Vortex bending and avalanche in a type-II superconductor observed with contactless ultrasonics

Katsuhiro Kawashima
Tokyo Engineering University, 1404-1 Katakura, Hachioji, Tokyo 192, Japan
(Received 17 February 1998)

Severe vortex bending as large as 90° and giant avalanche were observed over a large area of a disk-shaped type-II superconductor in the mixed state using a contactless ultrasonic technique. This phenomenon is greatly different from a conventional critical state model in which the vortices are assumed to stand perpendicular to the disk plane. Equations describing the contactless ultrasonics are derived using a dynamic equation of vortex motion that takes account of crystal displacements arising from ultrasonic wave propagation. Using the equations and the ultrasonic data, the variations of the bending angle and the vortex densities are obtained as a function of applied magnetic field.

INTRODUCTION

A hexagonal lattice of quantized vortex line penetrates a type-II superconductor in a dc magnetic field. The pinning of the vortex lattice by crystal defects keeps a zero resistance in the mixed state below a second critical field $H_{c2}$. The pathological behavior, however, of the vortex lattice in high-$T_c$ superconductors, transitions between crystal, glassy, and liquid phases, may limit applications of high-$T_c$ superconductors at "high temperature."

Study of the dynamical properties of vortices is important from a scientific as well as from a practical point of view. Among methods to study such vortex dynamics, ultrasonic methods have several advantages. One recently reported contactless ultrasonic method makes use of the mutual conversion between an electromagnetic field and ultrasonic waves. However, this method is not totally contactless since a quartz transducer and a delay rod are used either to detect or generate ultrasonic waves. A totally contactless ultrasonic method using two coils that can simultaneously generate and detect ultrasonic waves in a superconductor is reported here. A thickness resonance method was used to enhance the sensitivity. This method is abbreviated as EMAT-SC (electromagnetic acoustic transducer for superconductor).

For normally conducting metals in a dc magnetic field there is a similar contactless ultrasonic method that is known as EMAT. A type-II superconductor, a polycrystalline Nb disk, was used in the first place for a simpler platform of EMAT-SC. The sample was cooled in zero field, and then a uniform dc magnetic field perpendicular to the disk plane was applied. It was expected that only shear waves would be obtained for such a dc magnetic field, since the Lorentz force is, in principle, generated perpendicular to the magnetic field, and likewise is the crystal displacement associated with this shear wave. But, on the contrary and very unexpectedly, no shear wave, but rather only a longitudinal ultrasonic wave, was obtained in the weak-field regime. Such phenomena have never been observed before. An abrupt disappearance of the longitudinal waves, a sudden appearance of shear waves, and some more unusual phenomena followed as the field strength was varied. Equations describing the conversion efficiency between an ac magnetic field and ultrasonic waves are derived and changes of the near surface vortex bending angle and the vortex density with applied field are calculated using the ultrasonic data. It is found that the unusual ultrasonic phenomena are caused by severe vortex bending as large as 90° and by a giant vortex avalanche.

CONTACTLESS ULTRASONICS (EMAT-SC)

A disk shaped polycrystalline Nb plate (diameter: 19 mm; thickness: 0.52 mm) was cooled in zero field down to 4.2 K, and then a uniform dc magnetic field was applied perpendicular to the disk plane. A transmitting coil ($T$ coil, diameter: 8.7 mm; number of turns: 12) was placed near one surface with a gap of 0.2 mm. A similar receiving coil ($R$ coil) was placed in a symmetrical position on the other side of the sample (Fig. 1). Ac currents applied to the $T$ coil generate a radially polarized ac magnetic field parallel to the surface that in turn generates a surface shielding current in the surface of the sample in the mixed state. The amplitude of the ac magnetic field was $\approx 3$ mT that was estimated from a measured $T$-coil current. The shielding current exerts a Lorentz force on the vortices and the resulting deformation of the vortex lattice then propagates into the interior.

