Introduction to self-control problems

Examples of self-control problems

- **Procrastination** — planning to do a task (studying, doing housework, dieting, exercising, stop smoking, saving) tomorrow (or next week etc.), but in the next period further postponement seems desirable.

- **Addiction** — planning to indulge in some addictive activity (smoking, drinking etc.) at a moderate level. But when smoking, drinking etc. at a moderate level, further use seems desirable.

  Preference reversal, without full awareness of the self-control problems that such reversals lead to.

Why do people procrastinate?

- **Present-biased preferences**
  Extra weight on current well-being.

- **(Partial) naivete**
  Not (fully) aware of the self-control problems such present-biased preferences lead to.

- **Present-biased preferences** $\Rightarrow$ An incentive to postpone the task to the next period.

- **(Partial) naivete** $\Rightarrow$ The dec. mak. believes falsely that the task will actually be performed then.

When (partially) aware of self-control problems, how do people cope?

- **Commitments:**
  - Make public resolutions.
  - Join a health club.
  - Leave the ATM/credit card at home.
  - Higher tax payments in Jan-May and July-Nov.
  - Buy an apartment.

  The fact that people make commitments is empirical evidence of self-control problems.
Discounted-utility model with a constant discount rate (Samuelson, RES 1937)

Let \( (c_1, \ldots, c_T) \) be an intertemporal consumption profile. Under the "usual assumptions", preferences over \( (c_1, \ldots, c_T) \) can be represented by \( U^t(c_1, \ldots, c_T) \).

If additively separable between time periods:
\[
U^t(c_1, \ldots, c_T) = \sum_{\tau=t}^{\infty} D(\tau-t) u(c_\tau).
\]

If \( u \) does not depend on \( \tau \):
\[
U^t(c_1, \ldots, c_T) = \sum_{\tau=t}^{\infty} D(\tau-t) u(c_\tau).
\]

Discounted-utility model (cont.)

If \( D(x) \) is expon. decreasing: \( D(x) = \delta^x \) so that
\[
U^t(c_1, \ldots, c_T) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(c_\tau) = \sum_{\tau=t}^{\infty} \left( \frac{1}{1+\rho} \right)^{\tau-t} u(c_\tau),
\]
where \( \delta = \frac{1}{1+\rho} : \) discount factor, and \( \rho = \frac{1}{\delta} - 1 : \) discount rate

- Samuelson (RES 1937) himself did not believe that the discounted-utility model with a constant discount rate was descriptively valid.
- And it is not; still it is being used for convenience.

Empirical evidence contradicting discounted utilitarianism with a constant discount rate

- Choice between
  - \$100 now and \$200 in 2 years
- Choice between
  - \$100 in 6 years and \$200 in 8 years
- Indifferent between
  - \$15 now and \$30 in 3 months
  - \$60 in 1 year
  - \$100 in 3 years

Indicates a decreasing utility discount rate

Alternatives to exponential discounting

In continuous time:

- Exponential discounting: \( D(x) = e^{-\rho x} \), \( -\frac{D(x)}{D(x)} = \rho \)
- Hyperbolic discounting: \( D(x) = \frac{1}{1+kx} \), \( -\frac{D(x)}{D(x)} = \frac{k}{1+kx} \)
- Logarithmic discounting:
  \[
  D(x) = e^{-\rho \ln(1+kx)} = (1 + kx)^{-\rho}, \quad -\frac{D(x)}{D(x)} = \frac{\rho k}{1+kx}
  \]
Alternatives to exponential discounting (cont.)

In discrete time:

- Exponential discounting: (Samuelson, 1937)
  \[ U^t(c_1, \ldots, c_T) = u(c_t) + \sum_{t=1}^{\infty} \delta^{t-1} u(c_t) \]
  \[ 0 < \delta < 1 \quad D(x) = \delta^x, \quad x \to x+1: \frac{1}{\delta} - 1 \]

- Quasi-hyperbolic discounting (Present-biased preferences, \((\beta, \delta)\)-preferences):  
  \[ U^t(c_1, \ldots, c_T) = u(c_t) + \beta \sum_{t=1}^{\infty} \delta^{t-1} u(c_t) \]  
  (Elster, 1979) 
  \[ 0 < \beta \leq 1 \quad D(0) = 1, \quad 0 \to 1: \frac{1}{\beta\delta} - 1 \]
  \[ 0 < \delta < 1 \quad D(x) = \beta\delta^x, \quad x \to x+1: \frac{1}{\delta} - 1 \]

Decr. disc. rate leads to time-inconsistency

- A task to be performed at time 0, 1, 2, …, or not at all.
- Immediate cost: 25. Benefits at the next stage: 125.
- \((\beta, \delta)\)-preferences with \(\beta = 1/2\) and \(\delta = 4/5\).

