Instability of the Vortex Matter in Anisotropic Superconductors


All-Russian Electrical Engineering Institute, 12 Krasnokazarmennaya Street, 111250 Moscow, Russia
*Department of Physics, University of Oslo, P.O. Box 1048, Blindern, 0316 Oslo 3, Norway,
**Institute for Theoretical and Applied Electrodynamics RAS, 13/19 Izhorskaya Str., 125412 Moscow, Russia
***Institute for Radiophysics and Electronics NANU, 12 Proskura Str., 61085 Kharkov, Ukraine

A theory of the macroturbulent instability in the system containing vortices of opposite directions (vortices and antivortices) in hard superconductors is proposed. The origin of the instability is connected with the anisotropy of the current capability in the sample plane. The anisotropy results in the appearance of tangential discontinuity of the hydrodynamic velocity of vortex and antivortex motion near the front of magnetization reversal. The examination is performed on the basis of the anisotropic power-law current-voltage characteristics. The dispersion equation for the dependence of the instability increment on the wave number of perturbation is obtained, solved, and analyzed analytically and numerically. The instability is shown to be observed even at relatively weak anisotropy. The physical nature of the macroturbulent instability in the vortex matter in YBCO superconductors is verified by means of magnetooptic study of the instability in a single crystal prepared specially for this purpose. The instability develops near those sample edges where the oppositely directed flow of vortices and antivortices, guided by twin boundaries, is characterized by the discontinuity of the tangential component of the hydrodynamic velocity. This fact directly indicates that the macroturbulent instability is analogous to the instability of fluid flow at a surface of a tangential velocity discontinuity in classical hydrodynamics, and is related to the anisotropic flux motion in the superconductor.

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1. INTRODUCTION

Magnetic flux dynamics in type-II superconductors has been extensively studied since the end of the 50-s, starting with the pioneering work by A. A. Abrikosov. Primary attention was given to hard superconductors, whose magnetic characteristics are defined by the presence of vortex pinning centers. The main features characterizing the nonuniform penetration of magnetic flux into such systems were revealed and studied; various theoretical models of electrodynamic processes in superconductors were suggested. An avalanche of new research activity in this field was triggered by the discovery of high-$T_c$ superconductivity (HTS), which brought into play thermal fluctuations and the strong anisotropy of superconductors. Different types of phase transitions (melting, glass state transitions, etc.) in the flux line lattice (FLL) have been discovered, studied, and explained. Many of the newly obtained results are described in comprehensive review papers $^{1,2}$.

The use of high-resolution magneto-optical (MO) technique enabled an in-depth study of the dynamics of magnetic flux in superconductors. Among the most important features revealed by means of this method are the flux structures behaving strongly irregular both in time and space. Such structures arise usually due to the development of characteristic instabilities such as macroturbulence in 1-2-3 systems $^{3–5}$ and dendrite instability in magnesium deboride $^6$. Surprisingly, these dramatic instabilities in FLL have so far been investigated to a relatively small extent.

Perhaps, the macroturbulence is the most interesting phenomenon observed in the dynamics of the magnetic flux in HTS. It appears like a turbulentization of the FLL motion near the front of magnetization reversal that separates regions of vortices of opposite directions (vortices and antivortices). Note that the macroturbulence was only revealed in single-crystal samples of the 1–2–3 system. Its essence is as follows. When magnetic flux is trapped in a superconductor and a moderate field of the reverse direction is subsequently applied, a boundary of zero flux density will separate the regions containing vortices and antivortices. For definiteness, we apply the term “antivortices” to the vortices whose direction coincides with that of the external magnetic field, and the term “vortices”, to the vortices which were originally present in the sample, prior to switching on the magnetic field of negative sign. At some range of magnetic fields and temperatures, this flux-antiflux distribution becomes unstable. A disordered motion of magnetic flux arises at the front of magnetization reversal, which resembles a turbulent fluid flow. This process rapidly develops in time and is accompanied by the formation of fingers via which the antivortices penetrate into the region occupied by the vortices. In other words, the front of magnetization reversal
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takes on an irregular shape. The annihilation of vortices and antivortices occurs at the front, and the process of macroturbulence is soon terminated after a complete disappearance of the vortices. This pattern of penetration of the magnetic flux differs qualitatively from the steady-state slow motion of the front of magnetization upon initial turning on of the magnetic field, when vortices of only one direction are present in the sample.

The described instability cannot be understood within the framework of the critical state model or conventional models for the thermoactivated flux relaxation.

