Evaluation of seasonal and spatial variations of lumped water balance model sensitivity to precipitation data errors

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Abstract

Sensitivity of hydrological models to input data errors have been reported in the literature for particular models on a single or a few catchments. A more important issue, i.e. how model’s response to input data error changes as the catchment conditions change has not been addressed previously. This study investigates the seasonal and spatial effects of precipitation data errors on the performance of conceptual hydrological models. For this study, a monthly conceptual water balance model, NOPEX-6, was applied to 26 catchments in the Mälaren basin in Central Sweden. Both systematic and random errors were considered. For the systematic errors, 5–15% of mean monthly precipitation values were added to the original precipitation to form the corrupted input scenarios. Random values were generated by Monte Carlo simulation and were assumed to be (1) independent between months, and (2) distributed according to a Gaussian law of zero mean and constant standard deviation that were taken as 5, 10, 15, 20, and 25% of the mean monthly standard deviation of precipitation. The results show that the response of the model parameters and model performance depends, among others, on the type of the error, the magnitude of the error, physical characteristics of the catchment, and the season of the year. In particular, the model appears less sensitive to the random error than to the systematic error. The catchments with smaller values of runoff coefficients were more influenced by input data errors than were the catchments with higher values. Dry months were more sensitive to precipitation errors than were wet months. Recalibration of the model with erroneous data compensated in part for the data errors by altering the model parameters.

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1. Introduction

Conceptual catchment models are common tools for calculating runoff dynamics and water-balance at various scales and regions, and are widely used in water resource management, climate change impact
studies, etc. For engineering applications, models have to fulfill two conditions: the data necessary for calibration have to be readily available, and calibration must be easy. The latter condition leads to the requirement that only a few parameters are used and the model should be parsimonious with respect to the number of parameters. In the past, a number of such models have been defined (Alley, 1984; Vandewiele et al., 1992; Servat and Dezetter, 1993; Singh, 1995; Singh and Woolhiser, 2002; Guo et al., 2002; Xu, 2002; Singh and Frevert, 2002a,b). Since computer models are not a perfect representation of the reality and hydrological data are not error free, the result is uncertain. There are four important error sources in hydrological modelling (Refsgaard and Storm, 1996):

(a) Uncertainties in input data (e.g. precipitation, evapotranspiration, temperature, antecedent moisture condition),

(b) Uncertainties in data used for calibration, i.e. data used for comparison with simulated output (e.g. stream flow observations),

(c) Uncertainties in model parameters (non-optimal parameter values), and

(d) Uncertainties due to an imperfect model structure.

Error sources (a) and (b) depend on the quality of data, whereas (c) and (d) are more model specific. The disagreement between observed and simulated outputs depends on all four-error sources. To obtain a good fit between observed and simulated outputs the model parameters usually have to be calibrated. Normally there are problems in determining correct parameter values and only error source (c) can be minimised by parameter calibration. It is important that changes in one source do not compensate for errors in another source.

Of the various discrepancies in the model outputs, input (e.g. precipitation, evaporation) errors are perhaps the most important (Singh and Woolhiser, 1976). Earlier studies (e.g. Xu and Vandewiele, 1994; Paturel et al., 1995) showed that the errors in evaporation data influence model predictions to a much less degree than do precipitation data errors. The precipitation rate may be incorrect because of sampling error, because of spatial nonuniformity of the precipitation and the small number of sampling points or because of bias in the amount of rain caught by the gages or both (Sutcliffe, 1966; Paturel et al., 1995; Kobold et al., 2003). Errors in areal precipitation have both deterministic and stochastic components, i.e. systematic and random errors. Due to their random nature, random errors in the input data will undoubtedly produce random errors in the output of an input and output modelling system, and earlier studies (e.g. Xu and Vandewiele, 1994; Paturel et al., 1995) have confirmed this. Systematic errors in the data, on the other hand, will be reflected as incorrect values in the parameters of the model. This is because systematic error in precipitation will cause bias in the water balance, which will cause systematic error in the model parameter values. When the model is used for independent simulation, the existence of systematic errors may be much more serious than the effects of random errors. To be able to interpret model results correctly, the model user needs to have information about possible errors, their distribution and the consequences of using information with errors.