![FIG. 1. Schematic of EMAT-SC. Contactless generation and detection of ultrasonic waves for a type-II superconductor. Two coils are placed near the surfaces with 0.2 mm gaps. A uniform magnetic field $H_{ap}$ is applied perpendicular to the superconductor surface.](image-url)
sonic waves are generated through the coupling between the vortex and the pinning sites. The T-coil current frequency was scanned from 4.7 to 6.7 MHz. During the scanning, the thickness resonance condition \(kd = m\pi\) is satisfied for particular values of \(k\), where \(k\) is the ultrasonic wave number, \(d\) is thickness, and \(m\) is a positive integer, thereby generating a standing wave and greatly improving the generating efficiency. The generated ultrasonic wave moves the vortices at the opposite surface of the sample, and thereby generates an ac magnetic field in free space that is detected by the \(R\) coil. Ultrasonic data is obtained as amplitude vs frequency.

Typical data obtained are shown in Figs. 2(a)–2(c). \(L_1\) is the first-order longitudinal resonance and \(S_{31}\) and \(S_{32}\) are the third-order shear resonances. \(S_{31}\) and \(S_{32}\) are slow and fast shear waves generated by the birefringence effect of the sample anisotropy. These waves can be identified with a wave velocity \(V = 2fdlm\), where \(f\) is the resonance frequency. The obtained frequencies for \(L_1\), \(S_{31}\), and \(S_{32}\) from Fig. 2 are 4.94, 6.10, and 6.29 MHz, with corresponding velocities of 5140, 2120, and 2180 m/s, respectively. These velocities are a little larger, as expected from the difference in temperature, than the calculated longitudinal and shear wave velocities of 5050 and 2090 m/s obtained by Hill’s averaging method using the known single-crystal elastic constants of Nb at room temperature.

Figures 3(a) and 3(b) shows the shear \(S_{32}\) and longitudinal \(L_1\) wave amplitudes vs the applied dc magnetic field \(H_{ap}\). When \(H_{ap}\) is increased to 0.4 kG, a longitudinal wave appears above the noise level (indicated as \(L_1\)) and it grows larger as \(H_{ap}\) is increased. At about 0.9 kG it abruptly vanishes (\(L_2\) to \(L_3\)) and at the same time a large shear wave suddenly appears (\(S_3\)). As \(H_{ap}\) is further increased, the shear wave grows larger (\(S_3\) to \(S_6\)) and the longitudinal wave appears again (\(L_4\)), grows larger (\(L_4\) to \(L_5\)) and then decreases (\(L_5\) to \(L_6\)). Above the second critical field \(H_{c2} = 3.4\) kG, the EMAT-SC mechanism is taken over by the EMAT mechanism that will be explained later. Only the shear wave remains (\(S_6\)) and the longitudinal wave disappears (\(L_6\)), as expected. The shear wave continues to grow larger (\(S_6\) to \(Sm\)) as the dc magnetic field increases above \(H_{c2}\) to the maximum applied field 4.5 kG, following the square law (\(\propto H_{ap}^2\), the curved broken line), as expected for the EMAT mechanism. As \(H_{ap}\) is decreased from 4.5 kG to \(H_{c2}\), the shear wave decreases (\(Sm\) to \(S6\)) with a similar path as the path for the increase in field. As the magnetic field is further decreased below \(H_{c2}\), the shear wave decreases (\(S6\) to \(S7\)) but along a different path from that for the increase in field, showing a large hysteresis. Both waves experience more discontinuous changes as shown in Fig. 3 when the field is further decreased.

The data was found to be reproducible on repeated cycling of the magnetic field, although small variations (\(\pm 5\%\)) in the points of onset of the critical behavior were observed. The unusual behavior of the ultrasonic data will be explained later. Similar measurements were done using the same coils and the same sample in a normally conducting state at 12 K at which the EMAT mechanism is effective, and in an applied dc magnetic field that makes a 45° angle with the disk plane. T-coil currents generate eddy currents in a near surface region. These currents experience Lorentz forces in a static magnetic field. It has been found to be a good approximation for normally conducting metals that the Lorentz forces are directly applied to the solid and thereby generate ultrasonic waves. Generated ultrasonic amplitude is proportional to the static magnetic field. Ultrasonic waves in the static magnetic field induce currents that are detected by the \(R\) coil. The overall conversion efficiency therefore is proportional to the square of the static magnetic field, \(A = KH_{ap}^2\).

