Proton-proton scattering from low to LHC-energies

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- Coulomb scattering
- Low-energy strong interactions
- Regge poles and the pomeron
- The Froissart bound
- Feynman parton model
- Recent ideas and conclusion
Coulomb scattering

in CM-frame:

\[ Q^2 = 4p^2 \sin^2 \theta / 2 \]
Differential cross-section: \[ \frac{d\sigma}{d\Omega} = |f(Q)|^2 \]

In first Born approximation
\[ f(Q) = \frac{m}{2\pi} V(Q) \]

with
\[ V(Q) = \int d^3r \, V(r) \, e^{-iQ \cdot r} \]

in CM-frame where proton momentum \( p = mv \)

and reduced mass \( m = M/2 \).
Coulomb potential

\[ V(r) = \frac{e^2}{4\pi r} \]

with Feynman diagram

\[ V(Q) = \frac{e^2}{Q^2} \]

giving scattering amplitude

\[ f(Q) = \frac{\alpha}{4E \sin^2(\theta/2)} \]

where \( E = \frac{p^2}{2m} \) and \( \alpha = \frac{e^2}{4\pi} = 1/137 \)
Full Coulomb propagator

\[ \begin{array}{c}
\text{Full Coulomb propagator} \\
\hline
= \quad + \quad + \quad + \cdots
\end{array} \]

results in non-perturbative scattering amplitude

\[ f_C(\theta) = \frac{\alpha}{4E \sin^2(\theta/2)} e^{-i\eta \ln \sin^2(\theta/2)} \]

where parameter \( \eta = \alpha/v \) gives effective strength of the Coulomb interaction.
Coulomb cross-section is unmodified:

\[
\frac{d\sigma}{d\Omega} = \left( \frac{\alpha}{4E \sin^2(\theta/2)} \right)^2
\]

Probability to find two protons at zero separation:

\[
|\psi(0)|^2 = \frac{2\pi \eta}{e^{2\pi \eta} - 1}
\]

Becomes exponentially small when \( \eta > 1 \)

i.e. when \( p < 10 \text{ MeV} \).  Thus Coulomb interaction dominates for energies \( E < 1 \text{ MeV} \).
Low-energy strong interactions

\[ E < 100 \text{ MeV} \]

\[ V_{\text{eff}} = C \delta(\mathbf{r}) \]

described by effective Langrangian

\[
\mathcal{L} = i \psi^* \dot{\psi} + \frac{1}{2M} \psi^* \nabla^2 \psi - \frac{C}{2} (\psi^* \psi)^2
\]

Cross-section to lowest order \[
\frac{d\sigma}{d\Omega} = a^2
\]

where scattering length \[
a = \frac{m}{2\pi} C
\]
Higher order corrections

\[
\begin{aligned}
&\quad + \ldots + \quad + \ldots \\
&\quad + \ldots + \\
\end{aligned}
\]

given by bubble integral

\[
I_0(p) = \int \frac{d^3 k}{(2\pi)^3} \frac{2M}{p^2 - k^2 + i\epsilon}
\]

Full scattering amplitude:

\[
T = C \left[ 1 + CI_0 + (CI_0)^2 + (CI_0)^3 + \cdots \right]
\]

\[
= \frac{C}{1 - CI_0} = \frac{1}{1/C - I_0(p)}
\]
Bubble integral with cut-off regularization:

\[ I_0(p) = -\frac{M}{2\pi^2} \left( \Lambda + \frac{i}{2} \pi p \right) \]

Remove cut-off \( \Lambda \) by introducing renormalized coupling constant

\[ \frac{1}{C_R} = \frac{1}{C} + \frac{M \Lambda}{2\pi^2} \]

Renormalized coupling \( C_R \) goes to zero as \( \Lambda \to \infty \), no scattering on \( \delta \)-function potential in this limit. Theory is then trivial.

Thus

\[ T = \frac{4\pi}{M} \frac{1}{1/a + ip} \quad \text{with} \quad a = \frac{M}{4\pi C_R} \]

and differential cross-section becomes

\[ \frac{d\sigma}{d\Omega} = \frac{a^2}{1 + (ap)^2} \]
For proton-proton scattering with $E \ll 100$ MeV must include Coulomb repulsion.

**Blatt & Weisskopf: Theoretical Nuclear Interactions**

Full scattering amplitude:

$$f(\theta) = f_C(\theta) + f_S$$

$E = 3$ MeV
Coulomb-modified strong interactions:

\[ f_S = X + \left( X + \text{Coulomb-modified bubble} \right) + \cdots \]

Again form geometric series

\[ f_S = \frac{C_0}{1 - C_0 J_0(p)} \]

where Coulomb-dressed bubble now is

\[
J_0(p) = M \int \frac{d^3 k}{(2\pi)^3} \frac{2\pi \eta(k)}{e^{2\pi \eta(k)} - 1} \frac{1}{p^2 - k^2 + i\epsilon}
\]
Can be done analytically Kong and Ravndal (1998) and gives Coulomb-modified scattering length:

\[
\frac{1}{a_C} = \frac{1}{a(\mu)} - \alpha M \left[ \ln \frac{\mu \sqrt{\pi}}{\alpha M} + 1 - \frac{3}{2} C_E \right]
\]

Blatt and Jackson (1950) from quantum mechanics in a potential model.

