How to analyze the investment–uncertainty relationship in real option models?*

by

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KEYWORDS: investment, uncertainty, real options, stochastic control

JEL classification numbers: C61, D92, E22, G31

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How to analyze the investment–uncertainty relationship in real option models?†

Abstract

The real options tradition originally predicted a decreasing relationship between uncertainty and investment, through the positive effect of higher uncertainty on the trigger level for revenue relative to costs. An opposing effect on the probability of reaching the level has been identified, yielding a total effect with ambiguous sign. This paper makes three points. The “opposing” effect is not always opposing. Systematic risk cannot generally be assumed to increase with volatility. A probability is not the best measure of investment. The sign of the total effect is again ambiguous. This ambiguity is illustrated, depending on specification of model and parameters.

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1 Introduction

The theory of real options (Dixit and Pindyck 1994, Trigeorgis 1996) prescribes optimal investment rules under uncertainty. From these follow predictions for the macroeconomic relationship between measures of uncertainty on one hand and aggregate investment on the other. Originally the relationship was seen as decreasing, see, e.g., Pindyck (1991, p. 1123, p. 1131). More recently this has been challenged by Sarkar (2000, 2003), who derives a theoretical relationship which can be both increasing and decreasing.

This paper discusses the interpretation of Sarkar’s results, and takes a closer look at the “opposing effect” he identifies. The idea that uncertainty goes up can have several meanings. Moreover, it is not obvious how to measure the response in aggregate investment. Anyhow, the message that the relationship is not always monotone decreasing, survives.

†The author is grateful for helpful suggestions from an anonymous referee. This research was initiated while the author was visiting EPRU, Institute of Economics, University of Copenhagen, Denmark. He is grateful for their hospitality. A previous version appeared as EPRU working paper 03–17.
While Sarkar (2003) introduces a mean-reverting process for the output price, for which there are good arguments, most of the discussion here relates to the geometric Brownian motion (GBM) analysis in Sarkar (2000). The reason is that this is simpler and much more well known, and that most of the arguments relate to both analyses.

Another critical review of Sarkar’s results is found in Cappuccio and Moretto (2001). The present paper extends their analysis in several respects. More details are given in section 7 below.

2 The model

This follows Sarkar (2000), except that different values for the investment cost will be considered. Firm $i$ has the opportunity to invest $K_i$, whereby it will start a perpetual earnings stream $x_t$, where $t$ denotes time. The investment can at most take place once, at any time in the future. While $K_i$ is known and fixed\(^1\), the stream $x_t$ is a GBM with drift,

$$dx_t = \mu x_t dt + \sigma x_t dz_t, \tag{1}$$

where the drift parameter $\mu$ and the volatility parameter $\sigma$ are constants, and $dz_t$ is the increment of a standard Wiener process, $z_t$. Valuation follows the single-beta version of the ICAPM of Merton (1973), so that the present value of the perpetual earnings stream, if and when the investment is undertaken, is $x_t/\delta$, where $\delta \equiv r + \lambda \rho \sigma - \mu$. Here, $r$ is the riskless interest rate, $\lambda$ is the market price of risk, and $\rho$ is the correlation between $dz_t$ and the return on the market portfolio, all assumed to be constants.

The optimal time for firm $i$ to invest is the first time $x_t$ reaches a trigger level $x_i^*$ from below. Defining the constant $\alpha$ as

$$\alpha \equiv \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r - \delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{2}$$

the optimal $x_i^*$ is

$$x_i^* = \frac{K_i \alpha \delta}{\alpha - 1}, \tag{3}$$

\(^1\)For uncertainty in $K$, see McDonald and Siegel (1986).
cf. McDonald (2003, p. 393). It is assumed that the process starts out at \( x_0 < x^*_i \). Otherwise it would be optimal to invest immediately.

The probability of \( x_t \) reaching the critical \( x^*_i \) (at least once) within a time horizon \( T \), is

\[
\Pr(\text{invest in } i) = \Phi \left( \frac{\nu_T - \ln(x^*_i/x_0)}{\sigma \sqrt{T}} \right) + \Phi \left( \frac{-\nu_T - \ln(x^*_i/x_0)}{\sigma \sqrt{T}} \right) ,
\]

where \( \nu \equiv \mu - \sigma^2/2 \), and \( \Phi \) is the standard normal distribution function, cf. Etheridge (2002, p. 69f).

