Modal Logic over Higher Dimensional Automata
(extended abstract)*

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Higher dimensional automata (HDA) are a model of concurrency [5, 8] that can express most of the traditional partial order models like Mazurkiewicz traces, pomsets, event structures, or Petri nets. Modal logics, interpreted over Kripke structures, are the logics for reasoning about sequential behavior and interleaved concurrency. Modal logic is a well behaved subset of first-order logic; many variants of modal logic are decidable. However, there are no modal-like logics for the more expressive HDA models. We introduce and develop a modal logic over HDAs which incorporates two modalities for reasoning about “during” and “after”. This general higher dimensional modal logic (HDML) is shown to be decidable and we provide a complete axiomatic system for it.

With higher dimensional automata concurrent systems can be modeled at different levels of abstraction, not only as all possible interleavings of their concurrent actions. HDAs can model concurrent systems at any granularity level and make no assumptions about the durations of the actions. Moreover, HDAs are not constrained to only before-after modeling, but represent also the “during”.

Work has been done on defining temporal logics over Mazurkiewicz traces [4], and strong results like decidability and expressive completeness are known [7]. For general partial orders, temporal logics usually become undecidable [1]. For the more expressive event structures there are fewer works on modal logics.

There is hardly any work on logics for higher dimensional automata and, as far as we know, there is no work on modal logics for HDAs. In practice, modal logics, like temporal logics or dynamic logics, are desirable because they are generally decidable, as opposed to full first-order logic, which is undecidable.

That is why we introduce and develop a logic in the style of standard modal logic which has HDAs as models, hence, the name higher dimensional modal logic (HDML). This is our language for talking about general models of concurrent systems. We prove that HDML is decidable using an extension of the filtration technique. We associate to HDML an axiomatic system which is proven to be sound and complete for HDAs. The proof of completeness is rather involved and is based on a constructive method of building the canonical model for a consistent formula. The construction starts with a minimal model and gradually enlarges it using two constructions (for lifting and enriching the model) that may be associated to the two different diamond operators of HDML.

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HDML contrasts with standard temporal/modal logics in the fact that HDML can reason about what holds “during” the concurrent execution of some events. The related logic of [3] for distributed transition systems, has the same standard style of reasoning about what holds “after” some concurrent events have finished executing. As we show in the extended version, the “after” logics can be encoded in HDML, hence also the logic of [3].

Another purpose of our work is to provide a general framework for reasoning about concurrent systems at any level of abstraction and granularity, accounting also for choices and independence of actions. Thus, in the extended version of this abstract, significant work is put into showing (also through examples) that studying HDML, and particular variants of it, is fruitful for analyzing concurrent systems and their logics. In this respect we study variants of higher dimensional modal logic inspired by temporal logic and dynamic logic. We add to the basic language an Until operator and show how this variant of HDML, when interpreted over the class of HDAs corresponding to Kripke structures, can be particularized to LTL. A second variant decorates the HDML modalities with labels. This multi-modal variant of HDML together with the Until operator, when interpreted over the class of HDAs that encodes Mazurkiewicz traces, becomes LTL [7] (the linear time temporal logic over Mazurkiewicz traces).

Regarding expressiveness we are currently looking for the kind of bisimulation that is characterized by HDML. Standard bisimulations for concurrent systems like ST-bisimulation or split-bisimulation do not apply because of the during modality and the “during” behavior of HDML.

As future work we are investigating a tableaux system for HDML. We are also trying to understand better the relation of HDML with other logics for weaker models of concurrency like with the modal logic of [2] for event structures or other logics for Mazurkiewicz traces. Particularly interesting is how our results relate to the undecidability results of [1].

References