Intergenerational Equity

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Abstract
Axiomatic analysis of intergenerational social preferences over infinite streams of well-being faces the following dilemma. Equal treatment of generations combined with sensitivity for the interests of each generation rules out explicitly defined preferences that can rank any pair of infinite well-being streams. Hence, either intergenerational social preferences must be incomplete or equal treatment and sensitivity must be compromised. This critical review of axiomatic analyses of preferences over infinite streams evaluates different kinds of intergenerational social preferences by comparing their axiomatic basis as well as their performance in simple present-future conflicts. The scope for application is discussed, using real-world intergenerational conflicts (such as global warming) as a backdrop.
1. INTRODUCTION

Intergenerational equity is the subject of a wide range of study in philosophy, economics, and related disciplines. This review does not provide an all-encompassing survey of literature on intergenerational equity. Rather, its purpose is to present a critical review of contributions that present axiomatic analyses of intergenerational social preferences over infinite streams of well-being.

This line of contributions was initiated by Koopmans (1960) and Diamond (1965). Even though their seminal articles may also be seen as analyzing the preferences of an infinitely lived consumer, their work is relevant for resolving distributional conflicts between generations. With emerging real-world intergenerational conflicts (such as global warming) as a backdrop, a new set of analyses of intergenerational social preferences over infinite streams of well-being has appeared during the past few years. This review selectively covers these new developments.

1.1. Preferences over Infinite Streams of Well-Being

The contributions reviewed here consider an infinite but countable number of generations that follow each other in sequence. To each generation is assigned a level of generational well-being that indicates the situation under which people within this generation live. Hence, future development is given by an infinite stream of well-being. The reviewed papers raise the following normative question: How should different streams of well-being be ranked from a social point of view, taking into account the interests of all generations?

I adopt a consequentialist approach that abstains from judging, for example, the intentions and procedures lying behind each generation’s actions. Then the problem of giving an ethical basis for intertemporal choice is reduced to making comparisons between feasible intergenerational streams of well-being.

There may be good reasons to assign a smaller weight to future generations, such as the uncertainty of their existence (e.g., see Llavador et al. 2009). In this review I abstract from such reasons. Moreover, I assume that the population size is given and constant over time. Therefore, I offer no discussion of the consequences of letting population change exogenously or endogenously over time. In particular, the interesting question of optimal population size is not addressed (see Blackorby et al. 2005).

Many issues discussed in the reviewed contributions are from a purely formal viewpoint of interest exactly because there is an infinite number of future generations. In particular, how can an infinite number of generations be treated equally while still being sensitive to the interests of any single generation? This problem was addressed already by Diamond (1965), who shows that equal treatment cannot be combined with such sensitivity if the social preferences are required to be complete and continuous. Recent contributions—reviewed here—have established related (and stronger) impossibility results.

The analytical problem, requiring one to choose between sensitivity and equal treatment, on the one hand, and completeness and continuity, on the other hand, translates into a real normative dilemma in the social ranking of infinite well-being streams. It is a major concern of the contributions presented in this review.

1.2. Axiomatic Analysis of Intergenerational Preferences

Axiomatic analysis in social choice theory (see Thomson 2001 for an instructive guide to the axiomatic method) considers ethically attractive properties (called axioms) that may be
imposed on the social preferences and poses the following questions for any given set of axioms: Are the axioms compatible? If yes, what kind of social preferences satisfies all the axioms? Sensitivity for the interests of any one generation and equal treatment of different generations are such ethically attractive properties. Axioms of sensitivity reflect a concern for efficiency or nonwastefulness, whereas axioms of equal treatment capture notions of equity or justice.

Throughout, any social preference is a reflexive binary relation satisfying transitivity as a coherency axiom. As apparent below, completeness as a richness axiom and continuity as a robustness axiom may come into conflict with axioms of sensitivity and equal treatment, necessitating that efficiency and equity concerns be captured in alternative ways.

Finally, the reviewed classes of social preferences satisfy separability axioms, stating that the evaluation of two streams with a common part (e.g., head or tail) is independent of what the common part is.

Most of the results of this literature fall into two categories: Impossibility results, showing that a given set of axioms is incompatible, and characterization results, establishing that a given set of axioms determines a particular class of social preferences. As emphasized by Thomson (2001, pp. 360–61), impossibility results do not invalidate the axiomatic analysis; they reflect mathematical truths that cannot and should not be ignored.