The probability formula has three elements. Let

\[
Q \equiv \left( \frac{x^*_i}{x_0} \right)^{2\nu/\sigma^2} \equiv \left( \frac{x^*_i}{x_0} \right)^{-1} \left( \frac{x^*_i}{x_0} \right)^{2\mu/\sigma^2} ,
\]

and let the two \( \Phi \) expressions in (4) be \( \Phi_1 \) and \( \Phi_2 \), respectively. All probabilities to be discussed are conditional on some \( x_0 < x^*_i \). The first of these, \( \Phi_1 \), is the probability that \( x_T \) (at the horizon, \( T \)) exceeds \( x^*_i \), cf. equation (20.12) of McDonald (2003).

It follows that the other term in (4), now called \( Q\Phi_2 \), is the probability that \( x_t \) exceeds \( x^*_i \) during some interval(s) between 0 and \( T \), but returns to a value below \( x^*_i \) at \( T \).

Since \( x^*_i \) is increasing in \( \sigma \), the original point of view was that investment is decreasing in \( \sigma \): Some projects which would have been undertaken in the near future\(^3\) for a low value of \( \sigma \), will be postponed when it is realized that a higher \( \sigma \) applies. Metcalf and Hassett (1995) and Sarkar (2000) suggest an opposing effect: The probability of an increase in \( x_t \) up to a given trigger level \( x^*_i \) within some given time horizon, \( T \), is higher, the higher is \( \sigma \). This supposedly goes in the direction of an increasing relationship between uncertainty and investment, and Sarkar (2000) indeed demonstrates in a numerical example that the effect of \( \sigma \) on the probability has an ambiguous sign.

3 The probability of exceeding a trigger level

More can be said about the “opposing effect.” This is really the effect of \( \sigma \) on the probability in (4) when \( x^*_i \) is held constant (but \( \nu = \mu - \sigma^2/2 \) is allowed to vary with \( \sigma \)). Holding \( x^*_i \) constant simplifies the partial derivative sufficiently to get an idea of its sign. This turns

\(^2\)As \( T \) goes to infinity, this probability approaches unity or zero, depending on the sign of \( \nu \).

\(^3\)But see the discussion in section 7 below.
out to be ambiguous, not always positive, as the suggestion of an “opposing” effect seems to be based on.

A higher $\sigma$ gives a higher probability both for higher and lower outcomes, while reducing the probability for outcomes close to the expected path. The effects at $T$ (on $\Phi_1$) are easiest to grasp. Intuitively one would believe that the effect of $\sigma$ on the probability that $x_T > x_i^*$ depends on whether $x_i^* > E(x_T)$. To develop this idea, start with the normal distribution and consider the probability of $\ln(x_T)$ exceeding $\ln(x_i^*)$ in a case when $E[\ln(x_T)]$ does not depend on $\sigma$ (contrary to the present model, which has $E[\ln(x_T)] = \ln(x_0) + T(\mu - \sigma^2/2)$). It can be shown that this probability is increasing in $\sigma$ if and only if

$$\ln(x_i^*) \geq E[\ln(x_T)] \Leftrightarrow \ln \left( \frac{x_i^*}{x_0} \right) - T \left( \mu - \frac{\sigma^2}{2} \right) \geq 0. \quad (6)$$

But consider now the present model. The partial derivative $\partial \Phi_1 / \partial \sigma |_{x_i^*}$ is positive if and only if

$$\ln \left( \frac{x_i^*}{x_0} \right) - T \left( \mu + \frac{\sigma^2}{2} \right) \geq 0. \quad (7)$$

This does not always hold, but depends on parameter values. For given values of the other parameters, the expression can be made negative either by a sufficiently high $\mu$, by a sufficiently high $\sigma$, or (except if $\mu$ is far below zero) by a sufficiently high $T$. For the “opposing effect” this means that a higher $\sigma$ can easily lead to a lower probability of exceeding a given trigger level, $x_i^*$.

The condition $x_i^* > E(x_T)$ is equivalent to

$$\ln \left( \frac{x_i^*}{x_0} \right) - T \mu > 0, \quad (8)$$

which is implied by (7). However, there is no implication in the opposite direction, from (8) to (7). We may well have (as long as $\sigma > 0$) that $E(x_T)$ is somewhat less than $x_i^*$, but at the same time $\Phi_1$ is decreasing in $\sigma$ for a given $x_i^*$. This shows that the reversal of the “opposing” effect should not be ignored (or ascribed to an unreasonably high $E(x_T)$) as

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4So far only the distribution at $T$ has been considered. The effect via $Q\Phi_2$ is also ambiguous, and is considered below.