Previous studies of the issue have been focused on the effect of errors in input data (both rain-gauge data and weather radar data) on the quality of the results obtained with rainfall-runoff models on a particular catchment or a small number of catchments (e.g. Singh and Woolhiser; 1976; Xu and Vandewiele, 1994; Paturel et al., 1995; Kobold et al., 2003; Hossain et al., 2004). These studies have shown that hydrological models, no matter how physically based or conceptual, are sensitive to input data errors. A common feature is that, in most cases, the model system amplifies the initial systematic error to an extent, which depends on the phases of the hydrograph, and another finding is that hydrological models are more sensitive to precipitation errors than to evaporation errors (Xu and Vandewiele, 1994; Paturel et al., 1995). However, because of the small number of catchments used in the previous studies it was not possible to generalise and quantify how the runoff response (model result) changes with the change of catchment conditions and seasons for a given input data error. To the best of our knowledge, such an important issue has not been reported in the literature, which constitutes the main focus of the present study. The existing great diversity in catchment conditions and seasonality warrants the present study.
The objectives of the study, therefore, are to (1) evaluate the effect of precipitation data errors on model parameter estimates and model simulation results, and (2) examine and quantify how the effects change with seasons and catchments. To perform the study, both systematic and random errors are studied separately. A conceptual, lumped water balance model is chosen to demonstrate the study and the study region is central Sweden where good quality data from 26 catchments are available. Both the seasonal and the spatial effects of the systematic and random errors in precipitation data on model prediction and model parameters are evaluated.

This paper is organised as follows: after this brief introduction, the study region and data are described in Section 2; methodology including the evaluation criteria, the model, the error scenarios and the evaluation procedure are presented in Section 3; the results are shown in Section 4; and in Section 5 the discussions and conclusions are presented.

2. Study region and data

To perform a regional study, a large number of catchments in a region, say more than 20, with good hydrological and basin data is needed in order to provide the results that are statistically meaningful. In this study an area located in the central part of Sweden—the basin of Lake Mälaren with a drainage area of about 30,000 km² is selected (Fig. 1). The main reason for choosing this region is that it has 30-gauged sub-catchments ranging in size from 6 to 4000 km² representing one of the largest catchments in Sweden. In particular, the available data consist of daily precipitation from 41 stations, and daily temperature data from 12 stations and all with the observation period of at least 10 years. Land-use data are also available for the corresponding catchments. Among the 30 sub-catchments, 26 of them are used in this study, since earlier investigations (e.g. Seibert, 1995) on the basin’s characteristics and data show that some sub-catchments might have error in the determination of water divides.

The landscape of the area is dominated by large lakes and plains separated by high undulating ridges, rich in faults. The geology is characterised by granites in the northeast, sedimentary gneisses in the south and leptites and hålleflintas in the northwest with some small granite-dominated areas. The northwest is mostly forested (69.3%), while meadow and grain cultivation is predominant in the south (25%). Seibert
Table 1
The mean monthly hydrological data (1981–1991) and land-use data of the study region