Characterization results consist of two parts: (a) If a social preference satisfies a given set of axioms, then it is in a particular class. (b) If a social preference is in a particular class, then it satisfies a given set of axioms. Holding the class constant, part (a) becomes stronger if the axioms are weakened, whereas part (b) becomes stronger if the axioms are strengthened. Hence, the informational content of a characterization result is enhanced by using, if possible, a weaker set of axioms in part (a) than in part (b).

Owing to space constraints, I present characterization results as if-and-only-if results, where the set of axioms is not necessarily the weakest or strongest compatible with the class of social preferences considered. Rather, I choose sets of axioms that facilitate comparisons between different classes of social preferences. The axiomatic method makes an ethical debate of social preferences more transparent by reducing it to an evaluation of the underlying axioms.

Formal proofs are not presented here, as they are available in or can be adapted from the reviewed contributions.

1.3. Outline

In Section 2 I motivate why the reviewed contributions consider infinite streams of well-being and point to real-world distributional conflicts between the present and future generations. I argue for exploring how various social preferences resolve intergenerational distributional conflicts, as a complement to axiomatic analysis.

In Section 3 I introduce the formal setting in which the reviewed contributions are presented and state results showing the impossibility of combining equal treatment of generations with sensitivity for the interests of each generation, provided that one insists on a numerically representable social preference or a complete social preference that can be explicitly defined. However, I point out that equal treatment and sensitivity are not in conflict by themselves and can be used to justify the concept of sustainable development.
This impossibility sets the stage for Sections 4 and 5. In Section 4 I present axiomatic analyses of incomplete social preferences that combine equal treatment and sensitivity, resulting in classes of utilitarian and leximin (lexicographic maximin) preferences. I note that utilitarian and leximin preferences lead to quite different consequences even though they both treat generations equally. In Section 5 I present axiomatic analyses of numerically representable (and, hence, complete) social preferences. The main question is how to amend discounted utilitarianism, through imposing equity axioms compatible with sensitivity and numerical representability, so that the interests of future generations are respected.

In the concluding Section 6 I discuss the scope for applying the results to be reviewed. I argue that such axiomatic analyses may be relevant, suggest how normative prescriptions for intergenerational equity may be implemented even in democratic societies governed by the present generation, and indicate that this literature should offer useful structure in the discussion of climate policies.

2. RESOLVING INTERGENERATIONAL CONFLICT

In this section I first motivate why it is appropriate to analyze intergenerational distributional conflicts in models with an infinite number of generations and give real-world examples of such conflicts, then clarify what the notion of well-being is meant to capture, and finally argue that the evaluation of intergenerational preferences should not be based solely on the appeal of their underlying axioms, but also on their performance in intergenerational distributional conflicts.

2.1. Many Potential Future People

There are approximately 6.8 billion people currently alive (as of November 15, 2009). Approximately 100 billion people have ever lived. Hence, the ratio of people who have ever lived in the past to people living today is approximately 14 to 1. With 500 million years left of acceptable habitat for humans on Earth, population being stable at 10 billion with an average length of life equal to 73 years, the ratio of people who will potentially live in the future to people living now is approximately 10 million to 1.

These observations indicate that there are many people that may potentially live in the future. The fact is captured in economic models of intergenerational equity by having an infinite but countable number of future generations. Hence, in addition to creating analytical interest, the infinite number of generations is also a modeling choice that corresponds to an important feature of the underlying intergenerational distributional conflicts.

In a world with widespread inefficiencies, there may be potential for improving the situation for future generations without sacrificing present well-being. Nevertheless, there is a clear conflict of interest between generations in the management of the resource base: Unless the current generation refrains from increasing its own well-being by exploiting natural resources and degrading the quality of the natural environment, or makes sufficient compensating investments in manmade capital, the interests of future generations may be undermined.

As illustrations that behavior designed to respect the interests of future generations imposes a cost on people living now, consider the following. Abating greenhouse gas
emissions lowers current well-being but leaves future generations with a better climate. Preserving biodiversity lowers current well-being but widens options for future generations. Exploiting soil and water resources with caution also lowers current well-being but increases the potential for future food production. Finally, using antibiotics with care lowers current well-being but reduces future health problems. The current generation can also influence future well-being by its accumulation of manmade assets: Increased investments in produced capital lower current well-being but bequeath future generations with a larger productive capacity. These examples indicate that intergenerational distributional conflicts are real.