An explanation of the macroturbulence should focus on the experimental fact that the instability was reported for YBa$_2$Cu$_3$O$_{7−δ}$ and other 1–2–3 single crystals only, which are characterized by the anisotropy in the basal ab plane. This anisotropy can be related to a specific crystallographic structure of these HTS and, in particular, to twin boundaries.

An approach to understanding the mechanism of the macroturbulence was elaborated in Refs. 8,9 taking into account of the specific features of the flux motion in the anisotropic superconductors. The anisotropy gives rise to the motion of the flux lines at some angle with respect to the Lorentz force direction. For example, in the presence of twin boundaries, vortices and antivortices move preferably along these guiding boundaries (the so-called guiding effect). In general, the angle between the twins and the crystal grains is around 45°. As a result, the flux lines move at some angle with respect to the magnetization reversal front. In our opinion, it is exactly this circumstance that is of paramount importance to ascertain the nature of macroturbulence. The vortices and antivortices are forced to move towards each other in such a way that the tangential component of their velocity becomes discontinuous at the flux-antiflux interface. According to a classical paper of Helmholtz, a stationary hydrodynamical flow can be unstable and turbulent under such conditions. It was shown that a purely hydrodynamic approach which takes into account the anisotropic viscosity of the flux flow can indeed provide the basis for an understanding of the macroturbulence. In particular, the macroturbulence is usually observed within a rather narrow temperature window, typically 40–50 K. Here we consider the more realistic approach when the electric properties of a superconductor are described by non-linear $I − V$ characteristics. We study theoretically the instability of the magnetization reversal front under the assumption that the current-voltage characteristic is a power-law function with an exponent $m \geq 1$. 


2. THEORETICAL MODEL

Consider an infinite superconducting plate of thickness 2d in the external magnetic field \( H \) directed parallel to the sample surface along the \( z \)-axis. The \( x \)-axis is perpendicular to the plate, and the \( x \)-axis origin, \( x = 0 \), is placed in the plate center. Let magnetic field \( H \) first increase to the extent that the magnetic flux in the form of vortices completely fills the sample. Further, let this field decrease, pass through zero, and assume some negative value whose modulus is likewise higher than \( H_{c1} \). Vortices with opposite magnetic flux directions (antivortices) then penetrate into the surface regions of the plate on both its sides. It is clear from the problem symmetry that it suffices to consider one (e.g., the right, \( 0 < x < d \) ) half of the sample. The geometry of the problem is schematically shown in Fig. 1. Thermal activation causes slow magnetic flux flow. The annihilation of vortices and antivortices at the \( x = x_0 \) boundary separating the regions of their existence (see Fig. 1) results in additional penetration of antivortices. As a result, the total number of vortices in the center of the sample decreases, and the \( x = x_0 \) boundary slowly moves at rate \( U \) deeper into the sample (Fig. 1). Let us denote the densities of vortices and antivortices by \( N_1(x) \) and \( N_2(x) \), respectively. The relation between vortex density \( N_\alpha(x, y) \) (\( \alpha = 1, 2 \)) and magnetic induction \( B(x, y) \) in the corresponding superconductor region is

![Fig. 1. Flux distribution in one half of an infinite slab (\(|x| \leq d\)) containing trapped vortices of density \( N_1(x) \) in the central region \(|x| \leq x_0\), and antivortices of density \( N_2(x) \) penetrating from the outside.](image-url)
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obvious,

\[ N_\alpha(x, y) = s_\alpha B(x, y)/\Phi_0, \]  

(1)

where \( s_1 = 1 \), and \( s_2 = -1 \). The densities of vortices and antivortices should satisfy the continuity equation

\[ \frac{\partial N_\alpha}{\partial t} + \text{div}(N_\alpha \vec{V}_\alpha) = 0, \]  

(2)

where \( \vec{V}_\alpha \) denotes the hydrodynamic velocities of the vortices and antivortices.

In the hydrodynamic approximation, there are two equivalent approaches to find the vortex and antivortex velocities. The usual dynamic approach, where \( \vec{V}_\alpha \) is defined by the viscosity equation for FLL, which takes account of the Lorentz force, was applied in Refs. 8,9. The alternative approach implies the use of the \( I - V \) characteristics,

\[ \vec{J} = \vec{J}(\vec{E}), \]  

(3)

where \( \vec{J} \) and \( \vec{E} \) are the macroscopic current density and the electric field. We operate with the latter method in this paper. The velocities and electric field are interrelated by the usual equation,

\[ \vec{E} = \frac{1}{c} \vec{V} \times \vec{B}. \]  

(4)