<table>
<thead>
<tr>
<th>Station</th>
<th>Abbr.</th>
<th>Code</th>
<th>Area (km²)</th>
<th>Mean (mm)</th>
<th>Mean a prec. (mm)</th>
<th>Mean evap. (mm)</th>
<th>Lake runoff (%)</th>
<th>Forest (%)</th>
<th>Open field (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Åkesta Kv.</td>
<td>Ak</td>
<td>2216</td>
<td>727</td>
<td>60.1</td>
<td>39.6</td>
<td>21.6</td>
<td>4.0</td>
<td>69.0</td>
<td>27.0</td>
</tr>
<tr>
<td>Akers Krut.</td>
<td>Ar</td>
<td>2249</td>
<td>214</td>
<td>60.3</td>
<td>43.3</td>
<td>17.6</td>
<td>5.2</td>
<td>66.3</td>
<td>28.5</td>
</tr>
<tr>
<td>Bergsh.</td>
<td>Be</td>
<td>2300</td>
<td>21.6</td>
<td>55.6</td>
<td>40.2</td>
<td>16.3</td>
<td>0.2</td>
<td>69.5</td>
<td>30.3</td>
</tr>
<tr>
<td>Berg</td>
<td>Bg</td>
<td>2218</td>
<td>36.5</td>
<td>63.9</td>
<td>43.0</td>
<td>22.2</td>
<td>0.0</td>
<td>71.4</td>
<td>28.6</td>
</tr>
<tr>
<td>Backa Ö.</td>
<td>Bo</td>
<td>1374</td>
<td>834</td>
<td>73.8</td>
<td>40.7</td>
<td>32.8</td>
<td>7.5</td>
<td>68.7</td>
<td>23.7</td>
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<tr>
<td>Bernsh.</td>
<td>Bs</td>
<td>1573</td>
<td>595</td>
<td>78.0</td>
<td>43.4</td>
<td>34.9</td>
<td>8.6</td>
<td>77.3</td>
<td>14.1</td>
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<td>Dalkarsh.</td>
<td>Dl</td>
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<td>1182</td>
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<td>42.4</td>
<td>35.2</td>
<td>7.5</td>
<td>74.6</td>
<td>17.9</td>
</tr>
<tr>
<td>Fellingbr.</td>
<td>Fb</td>
<td>2205</td>
<td>298</td>
<td>62.6</td>
<td>39.9</td>
<td>24.6</td>
<td>6.0</td>
<td>63.8</td>
<td>30.2</td>
</tr>
<tr>
<td>Finntorp.</td>
<td>Ft</td>
<td>2242</td>
<td>6.96</td>
<td>65.9</td>
<td>43.9</td>
<td>22.1</td>
<td>4.7</td>
<td>95.3</td>
<td>0.0</td>
</tr>
<tr>
<td>Gränvad</td>
<td>Gr</td>
<td>2217</td>
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<td>0.0</td>
<td>41.1</td>
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<tr>
<td>Härnevi</td>
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<tr>
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<td>9.5</td>
<td>80.9</td>
<td>9.7</td>
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<tr>
<td>Käfalla</td>
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<td>5.2</td>
</tr>
<tr>
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</tr>
<tr>
<td>Lurbob</td>
<td>Lu</td>
<td>2245</td>
<td>122</td>
<td>60.8</td>
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<td>25.2</td>
<td>0.3</td>
<td>68.2</td>
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<tr>
<td>Odensvibr.</td>
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<td>2221</td>
<td>110</td>
<td>63.6</td>
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<td>6.3</td>
<td>71.0</td>
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<tr>
<td>Ransta</td>
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<td>0.9</td>
<td>66.1</td>
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<tr>
<td>Rällsälv</td>
<td>Rs</td>
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<td>298</td>
<td>79.3</td>
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<td>38.4</td>
<td>7.4</td>
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<td>13.8</td>
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<tr>
<td>Sävja</td>
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<td>722</td>
<td>59.7</td>
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<td>19.5</td>
<td>2.0</td>
<td>64.0</td>
<td>34.0</td>
</tr>
<tr>
<td>Skräddart.</td>
<td>Sd</td>
<td>2222</td>
<td>17.7</td>
<td>66.7</td>
<td>41.6</td>
<td>25.3</td>
<td>2.5</td>
<td>96.1</td>
<td>1.4</td>
</tr>
<tr>
<td>Skällnora b</td>
<td>Sn</td>
<td>1843</td>
<td>58.5</td>
<td>55.0</td>
<td>39.9</td>
<td>16.2</td>
<td>10.4</td>
<td>44.5</td>
<td>45.1</td>
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<tr>
<td>Stabby b</td>
<td>St</td>
<td>1742</td>
<td>6.18</td>
<td>56.4</td>
<td>36.2</td>
<td>18.7</td>
<td>0.0</td>
<td>95.6</td>
<td>4.4</td>
</tr>
<tr>
<td>Tärnsjöb</td>
<td>Ta</td>
<td>2299</td>
<td>13.7</td>
<td>59.7</td>
<td>39.5</td>
<td>21.8</td>
<td>1.5</td>
<td>84.5</td>
<td>14.0</td>
</tr>
<tr>
<td>Ulva Kv.</td>
<td>Ul</td>
<td>2246</td>
<td>976</td>
<td>61.2</td>
<td>43.9</td>
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<td>0.3</td>
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<td>Vatholma</td>
<td>Va</td>
<td>2244</td>
<td>293</td>
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<td>4.8</td>
<td>71.0</td>
<td>24.2</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td>66.8</td>
<td>40.9</td>
<td>25.8</td>
<td>4.3</td>
<td>71.2</td>
<td>24.5</td>
</tr>
</tbody>
</table>

a Actual evapotranspiration calculated by the model.

b Catchments used for independent testing of the regression equations.
(1995) has shown that the distribution of soil type parallels the landuse, i.e. areas of forest generally consist of sandy soil, whereas agricultural areas consist of clay soil. The mean annual precipitation and discharge are 800 and 310 mm, respectively. Excluding Lake Mälaren itself, 4.3% of the area is lakes (Table 1).