2.2. The Notion of Well-Being

Normative analysis of intergenerational equity must combine sensitivity for the interests of people living in the present with respect for the interests of the large number of people that may exist in the future. Such normative analysis raises several problems.

First, what contributes to individual well-being? Traditional welfare economic theory associates the maximal well-being that an individual can attain within her constraints with the choices that the individual makes. However, behavioral economists, psychologists, and moral philosophers argue that people might not maximize their well-being through their behavior because of self-control problems and deficiencies in their decision-making capabilities. For the purpose of this review, it is enough to assume that there exists some index of well-being.

The notion of well-being is assumed to include everything that influences the situation in which people live. Hence, it includes much more than material consumption. It is intended to capture the importance of health, culture, and nature. There are two important restrictions, however: Well-being does not include the welfare that people derive from their children’s well-being. Likewise, only nature’s instrumental value (i.e., recognized value to humans) is included in the well-being, not its intrinsic value (i.e., value in its own right regardless of human experience); i.e., an anthropocentric perspective is taken. The general rationale behind these restrictions is that there is an argument to be made in favor of distinguishing the discussion of intergenerational equity from the forces that can motivate our generation to act in an equitable manner.

The term utility refers to a specific cardinal scale for generational well-being and a utilitarian criterion makes use of such a scale. No specific view on what constitutes individual well-being is therefore implied by this terminology.

Second, how can individual well-being be aggregated into a concept of well-being for one generation? If generations followed each other in sequence, with all people within each generation having the same level of well-being, one could associate with every generation the well-being level of its members. In reality, generations overlap, and there is extreme inequality in well-being among people living at each moment of time. For the purpose of this review, I assume that there exists some index of generational well-being without addressing the question of how this aggregate measure is obtained.

This leaves us with the following problem: How can infinite streams of generational well-being be compared, when one combines sensitivity for the present with respect for the interests of the future? I look at literature that addresses the normative question of what current generations should do for the future rather than the positive question of what current generations actually do for the future.
This review discusses to what extent ethical social choice theory can guide us in the normative question of how intergenerational distributional conflicts should be resolved if all generations gathered behind a veil of ignorance (Vickrey 1945, Harsanyi 1953, Rawls 1971), not knowing in what sequence they would appear. What kind of criterion for intergenerational equity would we recommend if we did not know to what generation we belonged and considered intergenerational distribution from an impartial perspective?

The literature on Arrovian social choice (Arrow 1951) in an intergenerational setting (e.g., see Ferejohn & Page 1978, Bossert & Suzumura 2008) is not discussed here.

### 2.3. Reflective Equilibrium

Axiomatic analysis investigates on which fundamental ethical conditions various criteria for intergenerational equity are based, which in turn allows for the evaluation of their normative appeal.

The normative question of how to resolve intergenerational distributional conflicts, however, can be approached and answered in an alternative manner: By considering different kinds of technological environments (e.g., growth models without or with the restrictions imposed by natural resource constraints), one can explore the consequences of social preferences and compare the properties of the intergenerational well-being streams that are generated.

It is consistent with Rawls’ (1971) reflective equilibrium to do both: Criteria for intergenerational equity should not be judged only by the ethical axioms on which they build, but also by their consequences in specific environments. One may question the appropriateness of a deduced criterion for intergenerational equity if there is substantive discrepancy between the consequence of a criterion and our ethical intuition in relevant technological environments. This view has been supported by many scholars, including Atkinson (2001, p. 206), who writes

> the relation between economics and ethical principles is not linear but rather iterative. Examination of the implications of moral principles in particular models may lead to their revision. By applying ethical criteria to concrete economic models, we learn about their consequences, and this may change our views about their attractiveness,

and Dasgupta & Heal (1979, p. 311), who conclude “it is legitimate to revise or criticize ethical norms in the light of their implications.”

I use a particularly simple context to test the properties of various criteria of intergenerational equity. A completely egalitarian stream with constant well-being is compared with an alternative stream in which the current generation saves for the benefit of future generations or dissaves at the expense of future generations. This comparison brings forward essential properties of the criteria in a setting with an infinite number of generations.

