Conditional Value-at-Risk for Nonstationary Streamflow and Its Application for Derivation of the Adaptive Reservoir Flood Limited Water Level

Xiaoqi Zhang¹; Pan Liu, Aff.M.ASCE²; Chong-Yu Xu³; Bo Ming⁴; Aili Xie⁵; and Maoyuan Feng⁶

Abstract: The existing risk-based approaches for quantifying expected flood damage loss may not be suitable in the changing environment when streamflow becomes nonstationary. Conditional value-at-risk (CVaRα), a modified form of value-at-risk (VaRα), that takes account of both the magnitude and probability of damage loss has been widely applied to water resources issues. However, quantification of CVaRα under nonstationary conditions has not been addressed in the literature. This study proposes an approach that incorporates CVaRα in quantifying the flood damage loss under nonstationary conditions, and then CVaRαn associated with flood risk over a specified time horizon is deduced. To illustrate the concept of CVaRα, this study applies CVaRα into the adaptive flood limited water level (FLWL) optimization. With China’s Three Gorges Reservoir (TGR) as a case study, the results indicate that (1) the CVaRα not only can represent the possible flood damage loss over a time horizon in the future, but also can reflect the flood risk probability by choosing a suitable confidence level; and (2) it is reliable to take account of the CVaRα value as a constraint when studying the adaptive FLWL optimization problem. DOI: 10.1061/(ASCE)WR.1943-5452.0000906. © 2018 American Society of Civil Engineers.

Author keywords: Nonstationary; Flood limited water level; Flood risk; Conditional value-at-risk.

Introduction

Streamflow regime has been changed because of climate change and human activities (Greenwood et al. 1979; Obeysekera et al. 2011), thus Milly et al. (2008) stated that “stationarity is dead.” Numerous studies have demonstrated that hydrologic records could show nonstationarity in the form of increasing or decreasing trends (Lin et al. 2014; Strupczewski et al. 2001; Zhang et al. 2006), upward and downward shifts (Fortin et al. 2004; Li et al. 2015; Salas and Boes 1980), or their combinations (Cui and Li 2016; Kim et al. 2015; Villarini et al. 2009). Hydrologic probability distribution estimation methods that are based on stationary conditions need to be re-evaluated or modified in studying the long-term variation pattern of water resources and flood evolution (Feng et al. 2017a; Yan et al. 2016).

Reservoirs are one of the most effective engineering measures for water resource development and management, and reservoir operation systems contains many decision variables and multiple objectives (Li et al. 2010; Liu et al. 2006). Hence, adapting reservoir operation in a changing environment is of significant importance (Feng et al. 2017b; Mateus and Tullos 2017; Yang and Ng 2017). Specifically, the flood limited water level (FLWL), which is the key parameter for controlling the trade-off between activities of flood control and water conservation (Liu et al. 2015a; Ouyang et al. 2015; Yun and Singh 2008; Zhou et al. 2014), needs to be re-established to ensure flood safety when the reservoir inflow is altered. The FLWL is usually determined by flood routing of design floods, which is based on annual maximum flood samples, under the condition that the flood prevention risk does not increase (Liu et al. 2015a).

The probability of failure, which is routinely defined as the probability of exceeding a specific threshold of the variable (e.g., reservoir water level, downstream discharge), is widely used in engineering practices for flood risk analysis (Liu et al. 2015c; Read and Vogel 2015; Volpi et al. 2015). However, this concept relies on a flood frequency distribution assumed to remain stationary over time. Rootzén and Katz (2013) proposed design life level to quantify the risk of a given flood magnitude occurring over a specified time period in a changing climate. Salas and Obeysekera (2014) extended the evaluation formula of the risk under stationary conditions into a nonstationary framework by considering time-varying exceeding probabilities, which is widely accepted as the basis of studying flood risk probability under nonstationary conditions. The methods of incorporating nonstationarity into the assessment of flood frequency are usually done by first establishing the relationship between statistical parameters and time or meteorological factors (e.g., temperature, precipitation) (Condon et al. 2015; Du et al. 2015; Lima et al. 2015). Then, statistical parameters that use only time or meteorological variables as covariates are applied in various probability distributions to derive the time-varying exceeding probabilities (Jiang et al. 2017; Yan et al. 2017).
However, flood risk analysis in many literatures is only represented by two factors, i.e., extreme flood events and associated probability [which should be strictly defined as flood hazards by Apel et al. (2004)], which does not reflect the consequences of flooding (i.e., the flood damage loss).