3. Methodology

In this study, a monthly conceptual water balance model has been used to investigate seasonal and spatial effects of input-precipitation error on the model performance. Both systematic and random errors were considered. In the following sub-sections the water balance model, model fitting and evaluation criteria, sensitivity method, data error scenarios and the perform procedure are presented.

3.1. The water balance model

NOPEX-6 (Xu et al., 1996), a special version of the WASMOD system (Xu, 2002) is a typical water and snow balance model, developed for water balance investigations for the NOPEX (Halldin and Gryning, 1999) area and Nordic region. Earlier versions of the model have been applied to over 100 catchments in more than 20 countries (Vandewiele et al., 1992; Vandewiele and Ni-Lar-Win, 1998; Müller-Wohlfeil et al., 2003). The principal equations of NOPEX-6 are presented in Table 2. The model calculation step is chosen as a month, since for national and regional water resources assessment in developing countries, hydrological data on monthly time scale are more commonly available. The input data to the model are monthly values of areal precipitation, long-term monthly average potential evapotranspiration and air temperature. Precipitation \( p_t \) is first divided into rainfall \( r_t \) and snowfall \( s_t \) by a temperature index function; at the end of each month snowfall is added to the snowpack \( s_{pt} \), of which a fraction \( m_t \) melts and contributes to the soil moisture storage \( sm_t \). Parameters \( a_1 \) and \( a_2 \) are threshold temperatures, which determine the form of precipitation and the rate of snowmelt. Before rainfall contributes to the soil storage as ‘active’ rainfall, a small part is subtracted and added to the loss due to evapotranspiration. The soil water storage contributes to evapotranspiration \( e_{pt} \), to the fast component of flow \( f_t \) and to slow flow \( s_t \). Parameter \( a_5 \), used to convert long-term average monthly values to actual values of monthly potential evapotranspiration, can be eliminated from the model if potential evapotranspiration data are available or calculated using other methods. Parameter \( a_4 \) determines the actual evapotranspiration that is an increasing function of potential evapotranspiration and available water. The bigger the values for \( a_4 \), the greater the evaporation losses at all moisture storage state. The slow flow parameter \( a_5 \) controls the proportion of runoff that appears as ‘base flow’; higher values of \( a_5 \) produce a greater proportion of

Table 2
Principal equations of the monthly snow and water balance model NOPEX-6

<table>
<thead>
<tr>
<th>Term</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow fall</td>
<td>( s_t = p_t \left{1 - \exp\left[-(c_t - a_1)/(a_1 - a_2)\right]\right}^+ )</td>
</tr>
<tr>
<td>Rainfall</td>
<td>( r_t = p_t - s_t )</td>
</tr>
<tr>
<td>Snow storage</td>
<td>( s_{pt} = s_{pt-1} + s_t - m_t )</td>
</tr>
<tr>
<td>Snowmelt</td>
<td>( m_t = s_{pt} \left{1 - \exp\left[\left(c_t - a_2\right)/(a_1 - a_2)\right]\right}^+ )</td>
</tr>
<tr>
<td>Potential evap</td>
<td>( ep_t = (1 + a_5(c_t - c_{w}) - a_5)ep_{m} )</td>
</tr>
<tr>
<td>Actual evap</td>
<td>( e_t = \min\left[w_t \left(1 - e^{a_{ep}p_t}\right), ep_t\right])</td>
</tr>
<tr>
<td>Slow flow</td>
<td>( b_t = a_5 \left(sm_{t-1}\right)^2 )</td>
</tr>
<tr>
<td>Fast flow equation</td>
<td>( f_t = a_6 \left(sm_{t-1}\right)^2 (m_t + n_t) )</td>
</tr>
<tr>
<td>Total computed runoff</td>
<td>( d_t = b_t + f_t )</td>
</tr>
<tr>
<td>Water balance equation</td>
<td>( sm_t = sm_{t-1} + r_t + m_t - e_t - d_t )</td>
</tr>
</tbody>
</table>