### 3. COMBINING SENSITIVITY AND EQUAL TREATMENT

In this section I first introduce the formal setting. I then reproduce two sets of results on the possibility of combining equal treatment of generations with sensitivity for the interests of each generation. These results are used in later sections to organize the literature on intergenerational equity. Finally, I report a result showing how equal treatment combined
with such sensitivity can be used to justify the concept of sustainable development under certain domain restrictions.

Denote by $\mathbb{N}$ the set of natural numbers $\{1, 2, 3, \ldots\}$ and by $\mathbb{R}$ the set of real numbers. Denote by $x = (x_1, x_2, \ldots, x_n, \ldots) \in X$ an infinite stream of generational well-being, where $x_i \in Y$, with $Y \subseteq \mathbb{R}$, indicates the well-being of generation $t$ and $X = Y^{[\mathbb{N}]}$ is the domain of intergenerational well-being streams.

Following Diamond (1965), Svensson (1980), Chichilnisky (1996), Basu & Mitra (2003, 2007b), and Zame (2007), for example, I assume that $Y = [0,1]$ is the set of admissible well-being levels. A more general framework, as used by Koopmans (1960), is to assume that the well-being of generation $t$ depends on an $n$-dimensional vector $x_t$ that takes on values in a connected set $Y$. However, representing the well-being of generation $t$ by a scalar $x$, in the unit interval helps focus on intergenerational issues. Throughout this review I assume that the indicator of well-being is at least ordinally measurable and level comparable across generations; Blackorby et al. (1984) call this “level-plus comparability.”

For $x, y \in X$, write $x \geq y$ if and only if $x_i \geq y_i$ for all $i \in \mathbb{N}$, $x >_T y$ if and only if $x_i \geq y_i$ and $x \neq y$, and $x \gg y$ if and only if $x_i > y_i$ for all $i \in \mathbb{N}$. Denote by $X_T = (x_1, x_2, \ldots, x_T)$ and $T+1x = (x_{T+1}, x_{T+2}, \ldots, x_{T+T}, \ldots,)$ the $T$-head and the $T$-tail of $x$. Write $x \geq y$ if and only if $x_i \geq y_i$ for all $i \in \mathbb{N}$, where for all $x, y \in X$, if $x \geq y$ and $y \geq z$, then $x \geq z$.

A social welfare relation (SWR) $\succeq$ is a reflexive (i.e., $\succeq$ satisfies that, for all $x \in X$, $x \succeq x$) and transitive (i.e., $\succeq$ satisfies that, for all $x, y, z \in X$, if $x \succeq y$ and $y \succeq z$, then $x \succeq z$) binary relation on $X$, where for all $x, y \in X$, $x \succeq y$ entails that $x$ is deemed socially at least as good as $y$. Denote by $\sim$ and $\succ$ the symmetric and asymmetric parts of $\succeq$; i.e., $x \sim y$ is equivalent to $x \succeq y$ and $y \succeq x$ and entails that $x$ is deemed socially indifferent to $y$, whereas $x \succ y$ is equivalent to $x \succeq y$ and $\neg (y \succeq x)$ and entails that $x$ is deemed socially preferred to $y$ (where the logical quantifier $\neg$ indicates the negation of a statement).

If an SWR $\succeq$ is complete (i.e., $\succeq$ satisfies that, for all $x, y \in X$ with $x \neq y$, $x \succeq y$ or $y \succeq x$), then $\succeq$ is called a social welfare order (SWO).

An SWR $\succeq'$ is a subrelation to the SWR $\succeq''$ if, for all $x, y \in X$, (a) $x \succeq' y$ implies $x \sim'' y$, and (b) $x \succ' y$ implies $x \succ'' y$. An SWR $\succeq''$ extends the SWR $\succeq'$ if $\succeq'$ is a subrelation to $\succeq''$ and is an ordering extension of $\succeq'$ if in addition $\succeq''$ is an SWO.

A social welfare function (SWF) is a mapping $W: X \rightarrow \mathbb{R}$. An SWF $W$ numerically represents an SWR $\succeq$ if for all $x, y \in X$, $W(x) \geq W(y)$ if and only if $x \succeq y$. An SWF $W$ is monotone if $x \geq y$ implies $W(x) \geq W(y)$.