Both longitudinal and shear waves were obtained as expected from the EMAT mechanism, since the dc magnetic field has components parallel and perpendicular to the sample surface. Figure 4 shows the shear \(S_{32}\) and longitudinal \(L_1\) wave amplitudes vs the applied dc magnetic field \(H_{ap}\). Both sets of data could be fitted to the square law curves, \(A = KH_{ap}^2\). The ratio of the longitudinal wave to the shear wave amplitude is about 0.52 over the whole range of

---

**FIG. 2.** Thickness resonance peaks of ultrasonic waves obtained at 4.2 K. \(L_1\) refers to longitudinal waves and \(S_{31}\) and \(S_{32}\) to shear waves. (a) Only longitudinal waves were obtained just before an avalanche in an increasing magnetic field (at \(L_2\) and \(S_2\) in Fig. 3). (b) Both longitudinal and shear waves were obtained just before an avalanche in a decreasing magnetic field (\(S_7\) and \(L_7\) in Fig. 3). (c) Only shear waves were obtained right after an avalanche (\(S_8\) and \(L_8\) in Fig. 3).

**FIG. 3.** (a) Shear wave \(S_{32}\) and (b) longitudinal wave \(L_1\) amplitudes obtained at 4.2 K in a dc magnetic field, \(H_{ap}\). Closed circles and squares are for increasing fields and open circles and squares are for decreasing fields. The broken curved line in (a) corresponds to \(A = KH_{ap}^2\).
FIG. 4. Shear wave $S_{12}$ and longitudinal wave $L_1$ amplitudes obtained at 12 K in a dc magnetic field, $H_{ap}$. Circles are for shear waves and squares are for longitudinal waves. Closed circles and squares are for increasing fields and open circles and squares are for decreasing fields. The curved lines correspond to $A = KH_{ap}$

applied characteristics for both wave polarizations.

VOXEL DYNAMICS IN THE PRESENCE OF ULTRASONIC WAVES

In recent theoretical works, a generalized partial differential equation describing the linear response of a type-II superconductor in the mixed state was derived in which the equations for electrodynamics of the superconductor and the dynamic equation of vortex motion were used. Throughout this paper, a time harmonic variation $\exp(-i\omega t)$ is assumed for all fields. The electrodynamical equations are Maxwell's curl equations $\nabla \times E = -\partial B/\partial t$, $\nabla \times H = J$, the two-fluid equation $J = J_0 + J_s$, the constitutive relation $J_s = \sigma_{\text{NF}} E$, the supercurrent-source equation $\nabla \times J_s = -(B - \phi_0 \hat{B}_0)/\mu_0 \lambda^2$, and the vortex continuity equation (conservation of flux lines) $\partial B_s/\partial t = -\nabla \times (B_s \times \partial \eta/\partial t)$, where $\lambda$ is the London penetration depth, $J_0$ is the normal current density, $\sigma_{\text{NF}} = 1/\rho_{\text{NF}}$ is the local electrical conductivity of the normal fluid, $\rho_{\text{NF}}$ is normal fluid resistivity, $J_s$ is the supercurrent density, $B_0$ is the uniform magnetic field generated by an array of vortices, $B_s = n \phi_0 \hat{B}_0$ is the local vortex magnetic field, $n$ is the local areal density of vortices, $B_0 = B_0/B_0$ is the unit vector, $\phi_0$ is the flux quantum, and $\nu$ is the vortex displacement as measured from an equilibrium pinning site.

They obtained the following equation from the above equations:

$$\nabla^2 B = \frac{\mu_0}{\rho_{\text{NF}}} \frac{\partial B}{\partial t} + \frac{1}{\lambda^2} \left[ B - B_0 + \nabla \times (B_0 + \nu) \right].$$

(1)

The dynamic equation of vortex motion is

$$\eta \frac{\partial \nu}{\partial t} + \kappa_p \nu = J \times \phi_0 \hat{B}_0,$$

(2)

where $\eta$ is the viscous-drag coefficient, $\kappa_p$ is the restoring-force constant (Labusch parameter) of a pinning potential well, and a possible vortex-mass term is ignored here. $J \times \phi_0 \hat{B}_0$ is a rf Lorentz force per unit length of vortex line. They combined Eqs. (1) and (2) to obtain a single partial differential equation for the rf magnetic field $b = B - B_0$.