where: \( w_t = r_t + sm_{t-1} \) is the available water; \( sm_{t-1} = \max(sm_{t-1}, 0) \) is the available storage; \( n_t = r_t - ep_t(1 - e^{-a_{ep}p_t}) \) is the active rainfall; \( p_t \) and \( c_t \) are monthly precipitation and air temperature, respectively; \( ep_{m} \) and \( c_{w} \) are long-term monthly average potential evapotranspiration and air temperature, respectively; \( a_i (i = 1, 2, ..., 6) \) are model parameters with \( a_1 \geq a_2, 0 \leq a_4 \leq 1, a_5 \geq 0 \) and \( a_6 \geq 0 \).
‘base flow’. Values are expected to be higher in forest areas than in open fields and in sandy rather than clayey soils. The fast flow parameter $a_6$ increases with the degree of urbanisation, average basin slope and drainage density; lower values are likely for catchments dominated by forest.

### 3.2. Model fitting and evaluation criteria

For estimation of model parameters, an automatic optimization method was used (Xu et al., 1996). There are different objective functions in use in the literature, depending on the hypotheses postulated related to the nature of the residual, defined as the difference between calibrated and observed runoff. It is common to suppose that

$$ u_t = q_t - d_t $$  \hspace{1cm} (1)

where $q_t$ is the observed monthly river flow and $d_t$ is the computed flow. For statistical analysis, it is convenient to have homoscedastic deviations (i.e. common variance $\sigma^2$ for all deviations). If they are not, a transformation is usually needed. Previous studies (Vandewiele et al., 1992) show that taking a square root transformation is a good hypothesis, i.e.

$$ u_t = \sqrt{q_t} - \sqrt{d_t} $$  \hspace{1cm} (2)

with

$$ u_t \sim N(0, \sigma^2) $$  \hspace{1cm} (3)

i.e. $u_t$ is normally distributed with zero expectation and common variance $\sigma^2$, the so-called model variance. Moreover, deviations are assumed to be independent, i.e. for all $t$

$$ \text{EXPEC}(u_t, u_{t-1}) = 0 $$  \hspace{1cm} (4)

where EXPEC is the expectation operator.

The independence of $u_t$ has been discussed by Vandewiele et al. (1992), and it turned out to be a good hypothesis as compared with other transformations. In more recent studies of the model (Xu, 2001; Engeland et al., 2005), the homogeneity, normality and independence of the model residual series $u_t$ are tested by using both parametric and non-parametric methods. It turns out that the above assumptions (Eqs. (2)–(4)) are valid, i.e. the residuals are uncorrelated, normally distributed and have a constant variance (homogeneity).

For estimation of parameters, the maximum likelihood method was used. Because of the hypotheses in Eqs. (2)–(4), maximising the log-likelihood with respect to the model parameters is equivalent to minimising the sum of squares:

$$ SSQ = \sum (\sqrt{q_t} - \sqrt{d_t})^2 $$  \hspace{1cm} (5)

where the sum is extended over all months for which output $q_t$ as well as input data $p_t$ and $ep_t$ are available.

The quality of minimisation was checked by plotting SSQ versus each of the model parameters. In that way, it was possible to see whether a global minimum was reached. This is done for every model-basin combinations. Illustrations of this procedure can be found in Xu (2001).

In order to express the model performance, different quality measures, i.e. evaluation criteria, can be used. Evidently the model standard deviation $\sigma$, as given by Eq. (6) is an inverse measure of the quality of model performance.

$$ \sigma = \sqrt{\frac{\text{minimum SSQ}}{N-K}} $$  \hspace{1cm} (6)

where $N$ is the number of terms in the Eq. (2), and $K$ is the number of model parameters. The half width of a 95% confidence interval for $\sigma$ is approximately (Xu, 2001):

$$ \text{HWCI}(\sigma) = \frac{1.96\sigma}{\sqrt{2(N-K)}} $$  \hspace{1cm} (7)

The second criterion is the Nash and Sutcliffe (1970) efficiency (EF), which is dimensionless comparing the residual variance with the initial variance and is widely used in the literature.

$$ \text{EF} = 1 - \frac{U}{U_0} $$  \hspace{1cm} (8)

with $U_0 = \sum_{t=1}^{N} (q_t - \bar{q})^2$, $U = \sum_{t=1}^{N} (q_t - d_t)^2$, $q = \frac{1}{N} \sum_{t=1}^{N} q_t$, where $q_t$ and $d_t$ are the same as above.