The conditional value-at-risk ($CVaR_\alpha$), a modified form of value-at-risk ($VaR_\alpha$), is applied in this study to quantify the flood damage loss under nonstationary streamflow conditions. $VaR_\alpha$ is defined as the threshold at which the probability of a loss exceeding the threshold is $1 - \alpha$ for a given confidence level $\alpha$ (Piantadosi et al. 2008; Yamout et al. 2007). However, $VaR_\alpha$ takes no account of the magnitude of losses that would occur when the threshold is exceeded (Rockafellar and Uryasev 2002). Hence, it is incapable of distinguishing the size of the tail loss (Artzner et al. 1999; Yamout et al. 2007).

$CVaR_\alpha$ can deal with the limitations of $VaR_\alpha$ by considering the expected value of the loss conditional on the loss exceeding the $VaR_\alpha$ threshold (Piantadosi et al. 2008; Rockafellar et al. 2014; Rockafellar and Uryasev 2002). Therefore, $CVaR_\alpha$ is not only widely used for investment decisions and portfolio management in finance (Artzner et al. 1999; Piantadosi et al. 2008; Soltani et al. 2016), but also has been applied to water resources issues (Howlett and Piantadosi 2007; Webby et al. 2007). Webby et al. (2006) used $CVaR_\alpha$ to tradeoff objectives for amenity and environment flows against flood risk by setting extra drawdown levels for Lake Burley Griffin in Canberra when rainfall is expected. Yamout et al. (2007) compared the effect of incorporating $CVaR_\alpha$ into a water allocation problem with the frequently used expected value, and concluded that the expected value underestimated costs when compared with the $CVaR_\alpha$ model. Piantadosi et al. (2008) combined stochastic dynamic programming with $CVaR_\alpha$ to determine a policy for management of urban stormwater. Shao et al. (2011) proposed a $CVaR_\alpha$-based inexact two-stage stochastic programming model for analyzing the water resources allocation problem involving one reservoir and three competing water users. Soltani et al. (2016) established an objective function based on $CVaR_\alpha$ for planning agricultural water and return flow allocation in river systems. Yazdi et al. (2016) integrated ant colony optimization, artificial neural network, and three risk measures including expected flood discharge, $VaR_\alpha$, and $CVaR_\alpha$ for optimal design of detention dams under flood discharge uncertainties.

However, quantification of $CVaR_\alpha$ under nonstationary conditions and its application in reservoir flood control operation have not been addressed in the literature, which motivated this study. Therefore, the purpose of this study is to incorporate the $CVaR_\alpha$ in quantifying the flood damage loss in a nonstationary condition, and it may provide a new idea for evaluating the possible consequences of flooding. The specific objective of this study is to establish the loss function of flood damage with the help of $CVaR_\alpha$, and extend the $CVaR_\alpha$ value into the $CVaR_\alpha$ value, which represents the possible flood damage loss over a time horizon in the future, in a nonstationary framework. Then, this study incorporates the $CVaR_\alpha$ value in the application of the adaptive reservoir FLWL optimal model. The remainder of this paper is organized as follows. The “Methodology” section describes definitions of $VaR_\alpha$ and $CVaR_\alpha$, the method of assessing the flood damage with $CVaR_\alpha$, and the optimal model. Section “Case Study” addresses a case study of China’s Three Gorges Reservoir (TGR) and the nonstationary streamflow scenario generation for TGR. Then, results and discussions are shown in section “Results and Discussion.” Finally, conclusions are given in section “Conclusions.”

Methodology

Section “Flood Hazards Assessment with Probability” briefly reviews the traditional method of flood hazards assessment with probability under stationary and nonstationary conditions. The method of incorporating the $CVaR_\alpha$ into quantifying the flood damage loss and the derivation of $CVaR_\alpha$ for $n$ years are shown in section “Flood damage Assessment with $CVaR_\alpha$.” The optimal model of applying $CVaR_\alpha$ into reservoir flood control operation is established in section “Optimal Model.”

Flood Hazards Assessment with Probability

It is assumed that $Q_p$ is a design flood peak value of streamflows, and the probability of the inflow flood peak $Q$, which is a random variable, exceeding $Q_p$ is expressed as $p_i (i = 1, 2, \ldots)$. It is also assumed that $Q$ exceeding $Q_p$ will occur in the $i$th year for the first time.