5This possibility vanishes if $\nu = \mu - \sigma^2/2$ is held constant when $\sigma$ is varied, instead of holding $\mu$ constant.
far as $\Phi_1$ is concerned. The effect of $\sigma$ on $\Phi_1$ for a given $x^*_i$ may be negative even when $E(x_T)$ is less than the trigger level.

The differences between the three inequalities, (8), (7), and (6), are due to the facts that the $\ln$ function is non-linear (which explains the difference between (6) and (8)) and that $E[\ln(x_T)]$ depends on $\sigma$ by assumption (which determines (7)).

What about the other term in (4)? An increase in $\sigma$ seems to have an ambiguous effect on $Q\Phi_2$. This depends on

$$\frac{\partial(Q\Phi_2)}{\partial\sigma} \Bigg|_{x^*_i} = \frac{Q}{\sigma^2} \left\{ \varphi_2 \cdot \left[ \ln\left(\frac{x^*_i}{x_0}\right) + \sqrt{T} \left( \mu + \frac{\sigma^2}{2} \right) \right] - 4\Phi_2 \ln\left(\frac{x^*_i}{x_0}\right) \frac{\mu}{\sigma} \right\},$$

where $\varphi_2$ is the normal density corresponding to $\Phi_2$. No detailed discussion is offered here, whereas the total effects show up in the numerical analysis in section 6.

4 What does it mean to increase uncertainty?

Increased uncertainty in this model is obviously taken to mean a higher $\sigma$. There is a conceptual problem in analyzing the response to this, namely whether this increase is seen as a one-time, unexpected increase to a new, constant $\sigma$ value, in a particular historical situation (with a given set of potential projects), or something else. Taken literally the model does not allow for changes in $\sigma$. But the usual comparative statics analysis considers a starting situation in which some parameters are given, and makes the experiment of changing one parameter, regarding the others as fixed. Perhaps a model with stochastic volatility and optimal response to this would have been better suited to make predictions about the effects of changing volatility.

But even within the standard comparative statics of the present model, there is the question of which parameters are seen as fixed when $\sigma$ is increased.\footnote{The alternatives considered here are either $\rho$ being fixed or $\delta$ being fixed. A third possibility would be to keep $\nu$ fixed. This would be in line with estimating the drift on logarithmic data.} This point was also made by Cappuccio and Moretto (2001). Equations (2) and (3) above conceal the relationship $\delta = r + \lambda \rho \sigma - \mu$. Sarkar (2000) regards $r, \lambda, \rho$, and $\mu$ as constants when analyzing changes in $\sigma$. This is in line with the comparative statics analysis in Dixit and
Pindyck (1994, p. 179). However, McDonald and Siegel (1986) choose another assumption, and point out (their footnote 14) that there may be two interpretations of increased $\sigma$: One which is uncorrelated with the market portfolio and one which is not.

One great achievement of financial option theory, starting with Black and Scholes (1973), is that its results do not depend on systematic risk. When there is a rate-of-return shortfall (McDonald and Siegel, 1984), $\delta$, it is possible that systematic risk plays a role through the term $\lambda \rho \sigma$. But consider what happens if uncertainty is increased by addition or multiplication of a random variable which is stochastically independent of the other variables.

Let $dm$ be the increment of the market portfolio, while $dx$ is introduced above. For clarity, use now the notation $\sigma_x$ for what has been $\sigma$ above. For this discussion $dm$ and $dx$ are treated as any random variables, disregarding their infinitesimal dimension. Define $d\hat{x} \equiv dx/x$ and $d\hat{m} \equiv dm/m$. Their covariance is $\sigma_{xm}$, and their correlation is $\rho = \sigma_{xm}/\sigma_x \sigma_m$. Let superscript $o$ denote values at the outset, and superscript $a$ denote values after an increase in uncertainty. Two specifications of this increase will be considered.

First, let $\varepsilon$ be a random variable (with strictly positive variance) which is stochastically independent of $(d\hat{x}, d\hat{m})$, with $E(\varepsilon) = 0$. Let $d\hat{x}^o \equiv d\hat{x}^o + \varepsilon$. Then $\sigma_{xm}^o = \text{cov}(d\hat{x}^o + \varepsilon, d\hat{m}) = \text{cov}(d\hat{x}^o, d\hat{m}) = \sigma_{xm}^o$, while $\sigma_x^o = \sqrt{\text{var}(d\hat{x}^o + \varepsilon)} > \sqrt{\text{var}(d\hat{x}^o)} = \sigma_x^o$. Thus, if an increase in $\sigma_x$ takes the form of adding a random variable which is independent like this, then the covariance and the well-known $\beta$ (from the ICAPM) are unaffected. This means that the correlation is reduced, and that $\lambda \rho \sigma$, and thus $\delta$, are unaffected.