Proton-proton fusion:

\[
p + p \rightarrow D + e^+ + \nu_e
\]
Regge poles and the pomeron

$E > 1 \text{ GeV}$

\[ s = (p_1 + p_2)^2 \]
\[ = 2m_{\text{lab}} \]
\[ t = (p_1 - p_3)^2 \]
\[ = -Q^2 \]
\[ = -4p^2 \sin^2(\theta/2) \]

Differential x-section:

\[ \frac{d\sigma}{dt} = \pi |T(s, t)|^2 \]

Total x-section:

\[ \sigma_T = 4\pi \text{Im}T(s, 0) \]
Including Coulomb interaction

\[ T(s, t) \implies \frac{2\alpha}{t} F^2(t) e^{i\phi(t)} + T(s, t) \]

with proton form factor \( F(t) = \frac{1}{(1 - t/0.71)^2} \)

and calculable phase \( \phi(t) \)

\[
\text{ISR:} \quad \sqrt{s} = 24 \text{ GeV}
\]
Strong amplitude

\[ T(s, t) = \beta_R(t)s^{\alpha_R(t)-1} + \beta_P(t)s^{\alpha_P(t)-1} \]

Regge trajectory: \( \alpha_R(t) = 0.5 + \alpha'_R t \)

Pomeron trajectory: \( \alpha_P(t) = 1.0 + \alpha'_P t \)

\[ \sigma_T(s \to \infty) = 4\pi \text{Im} \beta_P(0) + \mathcal{O}(1/\sqrt{s}) \]

Total x-section should approach a constant value at high energies!
Pomeranchuk theorem (1958):

\[ \sigma_T(AB) = \sigma_T(\bar{A}\bar{B}) \]

as \( s \to \infty \)

Isaak Pomeranchuk (1913 - 1966)
Impact parameter representation from partial wave expansion:

\[ f(s, \theta) = \frac{1}{2ip} \sum_{\ell=0}^{\infty} (2\ell + 1) P_{\ell}(\cos \theta) \left[ e^{2i\delta_{\ell}(s)} - 1 \right] \]

Eikonal approximation: Replace sum over partial waves with integral over impact parameter \( b \):

1: \( \ell + 1/2 \rightarrow pb \)
2: \( \sum_{\ell=0}^{\infty} \rightarrow \int_{0}^{\infty} d\ell = p \int_{0}^{\infty} db \)
3: \( P_{\ell}(\cos \theta) \rightarrow J_{0}[(2\ell + 1) \sin \theta/2] \)
4: \( 2\delta_{\ell}(s) \rightarrow E(s, b) \)
\[ T(s, Q) = i \int \frac{d^2b}{2\pi} \left[ 1 - e^{iE(s,b)} \right] e^{iQ \cdot b} \]

Eikonal expansion:

\[ T(s, Q) = E(s, Q) + \frac{i}{2} E \otimes E - \frac{1}{6} E \otimes E \otimes E + \ldots \]

Hamer & Ravndal, 1970:

\[ E(s, Q = \sqrt{-t}) = \beta_R(t)s^{\alpha_R(t)-1} + \beta_P(t)s^{\alpha_P(t)-1} \]
Total, asymptotic cross-section becomes

\[ \sigma_T(s \to \infty) = 4\pi C \left[ 1 - \frac{C}{8\alpha'_P \log s} \right] \]

with \( C = \text{Im} \beta_P(0) \)

Serpukhov, 1970:

\( p_{lab} = 60 \text{ GeV} \)
\( \sqrt{s} = 10 \text{ GeV} \)

LHC, 2010:

\( \sqrt{s} = 14000 \text{ GeV} \)
ISR (1971 - 1984):

Total x-section increases at higher energies !!
Landshoff & Donnachie (1984)

Non-standard pomeron: \( \alpha_P(t) = 1.08 + \alpha'_P t \)

\[
\begin{align*}
\bar{p}p & : 21.70s^{0.0808} + 98.39s^{-0.4525} \\
p p & : 21.70s^{0.0808} + 56.08s^{-0.4525}
\end{align*}
\]

Violates Froissart bound!
Radius of proton: \( R \simeq 1\text{fm} = 10^{-13}\text{cm} \)

Classical x-section: \( \sigma_T = \pi R^2 = 30\text{mb} \)
Simplest inelastic process when one pion produced in overlap region where fraction $e^{-bm\pi}$ of total energy $\sqrt{s}$ is available:

$$e^{-bm\pi} \sqrt{s} \geq m_{\pi}$$

Thus

$$b_{\text{max}} = \frac{1}{2m_{\pi}} \log \frac{s}{m_{\pi}^2} \quad \text{(Heisenberg)}$$