On stationary conditions, $p_i$ is equal to a constant value $p$ whatever $i$ is, and hydrologic series are independent (i.e., $p_i$ subjects to independent identical distribution) (Gumbel 1961; Leadbetter 1983). Hence, the probability of exceedance event occurs in the $i$th year is as follows:

$$f(i) = P(I = i) = p(1 - p)^{i-1}, \quad i = 1, 2, \ldots$$

It is assumed that the project life of a hydraulic structure is designed for $n$ years, and the failure of the structure to facing a flood exceeding the design flood magnitude occurs before or at the $n$th year (Rootzén and Katz 2013; Salas and Obeysekera 2014), and hence the flood risk probability $R$ is shown as follows:

$$R = P(I \leq n) = p \sum_{i=1}^{n} f(i) = p \sum_{i=1}^{n} (1 - p)^{i-1} = 1 - (1 - p)^n \tag{1}$$

However, the exceedance probability of floods, namely $p_1, p_2, \ldots, p_i$, should vary through time under nonstationary conditions. Thus, the probability of exceedance event occurs in the $i$th year and the flood risk probability are, respectively, expressed as follows (Condon et al. 2015; Obeysekera and Salas 2014; Olsen et al. 1998):

$$f(i) = P(I = i) = (1 - p_1)(1 - p_2) \ldots (1 - p_{i-1})p_i, \quad i = 1, 2, \ldots$$

$$R = P(I \leq n) = p_1 + p_2(1 - p_1) + \ldots + p_n(1 - p_1)(1 - p_2) \ldots (1 - p_{n-1}) = \sum_{i=1}^{n} p_i \prod_{r=1}^{i-1}(1 - p_r) \tag{2}$$

Flood Damage Assessment with $CVaR_\alpha$

Definition of $VaR_\alpha$ and $CVaR_\alpha$

$VaR_\alpha$ is defined as the maximum loss with a given confidence level, $\alpha$, over a specified time horizon, which is derived by using cumulative probability distribution function of a random variable. $CVaR_\alpha$ is defined as the expected loss given that the loss exceeds $VaR_\alpha$. Let $L(x, \theta)$ be a loss function of a decision vector $x$ and a stochastic vector $\theta$, and $\varphi(x, \theta)$ is the cumulative distribution function of loss. The $VaR_\alpha$ and $CVaR_\alpha$ at a given confidence level $\alpha \in [0, 1]$ can be defined as follows (Rockafellar and Uryasev 2002):
discrete loss function. CVaRing the reservoir downstream floodplain. Let \( \theta \) tion for flood control of a decision vector \( \theta \), which represent the FLWL and the reservoir inflow with corre-

Because the derivation of CVaR\(_n\) is based on the relationship between the flood risk probability \( R \) [which is derived by (Salas and Obeysekera (2014)) and the confidence level \( \alpha \), the derivation of CVaR\(_n\) for \( n \) years assumes that the occurrences of annual flood damage loss could be regarded as the independent event, and no matter whether or not the CVaR\(_n\) for the \( i \)th year follows the same identical distribution. The CVaR\(_n\) for the \( i \)th year can be derived by Eq. (5). Whether or not the flood damage loss occurs in the \( i \)th year could also be regarded as the independent event which follows Bernoulli distribution, and hence, the CVaR\(_n\) for \( n \) years can be proposed as follows (detailed information regarding formula derivation is provided in the Appendix S1):

\[
CVaR_n = \frac{p_1 \cdot CVaR_{i1} + p_2 \cdot CVaR_{i2} + \cdots + p_n \cdot CVaR_{in}}{R}
\]

where \( \alpha = 1 - R \), and \( R = P(X \leq n) = 1 - (1 - \alpha)^n \) \((p_1 = p_2 = \cdots = p_n = p)\) under stationary conditions or \( R = P(X \leq n) = p_1 + p_2(1 - p_1) + \cdots + p_n(1 - p_{n-1}) \cdots (1 - p_{n-1})\) under non-

Optimal Model

The flowchart of the method for adaptive reservoir FLWL is given in Fig. 2, which has three schemes as follows.