A rf-vortex penetration depth $\lambda_{ac}$ was obtained from Eq. (3) as

$$\lambda_{ac}^2 = \frac{\lambda^2 + \delta^2_{\nu}/2}{1 - 2\lambda^2/\delta_{\text{NF}}},$$

(4)

where

$$\delta_{\text{NF}} = 2\rho_{\text{NF}}/\mu_0 \omega, \delta_{\nu} = 2\rho_{\nu}/\mu_0 \omega, \rho_{\nu} = B_0 \phi_0 \mu_\nu,$$

and $\mu_\nu = 1/(\eta + i \kappa_p/\omega)$ is the dynamic mobility. They pointed out that the same expression for $\lambda_{ac}$ holds for an arbitrary angle between $B_0$ and the superconductor surface.

A similar result was also obtained in Ref. 16. In Eq. (2), the vortex displacement $\nu$ is measured from an equilibrium pinning site that moves with the crystal displacement. In the presence of the crystal displacement $u$, the vortex displacement $\nu$ must therefore be replaced with the relative displacement $\nu - u$, yielding the following equation:

$$\eta \frac{\partial (\nu - u)}{\partial t} + \kappa_p (\nu - u) = J \times \phi_0 \hat{B}_0.$$

(5)

The first term of the left-hand side was introduced in Ref. 2 in a similar equation, and the second term was introduced in Ref. 4. Combining Eqs. (1) and (5) I obtain an equation for an rf magnetic field $b$ inside the superconductor, taking account of the crystal displacement arising from ultrasonic wave propagation.

$$\nabla^2 b = \sigma_{\text{NF}} \mu_0 \frac{\partial b}{\partial t} + \frac{1}{\lambda^2} \left[ b + \frac{i}{2} \delta^2_{\nu} \nabla \times (\hat{B}_0 \times [(\nabla \times b) \times \hat{B}_0]) + \nabla \times (B_0 \times \nu) \right].$$

(6)

An important special case is when $B_0$ is a constant vector for a planar geometry. Figure 5 shows the geometry wherein the rf field $b = \hat{b} \hat{z}$ is chosen to lie along $\hat{z}$, and $x$ measures the distance into the superconductor. The rf magnetic field is given in free space at $x = 0$ as $b = \mu_0 h_0 \hat{z}$. Where, $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors. A type-II superconductor of thickness $d$ is
inserted, and \( B_0 \) is taken to lie in the \( xz \) plane making an angle \( \alpha \) with the \( x \) axis. Longitudinal wave \( u_z \) and shear wave \( u_x \) propagate in the \( x \) direction. Equation (6) is simplified for this special case yielding

\[
\frac{\partial^2 b}{\partial x^2} = \frac{1}{\lambda_{ac}^2} \left( B_0 z \frac{\partial u_z}{\partial x} - B_0 \frac{\partial u_x}{\partial x} \right),
\]

where \( B_0 = B_0 \cos \alpha = n \phi_0 \cos \alpha, \quad B_0 = B_0 \sin \alpha = n \phi_0 \sin \alpha, \) and \( n \) is the vortex density. Equation (7) is used throughout the following analyses.

Ultrasonic waves generated by the rf magnetic field \( B = \mu_0 h_0 \hat{z} \) given at \( x = 0 \) can be obtained as follows. The equation for crystal displacement \( u \) is a plane wave equation

\[
\rho \frac{\partial^2 u}{\partial t^2} = \left( C + D \frac{\partial}{\partial t} \right) \nabla^2 u + F,
\]

where \( \rho \) is density, \( C \) is elastic constant, and \( F \) is the force per unit volume. The term proportional to \( D \) accounts for sound dissipation. \( F \) is considered to be a reaction force exerted on crystal (pinning sites) from vortices.

\[
F = -J \times B_0 = - (\nabla \times H) \times B_0 = - \frac{1}{\mu_0} \left( \nabla \times B \right) \times B_0.
\]

Combining Eqs. (8) and (9) yields the following equation for longitudinal wave in the special case of constant \( B_0 \) and a planar geometry:

\[
- \rho \omega^2 u_z = \left( C_i - i \omega D_i \right) \frac{\partial^2 u_z}{\partial x^2} + \frac{B_0 \omega}{\mu_0} \frac{\partial b}{\partial x},
\]

where \( C_i = \rho V_i^2, \ V_i \) is longitudinal wave velocity, and \( D_i \) accounts for dissipation of longitudinal wave. Since the crystal displacement \( u \) of the ultrasonic waves generated by the rf magnetic field is considered to be much less than the vortex displacement \( v \), the term on the right-hand side of Eq. (7) can be ignored. The rf magnetic field inside the superconductor with thickness \( d \gg |\lambda_{ac}| \) is therefore given by

\[
b = K_1 \exp(x/\lambda_{ac}) + K_2 \exp(-x/\lambda_{ac}).
\]