Using Eq. (1), we can rewrite Eq. (4) in the form,

\[ E_x = -\frac{N_\alpha s_\alpha \Phi_0}{c} V_{ay}, \quad E_y = \frac{N_\alpha s_\alpha \Phi_0}{c} V_{ax}. \]  

(5)

We suppose that the components of the current density vector along these directions are the odd functions of the electric field and for positive \( E \) can be presented as

\[ J_X = \frac{1}{\varepsilon} J_c \left( \frac{E_X}{E_0} \right)^{1/m}, \quad J_Y = J_c \left( \frac{E_Y}{E_0} \right)^{1/m}, \]  

(6)

where \( J_c \) is the critical current density along the \( Y \) axis, \( m > 1 \) is the corresponding exponent, \( \varepsilon < 1 \) is the anisotropy parameter for the critical current components and the value of \( J_c \) is defined as the current density \( J_Y \) at \( E = E_0 \) (usually, \( E_0 \) is accepted as 1 \( \mu \)V/cm). To simplify the model and to underline the main physical idea, we assume the same exponent \( m \) for both current components in Eq. (6).

As was mentioned above, the anisotropy in \( ab \) plane arises in 1–2–3 single crystals mainly due to the existence of twin boundaries. In a real

In the experimental situation, these boundaries make angles of about 45° with the axes $x$ and $y$. Therefore, we assume the angle between two coordinate systems, $\mathbf{xy}$ and $\mathbf{XY}$, to be equal to 45°. Now, using Eqs. (4), (5) and the Maxwell equation $\nabla \times \mathbf{H} = 4\pi \mathbf{J}/c$ we can derive the equations interrelating the vortex density and velocity components,

$$\frac{\partial N_\alpha}{\partial x} - \frac{\partial N_\alpha}{\partial y} = \frac{4\pi \sqrt{2} J_c}{c \Phi_0} \left[ \frac{N_\alpha \Phi_0}{c E_0 \sqrt{2}} (-V_{x\alpha} + V_{y\alpha}) \right]^{1/m}, \quad (7)$$

$$\frac{\partial N_\alpha}{\partial x} + \frac{\partial N_\alpha}{\partial y} = -\frac{4\pi \sqrt{2} J_c}{c \Phi_0} \left[ \frac{N_\alpha \Phi_0}{c E_0 \sqrt{2}} (V_{x\alpha} + V_{y\alpha}) \right]^{1/m}.$$

To solve the problem, we must formulate the boundary conditions at the sample surface, as well as at the interface between the domains occupied by the vortices and antivortices. We will first discuss the conditions at the sample boundaries. Ignoring the induction jump on the surface (this may be done in the case of fairly high values of $H$, $H \gg H_{c1}$), one can derive

$$N_2(d) = N_2(-d) = H/\Phi_0. \quad (8)$$

Since only the right-hand part $0 < x < d$ of the sample is treated, we will replace the condition $N_2(-d) = H/\Phi_0$ by the requirement,

$$V_1(0) = 0, \quad (9)$$

that immediately follows from the symmetry of the problem.

Now we turn to the boundary conditions at the moving vortex-antivortex interface. In general case, the position of the interface depends on time $t$ and the $y$ coordinate. The equation for this surface can be presented as $x = x_0(y, t)$.

The second boundary condition should define the rate of the vortex-antivortex annihilation. It is obvious that this rate goes to zero if the density of vortices or antivortices at the interface between them is zero. Then, following the conventional approach to describing such kinetic processes, we represent the rate of annihilation to be proportional to the product of the vortex and antivortex densities,

$$N_1(\vec{V}_1 - \vec{U})_n = RN_1 N_2, \quad (10)$$

where $\vec{U}$ is a velocity of the interface.

Finally, we assume that the average magnetic induction in the neighborhood of the interface is zero, i.e.,

$$N_1 = N_2 \quad (11)$$
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at \( x = x_0(y, t) \).

To find the base profile of the vortex density it is necessary to solve a complex set of partial differential equations (2)–(7) along with the boundary conditions (8)–(11) some of which are written at the moving interface. A peculiar complication consists in the fact that the velocity \( U(t) \) of the motion of the vortex-antivortex interface is unknown and should be found self-consistently. Unfortunately, the problem does not have simple (with \( U = \text{const} \neq 0 \)) automodel solutions because the interface moves non-uniformly under the conditions being studied. As the first approximation, let us calculate a stationary base profile of another system where the velocity \( U(t) \) equals zero. Assuming \( \partial N_\alpha/\partial t = 0 \) in Eq. (2), one can easily obtain the distributions \( N_1(x) \) and \( N_2(x) \),

\[
N_\alpha(x) = N_0 \sqrt{1 + s_\alpha C(x_0 - x)/d}, \tag{12}
\]

\[
C = 8\pi \sqrt{2dA/c_0N_0^2} \cdot \left[ \frac{\sqrt{2}\Phi_0RN_0^2}{cE_0(1 + \varepsilon^m)} \right]^{1/m},
\]