### 3.1. The Diamond-Basu-Mitra Impossibility Result

An uncontroversial ethical axiom on an SWR $\succeq$ is that any social preference must deem one well-being stream superior to another if at least one generation is better off and no generation is worse off.

**Axiom SP (strong Pareto):** For all $x, y \in X$, if $x >_T y$, then $x \succ y$.

This axiom captures efficiency concerns and ensures that the social preferences are sensitive to an increase in well-being for any one generation.

Another basic ethical axiom on an SWR $\succeq$ imposes equal treatment of all generations by requiring that any social preferences must leave the social valuation of a well-being stream unchanged when the well-being levels of a finite number of generations are...
permuted. [A permutation is a bijective mapping $\pi$ of $\mathbb{N}$ onto itself; it is finite whenever there exists $T \in \mathbb{N}$ such that $\pi(t) = t$ for any $t > T$.]

**Axiom FA (finite anonymity):** For all $x, y \in X$, if for some finite permutation $\pi$, $x_{\pi(t)} = y_t$ for all $t \in \mathbb{N}$, then $x \sim y$.

This axiom captures equity concerns in a setting in which there is no uncertainty about the existence of future generations. It is a basic fairness norm as it ensures that everyone counts the same in social evaluation. Invoking impartiality in this way is the cornerstone of ethical social choice theory reaching far beyond comparisons of infinite streams (e.g., see Sen 1970, chapter 5; Hammond 1976; d’Aspremont & Gevers 1977; Roemer 1996, p. 32; Mongin & d’Aspremont 1998).

In the setting of infinite streams, the finite anonymity axiom was first introduced by Diamond (1965). It has been used since in many contributions to formalize distributional concerns between an infinite number of generations (e.g., see Svensson 1980, where the term ethical preferences is associated with the strong Pareto and finite anonymity axioms). In the intergenerational context, the finite anonymity axiom implies that it is not justifiable to discriminate against some generation only because it appears at a later stage on the time axis.

It is of interest to consider the incomplete binary relation $\precsim$ that is generated by the strong Pareto and finite anonymity axioms, obtained when only these two axioms are assumed. Formally, we seek an SWR that satisfies axioms SP and FA and has the property of being a subrelation to any reflexive and transitive binary relation $\succeq$ satisfying axioms SP and FA. It turns out that such an SWR $\precsim$ exists and coincides with the well-known Suppes-Sen grading principle $\succeq^S$ (Suppes 1966, Sen 1970; e.g., see Svensson 1980 and Madden 1996 in the intergenerational context), which is defined as follows: For all $x, y \in X$, $x \succeq^S y$ if and only if, for some finite permutation $\pi$ of $\mathbb{N}$, $x_{\pi(t)} \succeq y_t$ for all $t \in \mathbb{N}$. As observed by Asheim et al. (2001, proposition 1), for example, it is indeed the case that the Suppes-Sen relation is generated by the strong Pareto and finite anonymity axioms: $\succeq^S$ satisfies axioms SP and FA and $\succeq^S$ is a subrelation to any SWR satisfying axioms SP and FA. Thus, in the intergenerational context, the Suppes-Sen grading principle can be given an ethical foundation in terms of two focal normative postulates for social preferences.

It follows from Arrow’s (1951) version of Szpilrajn’s (1930) extension theorem that an ordering extension of the Suppes-Sen grading principle exists (see Svensson 1980). However, such an SWO cannot be represented by an SWF.

**Proposition 1 (Basu & Mitra 2003, theorem 2):** Assume that the SWO $\succeq$ satisfies axioms SP and FA. Then there does not exist any SWF numerically representing $\precsim$.

Diamond’s (1965, p. 176) original theorem, a result he attributes to M.E. Yaari, establishes that no SWO can satisfy both continuity in the supnorm topology (see Section 5) and the strong Pareto and finite anonymity axioms. Because any supnorm continuous and strongly Pareitian SWO is numerically representable, Basu & Mitra (2003) strengthen the Diamond-Yaari theorem. Fleurbaey & Michel (2003, theorem 2) and Basu & Mitra (2007a, theorem 4) offer strengthenings of the Diamond-Yaari and Basu-Mitra theorems by weakening the strong Pareto axiom to the following weaker Pareitian axiom.

**Axiom WP (weak Pareto):** For all $x, y \in X$, if $x \gg y$, then $x \succ y$. 

...
Other related impossibility results include those presented by Hara et al. (2008).