1. The reservoir average annual hydropower generation under station-

2. Without increasing the reservoir flood risk probability, the re-

3. Based on the Scheme B1, the optimal adaptive FLWL model is established by adding the CVaR value as a new constraint (Scheme B2).

Objective Function

Maximization of hydropower benefits can be described by the average annual hydropower generation during flood season as follows:

\[
Max \bar{E}(Z_{x1}, Z_{x2}, \ldots, Z_{xn}) = \frac{1}{n} \sum_{j=1}^{n} E(Z_{xj})
\]

where \( Z_{xj} = \) decision vector (\( j = 1, 2, \ldots, n \)), representing the reser-

When the design life period is set as \( n \), then the CVaR\(_n\) value represents the flood damage loss for \( n \) years, and the probability that at least one flood damage occurs in a period of \( n \) years is \( R \). Thus, the confidence level is \( \alpha = 1 - R \). If the time horizon of \( n \) years is considered as a whole, the loss function for \( n \) years is determined when FLWL scheme is fixed. Whether or not the flood damage loss occurs could be regarded as the event which follows Bernoulli distribution, then the expected flood damage loss for \( n \) years could be represented as \( R \cdot CVaR_n \).

The relationship between the FLWL and the annual hydropower generation when the FLWL is set as \( \bar{x} \) is established by conventional generation operation. The relationship between the flood risk probability \( R \) for observed data. In this study, the annual hydropower generation in the \( i \)th year is calculated based on the observed data. The relationship between the FLWL and the annual hydropower generation is established by conventional generation operation.
Constraints

1. Cumulative flood risk probability:

\[ R^m_i(Z_{x1}, Z_{x2}, \ldots, Z_{xn}) \leq R^m_i(Z_{x1}^*, Z_{x2}^*, \ldots, Z_{xn}^*) \]  

(8)

where \( R^m_i(\cdot) \) is cumulative flood risk probability for the stationary condition in the \( j \)th year by using the reservoir conventional FLWL \( Z_{x1}^* = Z_{x2}^* = \cdots = Z_{xn}^* = Z_{x0} \), and \( R^m_i(\cdot) \) is cumulative flood risk probability for the nonstationary condition in the \( j \)th year by using the reservoir adaptive FLWL scheme, i.e., \( Z_{xj} \) represents the FLWL in the \( j \)th year (\( j = 1, 2, \ldots, n \)).

2. Conditional value-at-risk:

\[ CVaR^m_i(Z_{x1}, Z_{x2}, \ldots, Z_{xn}) \leq CVaR^m_i(Z_{x1}^*, Z_{x2}^*, \ldots, Z_{xn}^*) \]  

(9)

where \( CVaR^m_i(\cdot) \) value over a planning time horizon of \( n \) years with the reservoir conventional FLWL \( Z_{x1}^* = Z_{x2}^* = \cdots = Z_{xn}^* = Z_{x0} \) under stationary conditions corresponding to a confidence level \( \alpha \); \( CVaR^m_i(\cdot) \) value for \( n \) years under nonstationary conditions; and \( Z_{xj} \) = reservoir FLWL in the \( i \)th year (\( i = 1, 2, \ldots, n \)) for nonstationary conditions.

3. Reservoir water balance equation:

\[ V_{t+1} = V_t + (I_t - Q_t) \Delta t \]  

(10)

where \( \Delta t \) = time interval; \( I_t \) and \( Q_t \) = reservoir inflow and release during time period \( \Delta t \), respectively; and \( V_t \) = reservoir storage at time \( t \).

4. Reservoir storage limits:

\[ V_{\min} \leq V_t \leq V_{\max} \]  

(11)

where \( V_{\min} \) and \( V_{\max} \) = minimum and maximum allowable reservoir storages during flood season.

5. Release capacity limits:

\[ Q_t \leq Q_{\max}(Z_t) \]  

(12)

where \( Q_{\max}(Z_t) \) = reservoir maximum discharge when the reservoir water level is \( Z_t \) at time \( t \).

Eq. (8) and Eqs. (10)–(12) are the constraints for Scheme B1, whereas Eqs. (8)–(12) are the constraints for Scheme B2. The non-dominated sorting genetic algorithm-II is used as the optimization algorithm for the adaptive reservoir FLWL.