Neglecting unity in the radicand in Eq. (12) and using condition (8) one can evaluate the vortex density \( N_0 \) at the interface,

\[
N_0 = N_\alpha(x = x_0) \sim \left( \frac{H}{H_p} \right)^m \left( \frac{cE_0}{2^{(m+1)/2}\Phi_0R} \right)^{1/2}, \tag{13}
\]

\[
H_p = (8\pi dA\Phi_0/c)^{1/2}.
\]

Expression (12) is the solution of the stationary problem. We are interested in the base profile corresponding to the moving interface. This motion leads to a distortion of the profile with respect to Eq. (12). However, we assume the velocity \( U \) to be small with respect to the vortex velocity \( V_\alpha \), \( U \ll V_\alpha \).

Then, as usual, we assume that the vortex density can be represented as a sum of the unperturbed term and a fluctuation one. The linearized boundary conditions should be written at the perturbed interface. After simple but cumbersome transformations the dispersion equation for the increment \( \lambda \) at different values of the wave number \( k \) was obtained \(^{14}\).

The dependence of the increment \( \text{Re}\lambda \) on the wave number \( \kappa \) for \( m = 20 \) and the different values of the anisotropy parameter \( \varepsilon \) is shown in Fig. 2. The figure shows that the spectrum of perturbations strongly depends on the parameter of anisotropy.

Instability is observed at \( \varepsilon \leq \varepsilon_c \approx 0.43 \). Indeed, the increment \( \text{Re}\lambda \) becomes positive at some interval of the wave number \( \kappa \) and reaches the maximum value at finite \( \kappa \). If the anisotropy parameter is higher than \( \varepsilon_c \),
Fig. 2. The numerical solution $\text{Re}\lambda(\kappa)$ of the dispersion equation for $m = 20$, at different values of the anisotropy parameters $\varepsilon$: $\varepsilon = 0.5$ (curve 1), $\varepsilon = 0.45$ (2), $\varepsilon = \varepsilon_c = 0.43$ (3), $\varepsilon = 0.4$ (4), $\varepsilon = 0.38$ (5), and $\varepsilon = 0.2$ (6).

the increment $\text{Re}\lambda$ is negative at any value of $\kappa$. This implies that the vortex-antivortex system is stable with respect to small perturbations if the current anisotropy of a superconductor is not small enough. Of course, the critical value $\varepsilon_c$ of the anisotropy parameter depends strongly on the exponent $m$. The graphs in Fig. 3 illustrate the change in the value of $\varepsilon_c$ with an increase of $m$. The graph $\varepsilon_c(m)$ represents the separatrix dividing the phase space $(m, \varepsilon)$ into two parts corresponding to the stable (left-hand part) and unstable (right-hand part) states of the vortex system.

One of the most important results of this paper consists in the substantial weakening the requirement theoretically imposed on the anisotropy parameter $\varepsilon$ for the observation of the instability in the vortex-antivortex system. The effective parameter of the anisotropy determining the existence of the instability is $\varepsilon^m$ in the considered case of the nonlinear current-voltage characteristics. Since the exponent $m$ can reach several tens in real superconductors, the necessary condition for the instability is achieved for not too small values of $\varepsilon$.

This circumstance allows one to eliminate a seeming contradiction between our theory and experiments where the macroturbulence was reported to be observed even in detwinned YBCO single crystals. The MO image of the development of the macroturbulence in such a sample is shown in figure. The twinning structure in this sample is not observed in polarized light whereas the macroturbulence is clearly pronounced. However, a careful scan of the left top image in figure, where the initial magnetic flux distribution is shown. The penetration depth in the horizontal direction is clearly seen to be about 1.2 times higher than in the vertical one. This means that the anisotropy exists and the macroturbulent instability may appear.

Thus, our previous studies\textsuperscript{8–10} lead us to conclude that the macroturbu-
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Fig. 3. The separatrix dividing the stability and instability regions in the phase plane ($\varepsilon, m$). The solid and dashed lines correspond to different parameters.