Decr. disc. rate leads to time-inconsistency (cont.)

Utility of doing the task at

| \(\tau\) = 1 | 30     | 25     |
| \(\tau\) = 2 | 24     | 30     |

\(\beta = \frac{1}{2}\)
\(\delta = \frac{4}{5}\)

Time-consistency

**Definition:** Preferences are time-consistent if for any \(t, s \geq t\), and \((c_t, c_{t-1}, \ldots, c_s)\):

\[ U^t(c_t, \ldots, c_{t-1}, c_s, \ldots, c_T) \geq U^t(c_t, \ldots, c_{t-1}, c'_s, \ldots, c'_T) \]

\[ \Downarrow \]

\[ U^s(c_s, \ldots, c_T) \geq U^s(c'_s, \ldots, c'_T) \]

**Strotz’s main result:** With, for all \(s \in \{t, \ldots, T-1\}\)

\[ U^s(c_s, \ldots, c_T) = \sum_{\tau=s}^{T} D(\tau - s)u(c_{\tau}) \]

preferences are time-consistent if and only if

\[ D(x) = \delta^x \]
Behavior with time-inconsistent preferences

- **Naive behavior**: Choosing the best plan under the presumption that it will be followed.
- May end up with a third best alternative.

**Sophisticated behavior**: Choosing the best plan among those that will actually be followed.

Demand for commitment

Multi-self model of sophistication

- Let every decision node correspond to a different “self” or “agent” of the decision-maker
- Sophisticated behavior: SPE of the game where each self is a different player.
  (Peleg & Yaari, 1973)
- Backward induct.

Evidence from psychology on hyperbolic disc.

Typical procedure elicits indiff. points of the form $A$ at time $x \sim B$ at time $x + y$

- **Approach 1**: (hyperbolic disc. is a “better fit”)
  
  Assumption: There exists a disc. fn. $D(x)$ such that $(A \text{ at } x \sim B \text{ at } x + y) \iff D(x)A = D(x + y)B$
  
  Compare:
  - Exponential discounting: $D(x) = e^{-\rho x}$
  - Hyperbolic discounting: $D(x) = \frac{1}{1+kr}$

Hyperbolic discounting is virtually always better fits.
Approach 2: (evidence of “declining discount rates”)

Given an indifference point \( A \) now ~ \( B \) at \( x \),

\[
A = e^{-\rho(x)x} B
\]

Empirical finding: The aver. disc. rate is declining in \( x \).

Exponential discounting: \( D(x) = e^{-\rho x} \) \( \Rightarrow \) Constant \( \rho(x) \)

Hyperbolic discounting: \( D(x) = \frac{1}{x + k} \) \( \Rightarrow \) Declining \( \rho(x) \)

Two more types of evidence.

- Intertemporal pref. reversals: \( A \) now > \( B \) at \( y \)
- Preference for commitment: \( A \) at \( x \) < \( B \) at \( x + y \)

Outline of lectures on self-control problems:

- Second Lecture (24 Sep.): Multi-self vs. temptation.
  - Consequences of naivete and sophistication within the multi-self model.
  - Direct modeling of temptation.

- Third Lecture (30 Sep.): Applic. to the theory of cons.
  - E.g., why do people accumulate illiquate assets while running up expensive credit card debts?

- Fourth Lecture (7 Oct.): Dual-self models.
  - Are our decisions a fight between a short-sighted and farsighted self?

Additional references

- Read and van Leeuwen (1998), Predicting Hunger: The Effects of Appetite and Delay on Choice, *Organ Behav Hum Decis Process* 76, 189–205

- Read, Loewenstein & Kalyanaraman (1999), Mixing virtue and vice: combining the immediacy effect and the diversification heuristic *J Behavioral Decision Making* 4, 257–273


- Laibson (2010), Instant gratification, multiple selves and self control: How to control your selves, Harvard Univ.