The above two criteria measure, in different ways, how good the calculated hydrograph mimics the observed one. In order to have a measure of the bias of the total runoff volume, the third criterion is used in the study, which calculates the relative difference in
the total runoff volume and is expressed as:

\[ ER = \frac{\left( \sum_{i=1}^{n} d_i - \sum_{i=1}^{n} q_i \right)}{\sum_{i=1}^{n} q_i} \% \]  \hspace{1cm} (9)

3.3. Sensitivity analysis methods

Two sensitivity analysis methods are commonly used in the literature. Some authors used a mathematically defined ‘sensitivity coefficient’ method (McCuen, 1974; Beven, 1979; Rana and Katerji, 1998; Anderton et al., 2002). That is a coefficient determined by a certain sensitivity formulation, i.e. calculating partial derivatives of the model output variable against the input variables. The second method, a simple but practical way of sensitivity analysis is to calculate and plot the relative changes of an input variable (precipitation in our case) against the resultant relative change of the output variables (model parameter values or runoff in our case) as a curve (i.e. the ‘sensitivity curve’), then the corresponding relative change of the outcome can easily be read from the sensitivity curve for a certain relative change of the variable. This method has been used by many authors (Paturel et al., 1995; Xu and Vandewiele, 1994; Singh and Xu, 1997; Goyal, 2004). Due to the high non-linear nature of hydrological models, the simple sensitivity curve method is used in this study.

3.4. Data error scenarios

3.4.1. Systematic errors

Systematic errors (i.e. bias in the mean) may be present due to many factors (Eriksson, 1983; Sevruk, 1996), such as wind, placement of the instrument too near to an obstruction, and consistent misreading of a level by the operator might lead to consistent overestimates, etc. A source of bias may also be due to the data period used being too short and this happened to be a dry/wet period. Moreover, low spatial density (sparse coverage), and topographic effects may also introduce a systematic error. Earlier studies (e.g. Eriksson, 1983) show that the mean annual systematic error in the observed precipitation in Sweden is about 15%. The systematic errors considered in this study were successively taken equal to 0, ±5, ±10, and ±15% of the observed mean monthly precipitation. Such error scenarios are also consistent with the range of GCMs predicted precipitation changes in the region in the next century.

3.4.2. Random errors

Random errors are assumed to be (1) independent between months, (2) distributed according to a Gaussian law of zero mean and constant standard deviations that are 5, 10, 15, 20, and 25% of the standard deviation of the mean monthly precipitation.

3.5. The study procedure

The study was performed in the following steps:

First, the model was calibrated on all 26 catchments using the original input data and discharge data, the resulting model parameters were considered as ‘reference’ values. Second, the model was recalibrated using the corrupted precipitation data for all the catchments and the resulting model parameters were considered as ‘incorrect’ and compared with the reference values. Third, the model was run on all the catchments again using (1) the reference parameter values to simulate the discharge with corrupted precipitation data, and (2) the incorrect parameter values obtained in the second step to simulate the discharge with corrupted precipitation data. The resulting two discharge series were compared with the observed discharge series so that how the recalibration can compensate for the errors induced into the precipitation data could be examined.

4. Results and discussion

Following the above procedure, many error scenarios and catchment combinations were obtained. The results are presented in the following subsections. For illustrative purposes and for the sake of discussion, in some figures, the results from one or more catchments are shown and in other figures, the results obtained from the average of all catchments are shown.
4.1. Model calibration

In order to perform the regional sensitivity study of the model results to the precipitation data errors, the model was first optimised on the 26 catchments in the region using the observed hydrological data. In summary, the Nash and Sutcliffe efficiency (EF) and the relative error (ER) averaged over the 26 catchments are 0.8 and 0.51%, respectively. For illustrative purposes, the monthly observed and calculated runoff values averaged over the 26 catchments are shown in Fig. 2. It is seen that the model was capable of simulating the historical runoff series for the study region.