Total x-section

$$\sigma_T \leq \pi b_{\text{max}}^2 = \frac{\pi}{4m_{\pi}^2} \log^2 \frac{s}{m_{\pi}^2}$$

where

$$\frac{\pi}{4m_{\pi}^2} \simeq 15 \text{ mb} \quad \text{Fits need much smaller value.}$$
Froissart saturation: \( \sigma_T \propto \log^2 \frac{s}{m^2_\pi} \)
Feynman parton model

\[ f(x) |_{x \to 0} \propto \frac{1}{x^\alpha} \]
Feynman, 1970:

Assume completely absorptive scattering amplitude

\[ T(s, t) = iA(s, t) \]

so that scattering operator in impact-parameter representation

\[ S(s, b) = 1 - A(s, b) \]
Incoming parton wave function

\[ |\Psi\rangle = \sum_{n=0}^{\infty} C_n(x_1, x_2, \cdots, x_n) |P, n\rangle \]

Only wee partons contribute

\[ |C_n|^2 = \frac{|C_0|^2}{n!} \frac{c}{x_1} \frac{c}{x_2} \cdots \frac{c}{x_n} \]

where expect \( c \ll 1 \).

Normalization \( \langle \Psi | \Psi \rangle = 1 \) gives now:

\[ 1 = |C_0|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \left( c \int_{1/s}^{1} \frac{dx}{x} \right)^n = |C_0|^2 s^c \]
Each parton with energy \( xs \) scatters with amplitude \( \hat{S}(xs, b) \) so that scattered state becomes

\[
S(s, b) = \langle \Psi | S | \Psi \rangle = \exp \left( - c \int_{1/s}^{1} \frac{dx}{x} \left[ 1 - \hat{S}(xs, b) \right] \right)
\]

Self-consistency:

\[
\hat{S}(s, b) = S(s, b) \Rightarrow S(s, b) = \frac{1}{1 + F(b)s^c}
\]

where unknown function \( F(b) \) must decrease faster than any power - from unitarity.
\[ F(b) = e^{-b/a} \quad \text{with} \quad a \simeq 1 \text{ fm} \]

**Transition amplitude:**

\[ A(s, b) = 1 - S(s, b) = \frac{1}{e^{b/a-c \log s} + 1} \]

\[ b_{max} = ac \log s \quad \text{and total x-section} \quad \sigma_T \simeq \pi b_{max}^2 \]

\[ \sigma_T = \pi (ac)^2 \log^2 s \quad \text{Froissart!} \]
2: Better fit to differential x-section with

\[ F(b) = e^{-b^2/a^2} \]

which now gives total x-section

\[ \sigma_T = \pi c a^2 \log s \]
Recent ideas and conclusion

QCD:

Pomeron:

+ ... +

+ ....

BFKL
AdS/QCD:

4-dim Minkowski: QCD

5-dim Anti-de-Sitter: String theory

Reggeon:

Open string: \( J = 1 \)

Pomeron:

Closed string: \( J = 2 \)
LHC (CMS): TOTEM

\[ \gamma = 2.2 \text{ (best fit)} \]

\[ \gamma = 1.0 \]

\[ +1 \sigma \]

Cosmic ray data

\[ \sigma_{\text{tot}} \text{ (mb)} \]

\[ \sqrt{s} \text{ (GeV)} \]

Cosmic rays: hep-ph/0011167
A colourful connection

The scientists awarded this year’s Nobel Prize in Physics have solved a mystery surrounding the strongest of nature’s four fundamental forces. The three quarks within the proton can sometimes appear to be free, although no free quarks have ever been observed. The quarks have a quantum mechanical property called colour and interact with each other through the exchange of gluons – nature’s glue.

Inside the proton

The three quarks within the proton are held together by the powerful force mediated by the gluons, depicted here as coiled springs. As the distance between the quarks increases, so does the force between them.

The Standard Model and the four forces

The quarks and gluons of the strong (or colour) force are the third piece in the puzzle of nature’s four forces. The first piece, the electromagnetic force, is similar to the strong force but instead of gluons, particles of light, photons, are the force carriers. The gluons carry colour charge while the photons are electrically neutral. The second piece in the puzzle is the weak force, which controls some radioactive decays and energy production in the sun. This force differs from the other two because the force-carrying particles are very heavy. The fourth force, gravity, is the least understood even though it is experienced by us all. Gravitation is thought to be the force-carrying particles, but they have yet to be discovered. The Standard Model provides a description of all the forces apart from gravity.

The theory shows its true colours

The aftermath of a high-energy collision between a proton and an electron, as seen by the detector at the E821 experiment at Fermilab. The experiment is shown in cross-section, illustrating the destruction of proton and electron. The electron has a shock wave of radiation behind it, and the proton produces a shower of energetic particles. The diagram shows a portion of the shower and the experimental strong magnetic field.

A unified theory for all forces

This year’s prize paves the way for a more fundamental future description of the forces in nature. The electromagnetic, weak and strong forces have much in common and are perhaps different aspects of a single force. They also appear to have the same strength at very high energies, especially if supersymmetric particles exist. It may even be possible to include gravity if theories which treat matter as small vibrating strings are correct.