Case Study

The TGR, located in the Yichang City of China’s Hubei Province (Fig. 3), is used for a case study. The TGR is the largest water conservancy project ever undertaken in China, which combines multipurpose uses simultaneously, including flood control, hydropower generation, and navigation improvement. In particular, flood control is the most important role of the TGR because the downstream of TGR is the plain region of the middle and lower reaches of the Yangtze River. Equipped with 32 sets of 700-MW hydraulic turbo generators and 2 sets of 50-MW hydraulic turbo generators, the TGR is the largest hydropower station in the world, with the total yearly capacity of 22,500 MW.

The floodplain is an important flood control measure which can cooperate with the reservoir by appropriately diverting part of the flood volume, so as to reach the downstream flood control targets. Because of the fact that the streamflow in the middle and lower reaches of the Yangtze River does not match the flood discharge capacity of the river, there are four floodplains—Jingjiang, Chenguangji, Wuhan, and Hukou—with a total area of \( 1.18 \times 10^4 \) km\(^2\), arable land of \( 54.84 \times 10^8 \) m\(^2\), a
population of 6.12 million, and effective flood storage capacity of 63.30 billion m$^3$.

**Results and Discussion**

Three schemes used in the comparative analysis are presented: (1) the conventional FLWL scheme under stationary conditions (Scheme A); (2) the optimal adaptive FLWL scheme with the flood risk probability as a constraint under nonstationary conditions (Scheme B1); and (3) the optimal adaptive FLWL scheme with the $CVaR_\alpha$ value incorporated for nonstationary conditions (Scheme B2). Scheme A is used as a baseline for comparison with the other schemes. The conditional value-at-risk $\beta_n$ value derived under stationary conditions (Scheme A) is the upper limit value for the $CVaR_\alpha$ values under nonstationary conditions. The comparison between Scheme A and Scheme B1 is to show the necessity of the adaptive FLWL for nonstationary conditions, whereas the advantage of $CVaR_\alpha$ applications is given by comparing Scheme B1 with Scheme B2.

**Conventional Hydropower Generation under Stationary Condition (Scheme A)**

The average annual hydropower generation is derived by using conventional operation rules based on the observed data from 1882 to 2010 of the TGR. Table 1 shows the relationship between the FLWL and the average annual hydropower generation. Specifically, the average annual hydropower generation of the TGR is $410.80 \times 10^8$ kWh when the reservoir conventional FLWL is set as 145.0 m, which is the base scheme for comparison with Scheme B1 or Scheme B2. The relationship between the FLWL and the average annual hydropower generation is also used in Scheme B1 and Scheme B2 to assess the objective function for different FLWL schemes, in terms of considering uncertainty of reservoir inflow.

**CVaR\_\alpha under Stationary Condition (Scheme A)**

Fig. 4 shows the changing pattern of $CVaR_\alpha$ values with the reservoir FLWL of the TGR from 140.0 to 155.0 m and the confidence

<table>
<thead>
<tr>
<th>FLWL (m)</th>
<th>Eave ($\times 10^8$ kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>140.0</td>
<td>380.70</td>
</tr>
<tr>
<td>141.0</td>
<td>386.83</td>
</tr>
<tr>
<td>142.0</td>
<td>392.85</td>
</tr>
<tr>
<td>143.0</td>
<td>398.84</td>
</tr>
<tr>
<td>144.0</td>
<td>404.80</td>
</tr>
<tr>
<td>145.0</td>
<td>410.80</td>
</tr>
<tr>
<td>146.0</td>
<td>416.88</td>
</tr>
<tr>
<td>147.0</td>
<td>423.01</td>
</tr>
<tr>
<td>148.0</td>
<td>429.15</td>
</tr>
<tr>
<td>149.0</td>
<td>435.21</td>
</tr>
<tr>
<td>150.0</td>
<td>441.19</td>
</tr>
<tr>
<td>151.0</td>
<td>447.11</td>
</tr>
<tr>
<td>152.0</td>
<td>452.98</td>
</tr>
<tr>
<td>153.0</td>
<td>458.83</td>
</tr>
<tr>
<td>154.0</td>
<td>464.67</td>
</tr>
<tr>
<td>155.0</td>
<td>470.40</td>
</tr>
</tbody>
</table>