4.2. Sensitivity of model parameters to systematic errors in precipitation

The average influence of systematic errors on model parameter values is shown in Fig. 3 (left graph). The figure shows that (1) the least changes are obtained for parameters \( a_1 \) and \( a_2 \) which means that systematic errors in precipitation do not have a significant influence on parameters that determine the percentage of precipitation that falls as snow and the rate of snowmelt. (2) The most sensitive parameters to the errors in precipitation are, as expected, parameters \( a_4, a_5 \) and \( a_6 \) which determine the rate of actual evapotranspiration, slow flow and
fast flow, respectively. (3) A positive change in parameters \(a_5\) and \(a_6\) when there is a systematic underestimation in precipitation means that in such cases parameters \(a_5\) and \(a_6\) have to increase in order to minimise the values in the objective function (Eq. 2).

(4) While a rapid increase in parameter \(a_4\), as the systematic overestimation increases, means that a bigger portion of the extra precipitation added into the input data goes to the actual evapotranspiration rather than to runoff in the model, because the calculated runoff is controlled by the objective function. This is why the change of \(a_5\) and \(a_6\) is much smaller for precipitation with overestimation errors than with underestimation errors.

### 4.3. Sensitivity of model parameters to random errors in precipitation

As compared to systematic errors, random errors in precipitation have little effect on parameter values (Fig. 3 (right graph)). This figure reveals that (1) the change of parameter values affected by random errors in precipitation is also random, (2) the variation of percentage changes in parameters increases as the random precipitation errors increase, and (3) the random errors in precipitation have less effect on model parameters than do systematic errors.

### 4.4. Sensitivity of simulated runoff to systematic errors in precipitation

#### 4.4.1. Seasonal variation of runoff relative error

To examine the seasonal variation of simulation errors, systematic precipitation errors in a range of \(-15\) to \(15\%\) were introduced in the observed precipitation data. The study was done for all the catchments and for illustrative purposes the result obtained from catchment AK is shown for four selected months in Fig. 4, which can be generalized for all other catchment areas. One month is selected from each season of the year. It is seen from the figure that (1) the same precipitation error will cause different discharge errors depending on the season, (2) the same amount of overestimation error in precipitation causes slightly more discharge errors than does the underestimation error. For example, for an overestimation of \(15\%\), the runoff error varies from 25 to 55\% during the year. In January, because the precipitation is in the form of snow, the resulting discharge error is among the smallest. April is the month with a higher proportion of runoff and the relative runoff error for April is smaller as compared to summer and autumn months. (3) The slope of the relative error curve for each month is greater than one;

![Fig. 4. The monthly variation of runoff error owing to systematic precipitation errors. The 4 months are selected to represent the four seasons of the year](image-url)
it means that for all months the error in precipitation translates to a greater error in runoff.

The evolution of relative runoff errors was then studied continuously over a year for all years. For illustrative purposes, the monthly response of runoff to a systematic rainfall error of 10% is shown in Fig. 5 for catchment AK. This figure shows that (1) although the relative runoff errors are different from year to year, the shape of the relative runoff error curves remains the same in every case. (2) The largest relative errors are found during summer dry months and during the water rising phase in earlier autumn, which is a sensitive period for this type of rainfall-runoff algorithm. Smallest relative errors are found during winter months. (3) The result of relative runoff errors for underestimation errors in the precipitation data is similar as for overestimation except the maximum relative error is about 5–10% smaller.

4.4.2. Regional variation of runoff relative errors

Runoff coefficient, \( \text{RC} = \text{runoff/precipitation} \), is a result of combined factors of climate, geology and landuse, etc. which can be used as a physical parameter of the catchment. The effect of the same precipitation error on runoff for catchments with different runoff coefficients is shown in Fig. 6. Four catchments are chosen to show a relatively large difference in runoff coefficient in the region. It is seen that (1) the runoff response to the precipitation errors is dependent on the runoff coefficient of the catchment; the smaller the runoff coefficient, the bigger the relative runoff error for a given precipitation error. (2) The overestimation of precipitation produces a slightly bigger runoff relative error than does underestimation. For precipitation overestimation of 15%, the resulting runoff relative error between the catchments varies from 28 to 42%. (3) The slope of each line is greater than one, which means that the precipitation relative error is amplified by the model resulting in a greater relative error in the simulated runoff.