A similar effect follows from multiplicative uncertainty. Let $\psi$ be a random variable (with strictly positive variance) which is stochastically independent of $(d\hat{m}, d\hat{x})$, with $E(\psi) = 1$. In this case, redefine $d\hat{x}^o \equiv \psi d\hat{x}^o$. Then $\sigma_{xm}^o = \text{cov}(\psi d\hat{x}^o, d\hat{m}) = \text{cov}(d\hat{x}^o, d\hat{m}) = \sigma_{xm}^o$, while $\sigma_x^o = \sqrt{\text{var}(\psi d\hat{x}^o)} > \sqrt{\text{var}(d\hat{x}^o)} = \sigma_x^o$. Thus, if an increase in $\sigma_x$ takes the form of multiplication with a random variable which is independent like this, then the covariance is unaffected. Again the correlation is reduced, and $\beta, \lambda \rho \sigma$, and thus $\delta$, are unaffected.

These two examples show that there may be good reasons to consider the kind of increase in $\sigma$ which does not affect $\delta$: This is what happens if the increase is independent of the other random variables. It should be noted, however, that it is impossible to maintain
a fixed covariance if $\sigma_x$ is reduced to zero. In order for the similar type of independence to work when $\sigma_x$ is reduced, it is a condition that $d\hat{x}$, before the reduction, can be written as a sum of two stochastic variables, one of which is independent of the vector of the other and $d\hat{m}$.

5 How to measure the effect on investment?

The suggestion in Sarkar (2000) is to measure the effect on investment by the effect on the probability of investment during some time interval. This measure has the advantage of simplicity, i.e., one does not have to specify too many parameters in order to arrive at a number. The disadvantage is that it is not necessarily proportional to what one wants to measure, investment. Figure 1 in Sarkar (2000) shows that the probability is a concave function of $\sigma$, first increasing, then decreasing. Whether this carries over to investment is not so clear.

An alternative measure could be based on heterogeneity of investment projects. The simplest extension seems to be to assume that the investment cost varies across projects. The distribution of these costs will obviously be important. For some applications there may be reasons to introduce a particular distribution. For the present discussion a discrete approximation to a uniform distribution will be used. This allows a calculation of expected investment over some time horizon. If the projects are indexed by $i$, and the costs are $K_1, \ldots, K_n$, then the expected investment over a horizon of length $T$ is

$$E(\text{aggregate investment}) = \sum_{i=1}^{n} E(\text{inv. in project } i) = \sum_{i=1}^{n} \Pr(\text{inv. in project } i)K_i.$$  \hspace{1cm} (10)

The introduction of $K_i$ gives an additional non-linearity in the expression.

The simpler probability measure is more interesting if one is only considering the decision on a single investment project. The expected investment with a distribution of projects is more interesting for macroeconomic predictions about the effect of uncertainty on investment in a sector (or a region, a nation). A drawback is that some arbitrariness is introduced by looking at a particular set of projects. In particular one should not forget the dependence on history mentioned in the beginning of section 4.
6 Consequences for numerical results

As pointed out by Sarkar (2000), the sign of the effect on investment is ambiguous. While this was true when $\rho$ was assumed fixed, it is still true when $\delta$ is assumed fixed. Figures 1–3 show the probability of investment within 5 years as functions of the volatility, $\sigma$, and reproduce the numerical example in Sarkar (2000) with some modifications. In these calculations\(^7\) there is only one potential project with $K_1 = 1$. Figures 4–5 show expected investment with a distribution of project costs. The parameter values are identical to those in Sarkar (2000) except where noted. In particular, $r = 0.1$ and $x_0 = 0.1$ are used throughout.

Figure 1 reproduces Sarkar (2000) exactly, with $\mu = 0$, $\rho = 0.7$, $\lambda = 0.4$, which implies that $\delta$ increases as $\sigma$ is increased. The solid curve reproduces (and verifies) Sarkar’s Figure 1. As $\sigma$ goes from 0 to 0.6, $\delta$ is increasing from 10 percent to more than 26 percent. These are high numbers, and although $\delta$ can vary a lot empirically, most observations seem to be below 10 percent, cf. Milonas and Henker (2001) for crude oil and Heaney (2002) for copper, lead and zinc. $x^*_i$ is increasing monotonically from 0.1041 to 0.485 as $\sigma$ goes from 0.01 to 0.6.