Note: Eave means average annual hydropower generation.
level $\alpha$ from 99 to 99.99% (detailed information regarding the establishment of the loss function for the TGR is provided in the Appendix S2), and $c$ represents a unit cost for flood volume needs to be diverted into the downstream floodplain). It indicates that the changing tendency of the $\text{CVaR}_n$ value with FLWL behaves similarly under different confidence level values $\alpha$, and the relationship of $\text{CVaR}_n$ value and FLWL is approximately linear, which is corresponding to the relationship between $w_f$ and FLWL in Fig. 4. The higher the FLWL, the higher the $\text{CVaR}_n$ value at the same confidence level $\alpha$, which represents the greater flood damage loss. The higher the confidence level $\alpha$ (i.e., the larger inflow magnitude, which is derived from the mathematic relationship $\alpha + R = 1$), the larger the $\text{CVaR}_n$ value when the FLWL is fixed.

On stationary conditions, if the exceedance probability (design frequency) for each year under stationary conditions is set as 0.001, so the conditional value-at-risk $\beta_n$, corresponding to the FLWL $x = 145.0$ m and the design frequency $\alpha = 99.99\%$, is chosen to represent the $\text{CVaR}_n$ value for each year under stationary conditions. The cumulative flood risk probability $R_n$ for the time horizon from 2020 to 2039 is equal to 2%, which is estimated from Eq. (1). Thus, the $\beta_n$ value over the time horizon from 2020 to 2039 is equal to 33.06 constant $(c)$ billion yuan at 98% confidence level, which can be derived from Eq. (6).

**Optimization Results (Scheme B1 and Scheme B2)**

**Adaptive FLWL Considering the Flood Risk Probability under Nonstationary Condition: Scheme B1**

On a stationary condition, the exceedance probability for each year is equal to $p_0 = 0.001$, and then, the flood risk probability $R_n^*$ for $n$ years could be derived by Eq. (1), which is the upper limit value for constraining the increment of the adaptive reservoir FLWL for the nonstationary condition. In this study, the re-establishment of FLWL is derived based on flood routing of the nonstationary streamflow scenario (detailed information regarding the nonstationary streamflow scenario is provided in the Appendix S3). Fig. 5 shows the optimal results of FLWL scheme from 2020 to 2039 in Scheme B1, and it is assumed that every continuous five years choose the same FLWL value as one decision variable. The solid point and asterisk represent FLWL schemes for nonstationary and stationary conditions, respectively, whereas the dotted line and solid line represent the $\text{CVaR}_n$ values for $n$ years under nonstationary and stationary conditions, respectively. When the mean value of annual maximum data decreases from 2020 to 2039, the FLWL could be raised as in Fig. 5, and the flood risk probability $R_n^{**}$ for nonstationary conditions is derived by Eq. (2). The flood risk probability $R_n^{**}$ during 2020 to 2039 is 1.94% when the FLWL is re-established, which is not beyond the threshold $R_n^* = 1.98\%$. The average annual hydropower generation increases by 6.29% from that of conventional FLWL scheme. However, the $\text{CVaR}_n^*$ value over the time horizon from 2020 to 2039 is equal to 33.13 c billion yuan at 98% confidence level when adopting the optimal FLWL scheme derived from Scheme B1, where the confidence level value is chosen as the same as that of stationary conditions.

**Adaptive FLWL Considering CVaR$_n$ Value under Nonstationary Condition: Scheme B2**

Based on the Scheme B1, the optimal model of adaptive FLWL in Scheme B2 is established by adding the $\text{CVaR}_n^*$ value as a new constraint. The $\text{CVaR}_n^*$ for the nonstationary condition is derived by Eq. (6) with different FLWL schemes. The optimal adaptive FLWL model of the Scheme B2 is built to search for an optimal solution of FLWL schemes under the nonstationary streamflow scenario, which is the same as that of the Scheme B1. In Scheme B2, the optimal adaptive FLWL scheme should not only satisfy the restriction of the flood risk probability $R_n^{**} \leq R_n^*$ but also not cause the increment of the possible flood damage loss compared with $\beta_n$ values.