A more general comparison that combines all the scenarios and catchments is shown in Fig. 7. It is seen that (1) the largest relative change in runoff occurs for catchments with the lowest runoff coefficient and the runoff errors decrease as the runoff coefficient increases. (2) There exists a good correlation between the runoff relative error and runoff coefficient for both over- and under-estimation errors in precipitation. (3) A positive systematic error in precipitation produces a slightly bigger runoff error than a negative systematic error does.

4.5. Sensitivity of model quality measures to systematic and random errors in precipitation

The influence of systematic and random errors in precipitation data on model quality as measured by the model standard deviation, Nash and Sutcliffe efficiency and relative water balance error is shown in Fig. 8a–c, respectively. Fig. 8 shows the average results calculated from all catchments. Individual catchment gives a similar picture (not shown), but as expected from the discussion of the previous subsection, the model quality changes (adversely) slightly more for catchment with lower runoff coefficient or for season with lower flow. Fig. 8a shows that (1) the model standard deviation (an inverse measure of model quality, see Eq. (6)) increases faster for systematic errors than for random errors. For a 10% systematic error the model standard deviation changes significantly, whereas the model standard deviation changes significantly for 25% of the random error. (2) For systematic error
overestimation affects the model standard deviation more than does underestimation. The influence on the model standard deviation became significant when overestimation of precipitation went up to 10%. The same conclusion can be drawn from Fig. 8b, the only difference with Fig. 8a is that when model standard deviation increases the Nash and Sutcliffe efficiency (EF) decreases as the value of EF is proportional to the model quality which the former is an inverse measure of model quality. In Fig. 8c it is seen that the random precipitation error has almost no influence on the water balance error; this may be explained by the fact that random errors follow a normal zero mean distribution. The slight underestimation in runoff is perhaps due to the model imperfection error. The systematic precipitation error affects the simulated runoff remarkably, especially overestimation errors.

4.6. Sensitivity of simulated runoff to systematic errors in precipitation with and without recalibration

In order to investigate how much the calibration method in this kind of model can compensate for input data errors, the following studies were performed. Calibrate the model on original data and the calibrated runoff series is considered as a ‘reference’ data. Then, a systematic positive error of 10% is applied to precipitation. The model was run by using the calibrated parameter values and the corrupted precipitation data to produce a runoff series which is called ‘non-calibrated’ data. The model was run by using the calibrated parameter values and the corrupted precipitation data to produce a runoff series which is called ‘non-calibrated’ data. The model was calibrated with the corrupted precipitation data and the resulting runoff series is named as ‘calibrated’ data. For illustrative purpose, the mean monthly reference, non-calibrated and calibrated runoff series are plotted in Fig. 9 for catchment AK. It is seen that the calibrated runoff is closer to the reference runoff than the non-calibrated runoff with the same amount of
error being added to precipitation. This means that calibration of the model also compensates for input data errors and gives a better fit during the calibration period. However, it should be kept in mind that the calibration compensations are only valid during the calibration period and can result in bad simulation results in another range of conditions since the parameter values are incorrect.

5. Summary and conclusions

The contributions of the study are twofold. First, it confirms some results reported in the literature, such as (1) the resulting runoff relative error is much greater for dry months than for wet months, (2) the hydrological model amplifies the initial error for both over- and under-estimation of precipitation, (3) overestimation of precipitation affects runoff simulations more than does underestimation, and (4) recalibration of the model with corrupted input data can compensate for input data errors to some extent. The recalibrated runoff series is closer to the reference runoff than is the non-calibrated depth, although the input data are the same. Second, it reports some new findings, such as, (1) catchments with low runoff coefficients show a much greater relative error in runoff simulations than catchments with high runoff coefficients, (2) the model quality is significantly affected by the systematic precipitation data error, while random error in precipitation affects the model quality to a much less extent, (3) systematic error affects the model parameter values systematically, while random error affects model parameters randomly but in a much smaller range as compared with systematic error. As random error increases, parameter values become unstable.

The main conclusion of the study is that the response of the model parameters and model
performance depend, among others, on the type of error, the magnitude of error, physical characteristics of the catchment, and the season of the year. Any conclusion drawn from a sensitivity study performed on a particular catchment represents only a result of a case study and cannot be used for operational purposes.

It should be kept in mind that although the results of the study reveal a big range of model parameter and runoff response to a given precipitation error, this study is performed only in a relatively small region with similar climate. It is anticipated that the range of model response to a given precipitation error will be much greater when more catchments and regions are tested.

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References