The dotted curve has $\mu = 0.01$. This minor increase from $\mu = 0$ changes the probability curve dramatically, and it is now strictly decreasing. At low values of $\sigma$, the probability of investing is much higher, cf. the discussion in section 3. $x^*_i$ is increasing from 0.1006 to 0.308.

Figure 2 is based on calculations where again $\mu = 0$, but this time without any effect of $\mu$ on $x^*_i$, as $\delta$ is held fixed\(^8\) at 0.05. $\nu$ varies according to $\nu = \mu - \sigma^2/2$. $\nu$ is thus negative, and increasingly so as $\sigma$ increases. The effect of keeping $\delta$ fixed (and lower) is to increase the probability of investing. $x^*_i$ is increasing from 0.1001 to 0.308.

Figure 3 has the same parameter values as Figure 2, except that $K_i$ is now increased from 1 to 2. This gives a dramatically different curve, increasing with an inflection point. $x^*_i$ is increasing from 0.2002 to 0.616, exactly twice those values relating to Figure 2, cf.

\(^7\)The computer programs used are available at http://folk.uio.no/dilund/realopt.

\(^8\)For Figures 2–3 and 5, $\lambda$ and $\rho$ are not relevant for the calculations.
equation (3). This makes the probability of reaching the trigger level extremely low for low $\sigma$, and low even for higher $\sigma$ values.

Figures 4 and 5 reproduce the parameter vectors and the solid curves of Figures 1 and 2, respectively. But now two additional potential projects are added, with $K_2 = 2$ and $K_3 = 3$. The dashed curves show expected aggregate investment within 5 years when only firms 1 and 2 exist, while the dotted curves show expected aggregate investment when all three firms exist. To understand this in more detail, consider Figure 5. The difference between the dashed and the solid curve is the second term in the sum in (4). This is a probability multiplied by $K_2 = 2$. This particular probability is already shown in Figure 3. For $\sigma = 0.6$ it takes the value 0.06, and the difference between these two curves in Figure 5 is thus equal to $0.12 = 2 \cdot 0.06$ at $\sigma = 0.6$.

Since the probabilities are generally increasing functions of $\sigma$ for higher values of $K_i$, like in Figure 3, the differences between the curves are increasing in $\sigma$.

Hopefully a numerical exercise like this can shed some light on the mechanisms relating uncertainty to these measures of investment. It is quite clear that the relationship is neither straightforward to define nor (for many interpretations) strictly decreasing.

7 Discussion

There will, with probability one, be long periods of time during which $x_t$ is less than its previous maximum value. This is particularly pronounced under GBM, even with some positive drift. If all potential projects are known from many years back, there will be long periods without any investment at all in the model presented, cf. Lund (1993). This means that aggregate investment in $x_t$-yielding projects will be erratic. Perhaps a smoother aggregate investment can be obtained if there are many different earnings processes, not perfectly correlated, or if there is a stream of new potential investment projects.

The unreasonable features of GBM is one reason why Sarkar (2003) and other researchers have considered alternative stochastic processes. This paper will not discuss those in any detail. However, Sarkar (2003), after mentioning in the abstract the two
opposing effects of uncertainty on investment, proposes “incorporating a third factor, the
effect of mean-reversion on systematic risk.”

While there may well be a link between mean reversion and systematic risk, it is not
clear that this influences the relationship between $\sigma$ and investment. One should again take
care to specify what parameters are kept constant in the comparative statics. Changing $\sigma$
is in itself not necessarily an argument for a change in systematic risk.

The paper by Cappuccio and Moretto (2001) points out the possibility of keeping $\delta$
fixed in the analysis. They give no arguments why this may be a reasonable assumption,
and they do not mention its place in the previous literature. They describe the assumption
in the following terms (p. 11), “If we assume the dividend yield $\delta$ as the exogenously
fundamental market parameter, . . . .” As shown in section 4, this is somewhat misleading.
There can be given separate arguments (additive or multiplicative noise) for keeping $\sigma_{xm}$
and thus $\delta$ constant when $\sigma_x$ changes. Next, they describe the alternative assumption, $\rho$
constant, as follows (p.11), “On the contrary, if we rely on the capital asset pricing model
to evaluate the project price of risk . . . .” (which they define on p. 10 as $\lambda_M \rho$, or $\lambda \rho$ in
the notation of this paper). As shown in section 4, one may want to assume a constant $\delta$
even if one relies on the CAPM.