Fig. 6 shows the optimal results of FLWL scheme in Scheme B2. The solid point and asterisk represent the reservoir FLWL for nonstationary and stationary conditions, respectively, whereas the dotted line and solid line represent the $\text{CVaR}_n^*$ values for $n$ years under nonstationary and stationary conditions, respectively. To compare the $\text{CVaR}_n^*$ value with the $\beta_n^*$ value, both $\text{CVaR}_n^*$ and $\beta_n^*$ choose 98% as the confidence level value. For year 2020, $n$ is set as one, $\alpha = 1 - R_1 = 1 - (1 - p_1) = p_1$, and then $\text{CVaR}_1 = p_1 \cdot \text{CVaR}_{R1} / R_1$. For year 2021, $n$ is two, $\alpha = 1 - R_2 = 1 - (1 - p_1)(1 - p_2)$, and $\text{CVaR}_2 = (p_1 \cdot \text{CVaR}_{R1} + p_2 \cdot \text{CVaR}_{R2}) / R_2$. Thus, the value of $n$ is set from one to twenty corresponding to 2020 to 2039.

**Comparison of Scheme B1 and Scheme B2**

The difference between Scheme B2 and Scheme B1 is whether or not the $\text{CVaR}_n^*$ value for $n$ years is used as a constraint for adaptive FLWL optimization. Both of the optimal results of Scheme B1 and Scheme B2, compared with Scheme A, show the necessity of re-establishing FLWL when streamflow alters under nonstationary conditions. In this study, the reservoir FLWL of TGR could be raised when the nonstationary streamflow decreases, and then more hydropower generation benefit can be produced in case of adaptive reservoir FLWL.
Table 2. Comparison Results of Scheme B1 and Scheme B2 for FLWL Scheme and the Average Annual Hydropower Generation

<table>
<thead>
<tr>
<th>Year</th>
<th>Scheme A</th>
<th>Scheme B1</th>
<th>Scheme B2</th>
<th>Scheme A</th>
<th>Scheme B1</th>
<th>Scheme B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2020–2024</td>
<td>145.0</td>
<td>147.4</td>
<td>145.3</td>
<td>410.80</td>
<td>436.62</td>
<td>430.98</td>
</tr>
<tr>
<td>2025–2029</td>
<td>145.0</td>
<td>148.4</td>
<td>147.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2030–2034</td>
<td>145.0</td>
<td>149.7</td>
<td>149.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2035–2039</td>
<td>145.0</td>
<td>151.5</td>
<td>151.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Eave means average annual hydropower generation.

Table 2 and Fig. 7 show the comparison results of Scheme B1 and Scheme B2. On the stationary condition (Scheme A), the conditional value-at-risk \( \beta^n \) over 20 years from 2020 to 2039 is equal to 33.06 c billion yuan at 98% confidence level, and the cumulative flood risk probability is 1.98%. Under the nonstationary condition, the \( CVaR^n \) value over the time horizon from 2020 to 2039 is equal to 33.13 c billion yuan at 98% confidence level when the adaptive FLWL is re-established only considering the flood risk probability as a constraint (Scheme B1). Although the average annual hydropower generation increases by 6.29% compared with that under a conventional FLWL scheme, and the cumulative flood risk probability is 1.94%, the \( CVaR^n \) value in Scheme B1 is larger than that in Scheme A. In Scheme B2, the \( CVaR^n \) value over 20 years is equal to 33.05 c billion yuan at a 98% confidence level when considering the \( CVaR^n \) value and the flood risk probability as constraints, simultaneously. The average annual hydropower generation in Scheme B2 increases by 4.91% compared with that in the conventional FLWL scheme, whereas the cumulative flood risk probability is 1.939%.

The increment of FLWLs from 2020 to 2039 in Scheme B2 is smaller than that in Scheme B1 (in Table 2) when taking account of \( CVaR^n \) value for n years. However, Fig. 7 shows the \( CVaR^n \) values in Scheme B1 are larger than the threshold \( \beta^n \) in Scheme A in spite of both the flood risk probability \( R^n \) values in Scheme B1 and Scheme B2 satisfy the restriction determined by \( R^n \). Therefore, more possible flood damage loss would occur than under stationary conditions when adopting the optimal FLWL scheme of Scheme B1. Thus, it would be more reliable to take account of the \( CVaR^n \) value as a constraint when choosing the adaptive FLWL scheme.

