Application to the theory of consumption

Lectures in Behavioral economics
Fall 2014, Part 3

Relevance of behavioral economics

- Acc. to Vernon Smith ("Rational choice: The contrast between economics and psychology", *JPE* 99, 1991, 877-89) much beh. economic critique stems from the premises that
  1. ... rationality in the economy … derives from the rationality of the individual decision makers in the economy.
  2. ... individual rationality is a cognitively intensive, calculating process of maximization of the self-interest.
  3. ... economic theory can be tested by testing directly the economic rationality of the individuals isolated from interactive experience in social and economic environments.

Instead, V. Smith thinks that beh. economic critique is important if it predicts different economic behavior at an aggregate level.
Observed saving behavior

- Present-biased pref. can explain these phenomena.

- **Illiquid assets combined with credit card debt**
  - Illiquid assets are commitments.
  - Credit cards provide consumption smoothing, but facilitate tempting shopping splurges.

- **Co-movement between income and consumption**
  - Consequence of self-imposed liquidity constraints.
  - Becomes more binding at the time of retirement.

- **Laibson**: A model that explains empirical evidence.

Outline

- **Cons.-saving model w/illiquid asset**
  - Application of the multi-self approach, where sophisticates use illiquid asset as commitment.
  - Yields explanation of observed saving behavior.

- **A theoretical result in the cons.-saving model**

- **Empirical test of the cons.-saving model**
  - Angeletos et al. (2001).
  - Show that present-biased pref. fit data better.
Laibson’s (1997) consumption-saving model

Consumer makes decisions in periods 1,...,T
One liquid asset $x$ One illiquid asset $z$
Exogenous initial asset holdings $x_0, z_0 \geq 0$

In period $t$:
- earn labor income $y_t$
- earn asset income $R_t(x_{t-1} + z_{t-1})$
- choose consumption $c_t$ and a new asset allocation $x_t$ and $z_t$ such that
  $$c_t + x_t + z_t = y_t + R_t(x_{t-1} + z_{t-1})$$
  $$c_t \leq y_t + R_t x_{t-1}$$
  $$x_t, z_t \geq 0$$

Laibson’s (1997) consumption-saving model (2)

Period - $t$ intertemporal preferences:
$$U^t = E_t\left[u(c_t) + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} u(c_{\tau})\right]$$

3-period consumption-saving example

Budget constraint:
$$c_1 + \frac{c_2}{R} + \frac{c_3}{R^2} \leq y_1 + \frac{y_2}{R} + \frac{y_3}{R^2} \equiv W$$

Instantaneous utility:
$$u(c) = \ln c$$

Numerical illustration:
$$\beta = 0.8, \delta = 0.9, R = 1.1, \text{ and } W = \$1000.$$
Optimal period-0 behavior

\[ \max \beta \delta \left( \ln(c_1) + \delta \ln(c_2) + \delta^2 \ln(c_3) \right) \]

subject to \( c_1 + \frac{c_2}{R} + \frac{c_3}{R^2} \leq y_1 + \frac{y_2}{R} + \frac{y_3}{R^2} \equiv W \)

Solution:

\[ c_1^0 = \frac{W}{1 + \beta \delta + \delta^2}, \quad c_2^0 = \frac{\delta RW}{1 + \beta \delta + \delta^2}, \quad c_3^0 = \frac{(\delta R)^2 W}{1 + \beta \delta + \delta^2} \]

Numerical illustration:

\( c_1^0 = \$369.00, \quad c_2^0 = \$365.31, \quad c_3^0 = \$361.66 \)

Optimal period-1 behavior

\[ \max \ln(c_1) + \beta \delta \ln(c_2) + \beta \delta^2 \ln(c_3) \]

subject to \( c_1 + \frac{c_2}{R} + \frac{c_3}{R^2} \leq y_1 + \frac{y_2}{R} + \frac{y_3}{R^2} \equiv W \)

Solution:

\[ c_1^1 = \frac{W}{1 + \beta \delta + \beta \delta^2}, \quad c_2^1 = \frac{\beta \delta RW}{1 + \beta \delta + \beta \delta^2}, \quad c_3^1 = \frac{\beta (\delta R)^2 W}{1 + \beta \delta + \beta \delta^2} \]

Numerical illustration (optimal period -1):

\( c_1^1 = \$422.30, \quad c_2^1 = \$334.46, \quad c_3^1 = \$331.11 \)
Naive period-2 behavior

\[ \max \ln(c_2) + \beta \delta \ln(c_3) \]

subject to \( c_2 + \frac{c_3}{R} \leq W_2^N \equiv R(W - c_1^1) = \frac{\beta \delta R (1 + \delta) W}{1 + \beta \delta + \beta \delta^2} \)

Solution:

\[ c_2^N = \frac{W_2^N}{1 + \beta \delta}, \quad c_3^N = \frac{\beta \delta R W_2^N}{1 + \beta \delta} \]

Numerical illustration (naivete):

\[ c_1^1 = 422.30, \quad c_2^N = 369.46, \quad c_3^N = 292.61 \]

Sophisticated period-2 behavior

\[ \max \ln(c_2) + \beta \delta \ln(c_3) \]

subject to \( c_2 + \frac{c_3}{R} \leq W_2 \equiv R(W - c_1) \)