Their other main point is a suggestion to consider an American option with a finite
expiration date instead of an option which lasts forever, as in Sarkar (2000) and the present
that the expiration period and the period for calculation of the probability are different.

It is true that a finite expiration date is realistic for some real options, for instance due
to the expiration of a license. But it is not necessarily more realistic in general. A property
owner may typically postpone an investment decision forever. There is a long tradition for
considering infinitely-lived real options. For macroeconomic studies one would want some
typical situation, while for particular investment decisions, one will be able to determine
the details in each case.

If an infinitely-lived option is chosen, one might ask why to consider probabilities of
investing within some arbitrary time period, like five years. There are at least two reasons
for this. There is the technical reason that the probability of exceeding the trigger level ap-
proaches either unity or zero as the time period considered goes to infinity, cf. footnote 2. A pragmatic reason is that most investors and policy makers would be satisfied with a prediction for the next five to ten years, and that one can hardly expect a new volatility estimate to stay unchanged forever, cf. the discussion in the beginning of section 4. As a conclusion, it is not unreasonable to consider an infinitely-lived option instead of introducing an arbitrary expiration date. The cost is that one has to introduce a different (and equally arbitrary) calculation period for the probability, but some finite calculation period is a reasonable focus of interest, and will be the only interesting situation to analyze.

8 Conclusion

In the analysis of the effect of uncertainty on investment, there may or may not be an opposing effect to the traditionally observed negative effect. Furthermore, one must be careful in the definition of a change in uncertainty, even in comparative statics of a real options model based on GBM with constant volatility. It is not obvious that systematic risk is affected. One should also consider carefully what is a meaningful measure of investment in the model. The probability of a particular investment being undertaken within some time horizon is not proportional to expected aggregate investment, which is a possible measure of investment.

The numerical exercises undertaken here clearly show that depending on the specification of the model and its parameters and the measure of investment, the relationship between investment and uncertainty can be either strictly increasing, strictly decreasing, or non-monotone. This confirms the qualitative conclusion of Sarkar (2003).
References


Figure captions

Figure 1: Probability of investment in 5 years; $r = 0.1, \lambda = 0.4, x_0 = 0.1, K = 1; \rho$ fixed at 0.7; solid curve has $\mu = 0$; dotted curve has $\mu = 0.01$

Figure 2: Probability of investment in 5 years; $\mu = 0, r = 0.1, x_0 = 0.1, K = 1; \delta$ fixed at 0.05

Figure 3: Probability of investment in 5 years; $\mu = 0, r = 0.1, x_0 = 0.1, K = 2; \delta$ fixed at 0.05

Figure 4: Expected investment in 5 years; $\mu = 0, r = 0.1, \lambda = 0.4, x_0 = 0.1; \rho$ fixed at 0.7; curves show 1, 2, and 3 projects (cumulatively), with $K_i = i$

Figure 5: Expected investment in 5 years; $\mu = 0, r = 0.1, x_0 = 0.1; \delta$ fixed at 0.05; curves show 1, 2, and 3 projects (cumulatively), with $K_i = i$
Figure 1: Probability of investment in 5 years; $r = 0.1, \lambda = 0.4, x_0 = 0.1, K = 1$; $\rho$ fixed at 0.7; solid curve has $\mu = 0$; dotted curve has $\mu = 0.01$. 

Probability of investing as function of volatility

Pr(Invest by time T)

Volatility

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7

0.0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6
Figure 2: Probability of investing as function of volatility

Figure 2: Probability of investment in 5 years; $\mu = 0, r = 0.1, x_0 = 0.1, K = 1; \delta$ fixed at 0.05
Figure 3: Probability of investment in 5 years; $\mu = 0, r = 0.1, x_0 = 0.1, K = 2; \delta$ fixed at 0.05
Figure 4: Expected investment in 5 years; \( \mu = 0, r = 0.1, \lambda = 0.4, x_0 = 0.1; \rho \) fixed at 0.7; curves show 1, 2, and 3 projects (cumulatively), with \( K_i = i \).
Figure 5: Expected investment in 5 years; $\mu = 0, r = 0.1, x_0 = 0.1; \delta$ fixed at 0.05; curves show 1, 2, and 3 projects (cumulatively), with $K_i = i$