Examples of self-control problems

- **Procrastination** — planning to do a task (studying, doing housework, dieting, exercising, stop smoking, saving) tomorrow (or next week etc.), but in the next period further postponement seems desirable.

- **Addiction** — planning to indulge in some addictive activity (smoking, drinking etc.) at a moderate level. But when smoking, drinking etc. at a moderate level, further use seems desirable.

Preference reversal, without full awareness of the self-control problems that such reversals lead to.
Why do people procrastinate?

- **Present-biased preferences**
  Extra weight on current well-being.

- **(Partial) naivete**
  Not (fully) aware of the self-control problems that such present-biased preferences lead to.

- **Present-biased preferences** $\Rightarrow$ **An incentive to postpone the task to the next period.**
- **(Partial) naivete** $\Rightarrow$ **The dec. mak. believes falsely that the task will actually be performed then.**

When (partially) aware of self-control problems, how do people cope?

- **Commitments:**
  - Make public resolutions.
  - Join a health club.
  - Leave the ATM/credit card at home.
  - Higher tax payments in Jan-May and July-Nov.
  - Buy an apartment.

- **The fact that people make commitments is empirical evidence of self-control problems.**
Discounted-utility model with a constant discount rate (Samuelson, RES 1937)

Let \((c_t, \ldots, c_T)\) be an intertemporal consumption profile. Under the "usual assumptions", preferences over \((c_t, \ldots, c_T)\) can be represented by \(U^t(c_t, \ldots, c_T)\).

If additively separable between time periods:
\[
U^t(c_t, \ldots, c_T) = \sum_{\tau=t}^{\infty} D(\tau - t) u_\tau(c_\tau).
\]

If \(u\) does not depend on \(\tau\):
\[
U^t(c_t, \ldots, c_T) = \sum_{\tau=t}^{\infty} D(\tau - t)u(c_\tau).
\]

Discounted-utility model (cont.)

If \(D(x)\) is expon. decreasing: \(D(x) = \delta^x\) so that
\[
U^t(c_t, \ldots, c_T) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(c_\tau) = \sum_{\tau=t}^{\infty} \left(\frac{1}{1+\rho}\right)^{\tau-t} u(c_\tau),
\]
where \(\delta = \frac{1}{1+\rho}\): discount factor, and \(\rho = \frac{1}{\delta} - 1\): discount rate

- Samuelson (RES 1937) himself did not believe that the discounted-utility model with a constant discount rate was descriptively valid.
- And it is not; still it is being used for convenience.
Empirical evidence contradicting discounted utilitarianism with a constant discount rate

- Choice between $100 now and $200 in 2 years
- Choice between $100 in 6 years and $200 in 8 years
- Indifferent between $15 now and $30 in 3 months
  $60 in 1 year
  $100 in 3 years

Indicates a decreasing utility discount rate

Alternatives to exponential discounting

In continuous time: Disc. factor Disc. rate

- Exponential discounting: $D(x) = e^{-\rho x}$, $-\frac{D(x)}{D(x)} = \rho$
- Hyperbolic discounting: $D(x) = \frac{1}{1+kx}$, $-\frac{D(x)}{D(x)} = \frac{k}{1+kx}$
- Logarithmic discounting:

  $D(x) = e^{-\rho \ln(1+kx)} = (1+kx)^{-\rho}$, $-\frac{D(x)}{D(x)} = \frac{\rho k}{1+kx}$
Alternatives to exponential discounting (cont.)

In discrete time:

- Exponential discounting: \((\text{Samuelson, 1937})\)
  \[ U^t(c_t, \ldots, c_T) = u(c_t) + \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(c_\tau) \]
  \[ 0 < \delta < 1 \quad D(x) = \delta^x, \quad x \rightarrow x+1: \frac{1}{\delta} - 1 \]

- Quasi-hyperb. disc. (Present-biased pref., \((\beta, \delta)-\text{pref.})\):
  \[ U^t(c_t, \ldots, c_T) = u(c_t) + \beta \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(c_\tau) \]  \((\text{Elster, 1979})\)
  \[ 0 < \beta \leq 1 \quad D(0) = 1, \quad 0 \rightarrow 1: \frac{1}{\beta \delta} - 1 \]
  \[ 0 < \delta < 1 \quad D(x) = \beta \delta^x, \quad x \rightarrow x+1: \frac{1}{\delta} - 1 \]

Decr. disc. rate leads to time-inconsistency

- A task to be performed at time 0, 1, 2, …, or not at all.
  Immediate cost: 25. Benefits at the next stage: 125.