\( K_1 \) must be zero since \( \exp(x/\lambda_{ac}) \) grows larger as it propagates in the positive \( \hat{x} \) direction. \( K_2 \) is found from the electromagnetic boundary condition at \( x = 0 \), giving

\[
b = \mu_0 h_0 \exp(-x/\lambda_{ac}),
\]

where \( \text{Re}(-1/\lambda_{ac}) \) is chosen to be negative. Equations (10) and (11) yield for longitudinal waves

\[
\frac{\partial^2 u_z}{\partial x^2} + \frac{\rho \omega^2}{C_i - i \omega D_i} u_z = \frac{B_0 h_0}{C_i - i \omega D_i} \exp(-x/\lambda_{ac}).
\]

The solution is

\[
u_z = L_1 \exp(-\gamma x) + L_2 \exp(\gamma x) + L_0 \exp(-x/\lambda_{ac}),
\]

where \( L_0 = \lambda_{ac} B_0 h_0 / (C_i - i \omega D_i + \rho \omega^2 \lambda_{ac}), \ \gamma^2 = -\rho \omega^2 / (C_i - i \omega D_i), \) and \( \text{Re}(\gamma) \) is chosen to be negative. The coefficients \( L_1 \) and \( L_2 \) can be found from the stress-free boundary conditions \( \partial u_z / \partial x = 0 \) at \( x = 0 \) and \( x = d \). Under the conditions \( C_i \gg \omega D_i, \ \alpha d \ll 1, \ \kappa_d = m \pi \) (thickness resonance condition) the longitudinal wave \( u_z \) is given in a simple form as

\[
u_z = -\frac{i h_0 B_0 z}{\rho \omega^2} \frac{1}{\rho \omega^2 + \lambda_{ac}^2/2} \exp[i(k_i - \alpha_i) x] + \exp[-i(k_i - \alpha_i) x],
\]

where \( k_i = \omega / V_i, \ \alpha_i = D_i / \omega^2 / 2 \rho V_i^3 \) (attenuation coefficients). The factor \( 1/(2 \alpha d) \gg 1 \) represents the expected enhancement of the amplitude at resonance. The factor \( h_0 B_0 / \rho \omega^2 (1 + k_i^2 \lambda_{ac}^2) \) is similar to one obtained in Ref. 5 that dealt with a dc magnetic field perpendicular to the surface of a semi-infinite medium. A generated ultrasonic amplitude is proportional to \( B_0 \) \( \sin \alpha \), which explains the experimental results obtained in Ref. 3. The magnetic field \( b \) in free space at \( x = d \) generated by the ultrasonic wave can be obtained as follows. Plane longitudinal waves propagating in the \( x \) direction and reflected back at the stress-free surface \( x = d \) are described in a general form as

\[
u_z = L_3 \exp[i(k_i - \alpha_i) x] + L_4 \exp[-(i(k_i - \alpha_i) x].
\]

Equations (14) and (7) yield for longitudinal wave

\[
\frac{\partial^2 b}{\partial x^2} = \frac{1}{\lambda_{ac}^2} b = -\frac{B_0 z}{\lambda_{ac}^2} \exp[i(k_i - \alpha_i) x]
\]

\[
- L_4 \exp[-(i(k_i - \alpha_i) x],
\]

The solution is given as

\[
b = K_3 \exp(-x/\lambda_{ac}) + K_4 \exp(x/\lambda_{ac})
\]

\[
+ K_0 L_3 \exp[i(k_i - \alpha_i) x]
\]

\[
- L_4 \exp[-(i(k_i - \alpha_i) x],
\]

where

\[
K_0 = -B_0 z \exp[i(k_i - \alpha_i)/\lambda_{ac}] 1/k_i^2 \lambda_{ac}^2 (i(k_i - \alpha_i)^2 - 1/k_i^2 \lambda_{ac}^2).
\]

