameters are fixed, the uncertainties of the other free parameters are smaller, and the probability of an efficiency coefficient bigger than 0.6 is increased. Furthermore, the 95\% CI is narrower in Fig. 4(b) than that in Fig. 4(a), which indicates the reduction of model uncertainty. This result reveals that too many parameters (overparametrization) are one reason for equifinality in the hydrological model.

Fig. 3 Dotty plots of \( g_1 \) and \( K_{aw} \) plotted against likelihood value, while (a) the parameter \( WM_i \) of DTVGM is fixed to the optimal value, (b) the parameters \( WM_i, K_r, g_2 \) and \( WM \) of DTVGM are fixed to the optimal value.

Fig. 4 The 95\% CI of monthly simulated discharge from 1975 to 1982 due to: (a) one parameter fixed just the same with Fig. 3(a), (b) four parameters fixed just the same with Fig. 3(b), for the Chao River basin. The \( Q_{sim} \) is the 95\% CI of monthly simulated discharge and the \( Q_{obs} \) is the observed flow.
The impact of systematic errors of input data on equifinality

In the research, precipitation is used as an example of input data, and ±10% systematic errors are added to the precipitation. The same likelihood criteria and threshold value as used in Fig. 1 are used here. The simulated results are shown in Fig. 5 and Table 3 as follows:

![Graphs showing the impact of systematic errors on parameters g1 and Kaw](image)

**Fig. 5** Dotty plots of parameter g1 and Kaw for (a) precipitation reduced 10% systematically, (b) precipitation increased 10% systematically.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Original precipitation</th>
<th>Precipitation reduced by 10% systematically</th>
<th>Precipitation increased by 10% systematically</th>
</tr>
</thead>
<tbody>
<tr>
<td>R – 95CI</td>
<td>1.73</td>
<td>2.37</td>
<td>0.85</td>
</tr>
<tr>
<td>P – 95CI (%)</td>
<td>80.21</td>
<td>81.25</td>
<td>56.25</td>
</tr>
</tbody>
</table>

Table 3 The $R - 95CI$ and $P - 95CI$ due to different input data of precipitation for Chao River basin.

Figure 5 shows that the impacts of systematic error of input data on the two parameters are different. Parameter $Kaw$ displays are more influenced. Why are they different? It is because the hydrological model calibration makes simulated runoff fit to observed runoff. As discussed before, the $Kaw$ parameter controls the simulated evapotranspiration that is not included in the objective function, while the parameter $g1$ controls the simulated runoff which is constrained by the objective function. Accordingly, when the precipitation changed, the excess or missing water will be transferred to actual evapotranspiration, which results in the greater uncertainty of parameter $Kaw$. The difference in the impact of precipitation increase and reduction on the DTVGM model can also be seen from Table 3. The results indicate that the systematic error is likely to enhance the uncertainty of parameter values and reduces the accuracy of the simulation results. It should be noted that when the precipitation reduced by 10%, the response surface is more flat, and when the precipitation increased by 10% the overall simulation efficiency reduced. The results reveal that the systematic error of input data is another source of equifinality and uncertainty in DTVGM model.
The impact of insufficient constraint conditions on equifinality

The calibration of hydrological models is usually achieved with limited discharge data. In this study, the baseflow is used as additional data for model calibration in order to test the difference of multi- and single-objective functions. Many baseflow separation methods have been developed and applied to various catchments, such as the smoothed minimum method (Mazvimavi et al., 2004), digital filter method (Lyne & Hollick, 1979; Weeks & Boughton, 1987), streamflow hydrograph separation (Ysep, 1996), etc., of which the digital filtering method is most widely used. In this research, three digital filtering methods developed by Lyne & Hollick (1979), Weeks & Boughton (1987), and Chapman (1991) are used to calculate the daily baseflow, which is then summed to get the monthly baseflow and used to calibrate the baseflow simulated by the model.

![Figure 6](image_url)

**Fig. 6** Dotty plots of Nash and Sutcliffe coefficient against DTVGM parameter $g_1$, $g_2$, $K_{aw}$ and $K_r$ conditioning with GLUE based on 200,000 samples with threshold $R^2 \geq 0.06$ and $R^2_{base} \geq 0.5$ ($R^2_{base}$ is the Nash and Sutcliffe coefficient of monthly baseflow).

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Single-objective (total runoff)</th>
<th>Multi-objective (total runoff and baseflow)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R - 95CI$</td>
<td>1.73</td>
<td>1.06</td>
</tr>
<tr>
<td>$P - 95CI (%)$</td>
<td>80.21</td>
<td>78.08</td>
</tr>
</tbody>
</table>

Table 4 the $R - 95CI$ of the monthly simulated discharge from 1975 to 1982 and the $P - 95CI$ due to different constraint conditions of calibration for Chao River basin.

Figure 6 shows that the number of behavioural simulations reduced obviously after additional evaluation of the simulations with respect to the baseflow. However, the optimal result was not affected. As can be seen, the dotty plots of parameter $K_r$ changed significantly. That is because the parameter $K_r$ controls the baseflow directly. Table 4 shows that $R - 95CI$ reduced by 38.7%, from 1.73 to 1.06, while $P - 95CI$ reduced by 2.1%. It illuminates that the uncertainty of the model is reduced with almost no impact on accuracy of the result after additional evaluation of the simulations with respect to the baseflow. With the increase in calibrated data and multi-objective functions, the uncertainty of relevant parameters and model uncertainty reduced significantly, which puts forward higher requirements on hydrological observation and data collection.
CONCLUSION

The study result indicates that too many parameters in the model (overparametrization), systematic errors of input data, and constraint conditions that are too little are the sources of equifinality in model DTVGM. By quantitative comparison of the sensitivity of parameters, the parameter of minimum soil moisture is found to be the most insensitive parameter in the model DTVGM, and has little effect on simulated results. This kind of insensitive parameters can be fixed against the physical characteristics of the basin to reduce the equifinality of the hydrological model. Furthermore, the systematic errors of input data not only increase the predictive uncertainty caused by equifinality, but also reduce the accuracy of the model. So it is important to have some idea of the accuracy in the input data. It’s important to perform quality control on the input data before it is used to drive hydrological models. This paper also gives an example to reduce the equifinality in the model by using baseflow as an additional data for calibration. In this study, \( R - 95\% CI \) and \( P - 95\% CI \), as defined by equations (2) and (3), are used as two indices to compare the 95\% CI of the simulated result. The result illustrates that the uncertainty of parameters has been greatly reduced through the secondary model objective. This reduction in uncertainty of parameters translates into a marked reduction in the predictive uncertainty, most obviously in the constraint of the parameter \( Kr \) which is controlling baseflow simulation. So it is better to use as much information as possible to calibrate the model whenever the data are available.

It should be noted that although this study provides a comprehensive discussion about the sources of equifinality in the hydrological model and the impacts on parameters of the model are quantified using the GLUE methodology, it is only a preliminary study because only one model is used and the data are limited. More studies need to be done on other models and using more input and output data in order to generalize the findings obtained here.

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