The comparison results of the \( CVaR^n \) and the unconditional expected value are shown in Fig. 7. Both expected values in Scheme B1 and Scheme B2 are smaller than the upper limited values determined by Scheme A, whereas the \( CVaR^n \) values in Scheme B1 are larger than the threshold \( \beta^n \) in Scheme A. Thus, the \( CVaR^n \) value is stricter than the expected value when applying into the optimal adaptive FLWL under nonstationary conditions. The unconditional expected value is a special case of the \( CVaR^n \) value when the confidence level \( \alpha \) is infinitely close to 0 (The flood risk probability \( R \) is infinitely close to 100% can be deduced by relationship that \( \alpha + R = 1 \)). The higher the confidence level \( \alpha \), the larger the slope of linear function for the \( CVaR^n \) value with the FLWL, which could be derived by Fig. 4. Therefore, the slope of linear function for the expected value with the FLWL is not greater than that for the \( CVaR^n \) value with the FLWL. For the same increment (or decrement) of the expected value, the increment (or decrement) of the expected value is not greater than the \( CVaR^n \) value, and thus the \( CVaR^n \) value is more sensitive than the expected value when the FLWL changes.

Conclusions

This paper has incorporated \( CVaR^n \) in quantifying the flood damage loss by establishing the loss function, and the \( CVaR^n \) value can reflect the flood risk probability by choosing a suitable confidence level \( \alpha \). The larger the \( CVaR^n \) value, the larger the possible flood damage loss. Then, the \( CVaR^n \) value for n years is proposed to
represent the total possible flood damage loss over \( n \) years, and the \( CVaR^n \) value can be applied to reservoir flood control operation under nonstationary conditions.

The FLWL is the most significant parameter for the trade-off between activities of flood control and water conservation. When hydrologic records show nonstationarity in the form of increasing trends, the flood risk probability would be larger than that for stationary conditions; thus, it is necessary to choose the lower FLWL scheme to sacrifice power generation benefits in exchange for flood safety. When hydrologic records show decreasing trends, the flood risk probability would be smaller than that for stationary conditions, so the FLWL could be raised to gain more benefits. Therefore, it is essential to re-establish reservoir FLWL under nonstationary conditions when streamflow alters over time, whatever the change forms of nonstationary streamflow. The adaptive reservoir FLWL optimal model is derived to maximize the average annual hydropower generation without increasing flood risk probability and \( CVaR^n \) value.

The increment of FLWLs and the average annual hydropower generation over year 2020 to 2039 in Scheme B2 is slightly smaller than those in Scheme B1 when taking account of \( CVaR^n \) value. However, the \( CVaR^n \) value in Scheme B1 is beyond the threshold \( \beta_1 \), which is determined under stationary conditions, which means that more flood damage loss would occur than that of stationary conditions when adopting the optimal FLWL scheme of Scheme B1. Thus, it is more reliable to take account of the \( CVaR^n \) value when studying the adaptive FLWL optimization problem.

If reservoir streamflow reflects other forms of nonstationarity, such as increasing trend, the \( CVaR^n \) value under the nonstationary condition would be larger than that for stationary conditions. In Scheme B1, if the \( CVaR^n \) value for \( n \) years is greater than \( \beta_1 \) value for stationary conditions, the decrement of the \( CVaR^n \) value caused by the FLWL drawdown cannot compensate for the increment of the \( CVaR^n \) value induced by the increasing trend of inflow. Nevertheless, the more decrement of the reservoir FLWL will be done in Scheme B2 to guarantee that the \( CVaR^n \) value under nonstationary conditions is not beyond the threshold \( \beta_1 \). If the \( CVaR^n \) value for \( n \) years in Scheme B1 is less than \( \beta_1 \) value for stationary conditions, the optimal FLWL scheme gained from Scheme B1 would also be applicable for Scheme B2.

Thus, the application of the \( CVaR^n \) value to the adaptive reservoir FLWL optimal model demonstrated that the \( CVaR^n \) value is a more stringent constraint than the flood risk probability.

In this study, it is assumed that the unit cost of loss function is a constant, which should be further researched. The \( CVaR^n \) value for a planning time horizon is used as a constraint for the adaptive reservoir FLWL under the nonstationary condition in this study, and \( CVaR^n \) would also be of great use in the multiobjective optimization problems by considering the flood damage loss.

Acknowledgments

This study was supported by the National Key Research and Development Program (2016YFC0400097), the Excellent Young Scientist Foundation of NSFC (51422907), and the National Natural Science Foundation of China (51579180).

Supplemental Data

Figs. S1–S3 and Tables S1–S4 are available online in the ASCE Library (www.ascelibrary.org).

References


caption