The magnetic field in free space is given as \( b = K_5 \exp(ikx) \), where \( k_x \) is the free space wave number. \( K_5 \) must be zero since \( \exp(-x/\lambda_{ac}) \) grows larger as it propagates in the negative \( \hat{x} \) direction. The remaining unknown coefficients \( K_4 \) and \( K_5 \) can be found by enforcing the continuity of the tangential components of the electric and magnetic fields at the surface \( x = d \). Under the conditions \( k_i \gg \alpha_i, \ k_i \lambda_{ac} \ll 1 \), the magnetic field in free space at \( x = d \) is given as

\[
b = \frac{ik_x B_0 z}{1 + k_i^2 \lambda_{ac}^2} L_3 \exp[i(k_i - \alpha_i) d]
\]

\[
+ L_4 \exp[-(i(k_i - \alpha_i) d].
\]

Reference 4 gave a formula \( b \propto B_0 \exp(\omega \ell_{ac} / c) \) for the magnetic field, but it is clear that their formula is inappropriate when applied to ultrasonic waves reflected at the stress-free surface since it gives the value of zero in this important case. The magnetic field in free space at \( x = d \) generated by the standing longitudinal ultrasonic wave that is generated by
the incident magnetic field $h_0$ at $x=0$ is obtained by substituting Eq. (13) with $x=d$ into Eq. (17) as

$$b = \frac{k_e h_0 B_{0x}}{(1 + k_i^2 \lambda_{ac}^2)^2} \frac{1}{\rho V_i} \frac{(-1)^m}{\alpha_i d}.$$  \hspace{1cm} (18)

When an infinite plane transmitting coil that consists of straight wires directed along $\hat{y}$ and extends in the $yz$ plane at $x=0$ is used to generate an incident magnetic field and an ultrasonic wave, and when a similar receiving coil at $x=d$ is used to detect a magnetic field generated by the ultrasonic wave, an induced voltage $V_{acL}$ per unit area of the receiving coil can be obtained using Eq. (18) and Faraday’s law. The transmitting coil is assumed to carry a unit rf current and to have a unit number of wires per unit length, making $h_0=1$. The receiving coil has the same wire density.

For longitudinal waves, $V_{acL}$ is given by

$$V_{acL} = \frac{B_{0x}}{(1 + k_i^2 \lambda_{ac}^2)^2} \frac{1}{\rho V_i} \frac{(-1)^m}{\alpha_i d} = \frac{n^2 \phi_0^2 \sin^2 \alpha}{(1 + k_i^2 \lambda_{ac}^2)^2} \frac{1}{\rho V_i} \frac{(-1)^m}{\alpha_i d} = Q_l. $$ \hspace{1cm} (19)

For shear waves, $V_{acS}$ is given in a like manner by

$$V_{acS} = \frac{B_{0x}^2}{(1 + k_i^2 \lambda_{ac}^2)^2} \frac{1}{\rho V_s} \frac{(-1)^m}{\alpha_i d} = \frac{n^2 \phi_0^2 \cos^2 \alpha}{(1 + k_i^2 \lambda_{ac}^2)^2} \frac{1}{\rho V_s} \frac{(-1)^m}{\alpha_i d} = Q_s. $$ \hspace{1cm} (20)

Other equations for shear wave can be obtained in a like manner, and those equations are obtained simply by substituting $u_x, k_1, \alpha_f, V_f, C_f, B_{0x}$ in Eqs. (13), (17), and (18) with the corresponding shear wave values $u_s, k_s, \alpha_f, V_s, C_s, B_{0s}$, and also by changing the first signs of the right-hand sides of Eqs. (13) and (17) to the opposite signs. A transfer impedance $Z_l$ can be defined as a ratio of the received voltage $V_{acL}$ to the transmitting current $I_f=1$. $Z_l$ is therefore given as the same expression as $V_{acL}$. In the normal conducting state, $\lambda_{ac}$ reduces to $i \delta_{NQ}/2$ and the transfer impedance reduces to

$$|Z_l| = \frac{B_{0j}^2}{(1 + \beta^2)} \frac{1}{\rho V_i} \frac{1}{\alpha_i d} \frac{1}{\alpha_i d}, $$ \hspace{1cm} (21)

where $\beta = k_i^2 \delta_{NQ}/2$, $i$ is either $l$ or $s$, and $j$ is either $z$ or $x$. This is exactly the same as the expression derived in Ref. 11 for the resonance mode EMAT with a normal-state metal from a coupled Maxwell equation and an elastic wave equation formulation. This means that Eqs. (19) and (20) can be used for EMAT as well as EMAT-SC. However, for EMAT, $B_{0z}$ or $B_{0s}$ simply means a parallel or perpendicular component of an applied magnetic field, equal to the magnetic field inside a nonmagnetic metal.