Solution:

\[ c_2(c_1) = \frac{W_2}{1 + \beta \delta}, \quad c_3(c_1) = \frac{\beta \delta R W_2^2}{1 + \beta \delta} \]

\[ c_3(c_1) = \frac{\beta \delta R^2 W_2}{1 + \beta \delta} \]
Sophisticated period-1 behavior

\[
\max \ln(c_1) + \beta \delta \ln(c_2(c_1)) + \beta \delta^2 \ln(c_3(c_1))
\]

\[
\max \ln c_1 + \beta \delta \ln \left( \frac{R(W-c_1)}{1+\beta \delta} \right) + \beta \delta^2 \ln \left( \frac{\beta \delta R^2 (W-c_1)}{1+\beta \delta} \right)
\]

Solution: \[c_1^S = \frac{W}{1+\beta \delta + \beta \delta^2},\]

\[c_2^S = c_2(c_1) = \frac{1}{1+\beta \delta} \cdot \frac{\beta \delta R(1+\delta)W}{1+\beta \delta + \beta \delta^2},\]

\[c_3^S = c_3(c_1) = \frac{\beta \delta R}{1+\beta \delta} \cdot \frac{\beta \delta R(1+\delta)W}{1+\beta \delta + \beta \delta^2}\]

Numerical illustration (sophistication):
\[c_1^S = $422.30, \quad c_2^S = $369.46, \quad c_3^S = $292.61\]

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Sophisticates can implement optimal period-1 behavior by using the illiquid asset

If \( \beta = 0.8, \delta = 0.9, R = 1.1, Y_1 = 1000, \) and \( Y_2 = Y_3 = 0, \) then

In period 1:
- consume $422.30
- save $304.05 in the liquid asset
  (which yields $334.46 in period 2)
- save $273.64 in the illiquid asset
  (which yields $331.11 in period 3)

- Sophisticates strictly prefer the use of illiquid assets.
- Naifs do not recognize the commitment value of illiquid assets.
The illiquid asset is not a perfect commitment technology.

- You cannot prevent yourself from consuming current income.
  - If $\beta = 0.8$, $\delta = 0.9$, $R = 1.1$, $Y_1 = 500$, $Y_2 = 550$, and $Y_3 = 0$, then the illiquid asset does not help at all.

- An illiquid asset does not work as a commitment device if you can borrow against its future payoff.
  - Credit cards (and other liquidity enhancing instruments) may undermine the commitment value of illiquid assets.

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A theoretical result in the consum.-saving model

- Assume no illiquid asset

Period $-t$ intertemporal preferences:

$$U^t = E_t \left[ u(c_t) + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} u(c_{\tau}) \right]$$

Optimality condition if $\beta = 1$:

$$u'(c_t) = E_t \left[ R \delta u'(c_{t+1}) \right] = E_t \left[ R^s \delta^s u'(c_{t+s}) \right]$$

Generalized opt. condition for sophisticates with $\beta < 1$:

$$u'(c_t) = E_t R \left[ \left( \frac{dc_{t+1}}{dx_{t+1}} \right) \beta \delta + \left( 1 - \frac{dc_{t+1}}{dx_{t+1}} \right) \delta \right] u'(c_{t+1})$$

- A source of "under-saving". 

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Proof of a theoretical result in the cons.-saving model

\[ u'(c_t) = \beta \delta R \frac{dU^{t+1}}{dx^{t+1}} \]

- cash on hand: \( x_t = y_t + R(x_{t-1} - c_{t-1}) \)
- choose consumption: \( c_t \leq x_t \)

\[ u'(c_{t+1}) = \beta \delta R \frac{dU^{t+2}}{dx^{t+2}} \Rightarrow \frac{dU^{t+2}}{dx^{t+2}} = \frac{u'(c_{t+1})}{\beta \delta R} \]

\[ \frac{dU^{t+1}}{dx^{t+1}} = \frac{dc^{t+1}}{dx^{t+1}} u'(c_{t+1}) + \delta R \left( 1 - \frac{dc^{t+1}}{dx^{t+1}} \right) \frac{dU^{t+2}}{dx^{t+2}} \]

\[ = \left( \frac{dc^{t+1}}{dx^{t+1}} + \left( 1 - \frac{dc^{t+1}}{dx^{t+1}} \right) \frac{1}{\beta} \right) u'(c_{t+1}) \]

\[ u'(c_t) = R \left( \frac{dc^{t+1}}{dx^{t+1}} \right) \beta \delta + \left( 1 - \frac{dc^{t+1}}{dx^{t+1}} \right) \delta u'(c_{t+1}) \]

Empirical test of the consumption-saving model

- Compare data with simulations \( \text{Angeletos et al. (2001)} \)
  under exponential disc. and hyperbolic disc. (sophisticates)

- Households with liquid assets > 1 month’s income:

  Exponent. simulation: 73%
  Hyperbolic simulation: 40%
  Data: 43%

- Households with positive credit-card borrowing:

  Exponent. simulation: 19%
  Hyperbolic simulation: 51%
  Data: 70%

- Mean credit-card borrowing (all households):

  Exponent. simulation: $900
  Hyperbolic simulation: $3400
  Data: >$5000

- Cons.-income comovement (income coefficient):

  Exponent. simulation: 0.032
  Hyperbolic simulation: 0.166
  Data: \( \approx 0.2 \)
Simulated Mean Income and Consumption

Simulated Total Assets, Illiquid Assets, Liquid Assets, and Liquid Liabilities for Households with Exponential Discount Functions
Mean Illiquid Assets of Households with Exponential and Hyperbolic Discount Functions

Mean Liquid Assets and Liabilities
Additional references