Fig. 4. The MO image of the development of the macroturbulence in a detwinned YBCO single crystal.

lent instability arises due to the tangential discontinuity of the hydrodynamic velocity at the vortex-antivortex interface resulting from the guiding effect. Nevertheless, a certain dissatisfaction persisted since an experimental investigation of this physical picture has not been performed. This motivates the present study devoted to the experimental investigation of the possible nature of the instability. The main idea of this investigation is to study the behavior of the flux-antiflux interface in a crystal cut out in such a way that the anisotropy effects would be present mainly near some edges of the

Fig. 5. Polarized light image of the sample, which reveals that it consists of essentially singly oriented twin boundaries parallel to the long side (hypotenuse).

3. EXPERIMENTAL STUDY

To realize such an experiment, the sample was shaped into a triangular plate with one edge cut parallel to the twin boundaries. Hence, flux guiding and macroturbulence are not expected for the interface running along this edge, but should be present for the other edges.

The YBa$_2$Cu$_3$O$_{7-\delta}$ single crystals were grown using a technique described in Ref. 15. The crystals with dimensions up to $5 \times 5$ mm$^2$ parallel to the $ab$ plane and about 10–20 $\mu$m along the $c$ axis. Then, several crystals having large domains with aligned twin boundaries were chosen. After a selection of such domains, we prepared two samples and shaped them by laser cutting into a nearly right-angled triangular plate. The polarized light microscope image of one of the samples is shown in Fig. 5. The size of the sample along the hypotenuse is about 1.1 mm. The crystallographic $ab$-plane coincides with the sample plane. It is clearly seen that the twin boundaries are directed along the hypotenuse. The twin spacing is approximately 2 $\mu$m. The critical temperature of the samples is 91 K and the width of the transition is about 0.3 K.

The study of the magnetic flux penetration and the macroturbulence was carried out by the conventional magnetooptic imaging technique. The image in Fig. 6 demonstrates the distribution of trapped magnetic flux after cooling the sample down to 30 K in an external transverse magnetic field $H = 1$ kOe, which was subsequently switched off. The brighter regions of
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Fig. 6. MO image of the trapped magnetic flux. $T = 30$ K.

In order to search for instability, the sample was first cooled in a transverse magnetic field $H$. Then the field was abruptly reversed and MO images were recorded and analyzed. Various sample temperatures and reverse fields were used. The most pronounced unstable behavior was observed at $H = 1$ kOe and $T = 30$ K, and is illustrated by the series of MO images in Fig. 2. The manifestation of instability as seen through the oculars of the microscope can be described as follows. We observed a specific contrast vortex-antivortex interface (the dark lines in the images) before building up the instability.

The images (a-d) in Fig. 7, obtained at 0.1, 0.2, 0.3 and 10 seconds after the field reversal, show the consecutive stages of the development of the instability. The key point here is to observe the significant difference in the interface geometry, and its motion away from the sample edges as function of time. First, one notes that except for the hypotenuse, the interface elsewhere is very sharply defined, a usual feature of turbulent behavior. Second, along the hypotenuse the interface remains essentially static whereas substantial motion takes place elsewhere, e.g. for the interface along the upper cathetus, where the velocity is estimated to 3 mm/sec at the initial stage of the development of the instability. Note that the fast change of interface position occurs after the field reversal, when the critical profile has been established, and can be interpreted as the development of the instability only. Unstable motion appears clearly also from the short edge in the lower left part of the crystal. Also along the left cathetus the interface moves, although with a slightly smaller velocity. We conclude therefore that we experimentally have...
found that the instability giving rise to the bending flux-antiflux interface occurs only along edges oriented with some angle $\theta \neq 0, \pi/2$ with respect to the twin boundaries, i.e., where the guiding effect leads to the tangential discontinuity of the hydrodynamic vortex velocity. By performing MO imaging at different temperatures we found that the instability exists in the present sample in the temperature window of $15 \text{ K} < T < 45 \text{ K}$. The effect is well reproducible after several cycling magnetic field and temperature.

![Fig. 7. Evolution of the magnetic flux distribution under condition of the development of instability. Images (a)-(d) were obtained in 0.1, 0.2, 0.3, 10 seconds after the reverse of the external magnetic field, respectively. $T = 30 \text{ K}$, $H = 1 \text{ kOe}$. The flux-antiflux interface is the dark line, e.g. as pointed at by the white arrow.](image)

Thus, the present study can be considered as a good experimental confirmation for the possible ascertainment of the physical nature of the instability in type-II superconductors. The specific anisotropy for $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ superconductors provides the guiding effect in the vortex motion. As a result, the discontinuity of the tangential component of the flux-line velocities ap-
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pear at the vortex-antivortex interface. This leads to the development of the turbulence similar to the case of the classical dynamics of fluids.

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