**VORTEX BENDING AND AVALANCHE**

The transmitting and receiving coils used in the experiment are circular disk coils. The equations obtained in the last section must therefore be modified. Three different cases (A), (B), and (C) shown in Fig. 6 are considered. In the case (A), a dc magnetic field generated by an array of vortices has a $B_{0x}$ component that is perpendicular to the sample surface and the coil plane. Shear waves polarized parallel to the $z$ and to the $y$ axes are generated and detected in this case. A small area of a disk coil can be assumed to be a part of a plane coil if the gap between the disk coil and the sample surface is small enough. An induced voltage in the receiving disk coil is, therefore, given using Eq. (20) simply as

$$V_A = \frac{1}{2} Q_x \pi R^2 \frac{N}{R} G_s,$$

$$= \frac{1}{2} \pi \rho R \frac{B_{0x}^2}{(1 + k_i^2 \lambda_{ac}^2)^2} \frac{1}{\rho V_s} \frac{(-1)^m}{\alpha_i d} G_s, $$ \hspace{1cm} (22)

where $R$ is a coil radius and $N$ is the number of turns. The factor $1/2$ is derived by averaging $y$ components ($z$ components) of small wire elements that are effective in generating and detecting shear waves polarized in the $z(y)$ direction. $G_s$ is a correction term for shear waves that takes account of the geometrical factors of the circular disk coils. In the case (B), the dc magnetic field generated by vortices only has radial components $B_{0R}$. This appears to be a hypothetical case but it is realized in the experiment in the case of the 90° vortex bending. A longitudinal wave is generated and detected in this case. An induced voltage in the receiving disk coil is given using Eq. (19) as

$$V_B = \frac{1}{2} Q_L \pi R^2 \frac{N}{R} G_L = \pi \rho R \frac{B_{0R}^2}{(1 + k_i^2 \lambda_{ac}^2)^2} \frac{1}{\rho V_i} \frac{(-1)^m}{\alpha_i d} G_L,$$

$$= \frac{1}{2} \pi \rho R \frac{B_{0R}^2}{(1 + k_i^2 \lambda_{ac}^2)^2} \frac{1}{\rho V_i} \frac{(-1)^m}{\alpha_i d} G_L, $$ \hspace{1cm} (23)

where $G_L$ is a geometrical correction term for longitudinal waves. In the case (C), the dc magnetic field has only a parallel component $B_{0z}$. Longitudinal waves are generated and detected in this case. The induced voltage in the receiving coil is given using Eq. (19) as

$$V_C = \frac{1}{2} Q_L \pi R^2 \frac{N}{R} G_L = \frac{1}{2} \pi \rho R \frac{B_{0z}^2}{(1 + k_i^2 \lambda_{ac}^2)^2} \frac{1}{\rho V_i} \frac{(-1)^m}{\alpha_i d} G_L.$$  \hspace{1cm} (24)

The factor $1/2$ is derived by averaging $y$ components of small wire elements that are effective in generating and detecting longitudinal waves. $B_{0z}$ corresponds to $n \phi_0 \cos \alpha$ and both $B_{0R}$ and $B_{0z}$ correspond to $n \phi_0 \sin \alpha$. Equations (22), (23), and (24) can be used for EMAT as well as for EMAT-SC by replacing the superconductor values with the corresponding normal conductor values. The main interaction between vortices and ultrasonic waves takes place in the near surface region, and $\alpha$ therefore can be considered to be a vortex.
VORTEX BENDING AND AVALANCHE IN A TYPE-II . . .

bending angle since vortices have only axial directions at the midplane of the sample by the symmetry. On the assumption that all the vortices have the same bending angle \( \alpha \) in the relevant area, \( \alpha \) and density \( n \) can be obtained using Eqs. (22), (23), and (24).