- \((\beta, \delta)-\text{preferences with } \beta = 1/2 \text{ and } \delta = 4/5.\)

Better to do it now than never.

Even better to do it at the next stage.
Decr. disc. rate leads to time-inconsistency (cont.)

Utility of doing the task at

| $\tau = 1$ | 30 | 25 |
| $\tau = 2$ | 24 | 30 |

$\beta = \frac{1}{2}$

$\delta = \frac{4}{5}$

Time-consistency

**Definition**: Preferences are time-consistent if for any $t, s > t$, and $(\overline{c}_t, \ldots, \overline{c}_{s-1})$:

$$U^t(\overline{c}_t, \ldots, \overline{c}_{s-1}, c_s, \ldots, c_T) \geq U^t(\overline{c}_t, \ldots, \overline{c}_{s-1}, c'_s, \ldots, c'_T)$$

$\uparrow$

$$U^s(c_s, \ldots, c_T) \geq U^s(c'_s, \ldots, c'_T)$$

**Strotz's main result**: With, for all $s \in \{t, \ldots, T - 1\}$

$$U^s(c_s, \ldots, c_T) = \sum_{\tau = s}^{T} D(\tau - s)u(c_\tau),$$

preferences are time-consistent if and only if

$$D(x) = \delta^x$$
**Behavior with time-inconsistent preferences**

- **Naive behavior:** Choosing the best plan under the presumption that it will be followed.

- May end up with a third best alternative.

- **Sophisticated behavior:** Choosing the best plan among those that will actually be followed.

- Demand for commitment
Multi-self model of sophistication

- Let every decision node correspond to a different “self” or “agent” of the decision-maker.
- Sophisticated behavior:
  SPE of the game where each self is a different player. (Peleg & Yaari, 1973)
- Backward induct.

Evidence from psychology on hyperbolic disc.

Typical procedure elicits indiff. points of the form

$A$ at time $x \sim B$ at time $x + y$

- **Approach 1:** (hyperbolic disc. is a “better fit”)

Assumption: There exists a disc. fn. $D(x)$ such that

$(A \text{ at } x \sim B \text{ at } x + y) \iff D(x)A = D(x + y)B$

Compare: Exponential discounting: $D(x) = e^{-\rho x}$

Hyperbolic discounting: $D(x) = \frac{1}{1 + kx}$

Hyperbolic discounting is virtually always a better fit.
**Approach 2:** (evidence of “declining discount rates”)

Given an indifference point \((A \text{ now}) \sim (B \text{ at } x)\),
define the average discount rate to be \(\hat{\rho}(x)\) such that

\[
A = e^{-\hat{\rho}(x) x} B
\]

Empirical finding: The aver. disc. rate is declining in \(x\).

Exponential discounting: \(D(x) = e^{-\rho x} \Rightarrow \text{Constant } \hat{\rho}(x)\)

Hyperbolic discounting: \(D(x) = \frac{1}{1+kx} \Rightarrow \text{Declining } \hat{\rho}(x)\)

**Two more types of evidence.**
- Intertemporal pref. reversals: \(A \text{ now } \succ B \text{ at } y\)
- Preference for commitment: \(A \text{ at } x \prec B \text{ at } x+y\)

Outline of lectures on self-control problems:

- Second Lecture (24 Sep.): Multi-self vs. temptation.
  - Consequences of naivete and sophistication within the multi-self model.
  - Direct modeling of temptation.

- Third Lecture (30 Sep.): Applic. to the theory of cons.
  - E.g., why do people accumulate illiquate assets while running up expensive credit card debts?

- Fourth Lecture (7 Oct.): Dual-self models.
  - Are our decisions a fight between a short-sighted and farsighted self?
Additional references


- Laibson (2010), Instant gratification, multiple selves and self control: How to control your selves, Harvard Univ.