Mass transport induced by internal Kelvin waves beneath shore fast ice

E. Støylen and J. E. Weber
Department of Geosciences, University of Oslo,
P.B. 1022 Blindern, N-0315 Oslo, Norway

Background and scope
Coastally trapped Internal Kelvin waves may reach large amplitudes, and are always propagating in one direction. When induced by tides the motion is steady, motivating study of the second order wave drift. This drift may yield systematic transport of densities, and a rigid ice lid at z=H1. The under-ice pressure P_s is not necessarily constant.

Mathematical model
We look at a two-layer reduced gravity model under idealized basin.

Second order theory
The upper layer currents are integrated in the vertical between material surfaces ξ and H_1, in order to retain the Lagrangian drift properties of the flow. The resulting flux terms are separated in a fluctuating (tilde) and a mean (overbar) part:

\[ U = \overline{U} + \tilde{U} \]

The mean contribution is to second order in wave steepness. It consists of the general Stokes drift in addition to a mean Eulerian term dependent on friction:

\[ \overline{U} = \overline{U}_E + \overline{U}_K \]

The Stokes term is derived directly from the linear wave. The vertically integrated equations are averaged over a wave period to obtain the following set of equations solving the Eulerian motion (subscript indicates partial derivatives):

\[ -f\tilde{\overline{V}_E} - c_s^2 \tilde{\overline{V}_E} = -(3/2) c_s \alpha \overline{U}_E - c_D \frac{\overline{U}_E \overline{V}_E}{H_1} \]

Here we assume that friction acts on the mean motion through some drag coefficient c_D. This problem resembles the steady barotropic storm surge problem; the forcing here is divergence of Stokes flux (radiation stress). Also note a forcing term in the continuity equation.

The resulting Eulerian fluxes are

\[ \overline{U}_E = \sqrt{\frac{ah_1}{c_p}} A e^{\alpha x/y} \]

\[ \overline{F}_E = \alpha c_D A \left[ \frac{a h_1}{2 c_p} \left( e^{-2 \beta x/y} \right) e^{\alpha x/y} + \frac{ah_1}{c_D} \left( e^{-2 \beta x/y} \right) e^{\alpha x/y} \right] \]

Numerical simulation
We utilize a linear vertically integrated reduced gravity model in a circular geometry. We assume known wave motion, and insert the resulting radiation stress terms as geometrically dependent forcing terms over the domain. The variables are the Eulerian fluxes, and the mean interface displacement.

Results are shown in Fig. 2. The black lines in a) indicate the wave generation area, and max wave amplitude is just to the right of this area. The interface maximum is close to the wave entrance, decreasing along the coast, and more abruptly toward the interior of the basin. The currents generally follow the isobars, trapped near the coast. Close to the edge of the trapping region there is a current component normal to the coast, in accordance with theoretical results. Outside the trapping area, a small return current is observable.

Realistic topographies
In order to study the internal Kelvin wave generation, numerical model runs are made for a two-layer system, solving the linear wave motion from oscillatory interface boundary conditions. Fig. 3 a) show the result from a semi-enclosed rectangular basin with a shallow and narrow sound at x=40km, and open boundary at x=0. Fig. 3 b) show result for a topography resembling the Van Mijen fjord in Svalbard. The two-layer density profile applied do not resemble the physical conditions in this fjord, we merely compare results from the two geometries to compare the resulting wave patterns. The striking similarities indicate that the internal Kelvin wave is a robust phenomenon with respect to topographies.

Conclusions
• Eulerian mean currents may be comparable to the conventional Stokes drift in magnitude.
• Due to damping of wave amplitude, a mean flow normal to the coast arises. This is verified numerically.
• Internal Kelvin wave propagation is not easily disturbed by topography, thus transport mechanisms from these waves may be important in stratified coastal environments, e.g. the Barents Sea and Arctic fjords.