\[
\tan^2 \alpha = \frac{V_{pl}(sc)}{V_A(sc)} \left[ \frac{V_c(nc, \alpha = 45^\circ)}{V_A(nc, \alpha = 45^\circ)} \right],
\]

\[
n^2 = (1 + \tan^2 \alpha) \frac{V_A(sc)}{V_A(sc, B_{0x} = B_{c2}, \alpha = 0^\circ)} \left( \frac{B_{c2}^2}{\phi_0} \right),
\]

where \( sc \) and \( nc \) means the superconducting and normalconducting states, respectively, and the conditions \( |k^2 \lambda_{ac}^2| \ll 1, |k^2 \lambda_{ac}^2| \ll 1 \) are used. It is also assumed that the ultrasonic velocities and dissipations in the superconducting (4.2 K) and normally conducting state (12 K) are nearly equal. Using Eqs. (25) and (26) and the ultrasonic data, the variations of the bending angle \( \alpha \) and the vortex density \( n \) are calculated as a function of applied magnetic field, \( H_{ap} \).

The term \( [V_c(nc, \alpha = 45^\circ)/V_A(nc, \alpha = 45^\circ)] \) in the right-hand side of Eq. (25) was set to be 0.52, which was obtained in the experiment. The results are shown in Figs. 7(a) and 7(b). While \( H_{ap} \) is between 0.4 K and 0.9 K, only a longitudinal wave is detected without any trace of a shear wave (L1-L2 and S1-S2 in Fig. 3). When only the longitudinal wave is generated and detected the Lorentz force in the near surface region since the Lorentz force is perpendicular to it. This implies severe vortex bendings as large as 90° (P1-P2 in Fig. 7) in the near surface region over a large area. The corresponding area is at least a circular area of 8.7 mm diameter facing the coil. A sketch of how the 90° bending could be arranged over the disk area is shown in Fig. 8, for which the vortex lines are bent toward the center of a disk sample. The other side of the disk surface is not shown but a symmetrical figure can be imagined. It is considered that this 90° bending is caused by strong circumferential screening currents required to keep the sample diamagnetic. This picture is justified by considering the rotational symmetry of the whole system and the local magnetic-field directions generated by the circumferential screening currents in the increasing static magnetic field. The 90° vortex bending is greatly different from a conventional critical state model in which the vortex lines are assumed to stand perpendicular to the disk plane. 90° vortex bending was reported before, but only for a restricted, narrow, circumferential ring area. The 90° bending over a large surface area of a thick sample is reported here. When only shear waves are observed, and only in this case, the vortex lines are assumed to be perpendicular to the disk plane. The vortex density at 90° bending cannot be calculated by Eqs. (25) and (26) since the shear wave cannot be obtained. It was therefore calculated by Eq. (19) using a density and an angle obtained in another field strength (1.57 K, \( Pn \) in Fig. 7) in which both waves were obtained. At 0.9 K a giant avalanche occurs with a sudden change in the vortex density from 5.5 \( \times 10^{12}/m^2 \) to 3.6 \( \times 10^{13}/m^2 \) (N2 to N3) and in the bending angle from 90° to 0° (P2 to P3). After the avalanche the vortex density gradually increases and the bending angle also increases, reaching another maximum of 4.4° (\( Pn \)). As \( H_{ap} \) is further increased, the vortex density continuously increases, reaching the theoretical maximum of \( B_{c2}/\phi_0 \) (N6) and the bending angle decreases to 0° (P6). Another avalanche and vortex bendings follow as \( H_{ap} \) is decreased. Before every avalanche the bending angle increases and right after the avalanche it becomes almost 0° and then increases again, leading to another avalanche. \( \alpha \) and \( n \) are thus obtained as averaged values in the circular area. However, the EMAT-SC method is the only method, so far, that can measure these parameters in situ, over a large flat area and over a wide range of magnetic fields. The derived equations for EMAT-SC include the effects of vortex dynamics through the quantity \( \lambda_{ac} \), so that the equations are useful for the ultrasonic investigation of vortex dynamics in high-\( T_c \) superconductors in the linear response regime.

ACKNOWLEDGMENT

The author thanks O. B. Wright for discussions.