Svensson (1980) shows that there exist topologies allowing for continuous social preferences that are sensitive to the interests of each generation and treat all generations equally. However, under relevant technological assumptions, such topologies undermine the compactness of the set of feasible allocations. Lauwers (1997a) presents a general discussion of these issues.

3.2. The Lauwers-Zame Impossibility Result

When Arrow’s (1951) version of Szpilrajn’s (1930) extension theorem is applied to show that an ordering extension of the Suppes-Sen grading principle exists, the argument relies on the axiom of choice. Hence, this result does not help in constructing an explicitly defined SWO that extends the Suppes-Sen grading principle. Fleurbaey & Michel (2003, p. 794) conjecture that no ordering extension of the Suppes-Sen grading principle can be explicitly defined.

The issue of whether an ordering extension of the Suppes-Sen grading principle can be explicitly defined was resolved by Lauwers (2010) and Zame (2007), who essentially show that this is not possible and thereby confirm the Fleurbaey-Michel conjecture. Zame’s (2007) result is stated as follows.

Proposition 2: No definable SWO \( \succeq \) can be proved (on the basis of the Zermelo-Fraenkel axioms and the axiom of choice) to satisfy axioms SP and FA.

The Diamond-Basu-Mitra and Lauwers-Zame impossibility results constitute an ethical dilemma. If we only consider SWRs that can be explicitly defined, we are forced to make a choice between the following alternatives: either require that the SWR satisfies the strong Pareto and finite anonymity axioms or require that the SWR be complete. In the former case, the SWR must be incomplete (and, hence, not numerically representable). In the latter case, efficiency and equity concerns must be captured by conditions other than the strong Pareto and finite anonymity axioms.

Section 4 is devoted to equitable and strongly Paretian SWRs, for which the term equitable means that the finite anonymity axiom is satisfied. Section 5 presents equitable SWOs that do not satisfy the finite anonymity axiom; hence, equity is captured by alternative axioms.

3.3. Justifying Sustainability

In relation to Diamond-Basu-Mitra and Lauwers-Zame impossibility results, it is worth noting that the strong Pareto and finite anonymity axioms are not in conflict by themselves. It is of interest to explore the consequences of imposing these axioms, i.e., to explore the consequences of the Suppes-Sen grading principle.

Asheim et al. (2001) show that the Suppes-Sen grading principle (and, thus, the strong Pareto and finite anonymity axioms) has far-reaching implications in technological environments that satisfy the following productivity condition.

Condition of immediate productivity: A set of feasible well-being streams satisfies immediate productivity if, for all feasible \( x \) with \( x_t > x_{t+1} \) for some \( t \), the well-being stream \( (x_1, x_2, \ldots, x_{t-1}, x_{t+1}, x_t, x_{t+2}, \ldots) \) is feasible and inefficient.
With this domain restriction, we can make the following argument. If a well-being stream is not nondecreasing—i.e., there exists some \( t \) such that \( x_t > x_{t+1} \)—then it is feasible to save the additional well-being of generation \( t \) for the benefit of generation \( t + 1 \) such that \( t + 1 \)'s gain is larger than \( t \)'s sacrifice. By axiom FA the new stream would have been socially indifferent to the old one even if the additional utility of \( t \) were transferred to \( t + 1 \) without any net productivity (as this would have amounted to a permutation of the well-being levels of generations \( t \) and \( t + 1 \)). By axiom SP it follows that the new stream is (strictly) preferred as \( t + 1 \)'s gain is larger than \( t \)'s sacrifice.

This argument means that only nondecreasing well-being streams are undominated by an SWR satisfying the strong Pareto and finite anonymity axioms in technological environments satisfying the condition of immediate productivity. Because any nondecreasing utility stream is sustainable—according to any common definition of the notion of sustainable development—the strong Pareto and finite anonymity axioms thus justify sustainability.

4. EQUITABLE AND PARETIAN PREFERENCES

Following Sidgwick (1907), Pigou (1932), and Ramsey (1928), there is a long tradition in economics for considering the unfavorable treatment of future generations as ethically unacceptable. The quote from Pigou (1932, part I, chapter 2) in which he explains the preference for present pleasure over future pleasure by our defective telescopic faculty is well-known. Likewise, Ramsey (1928, p. 543) assumes that “we do not discount later enjoyment in comparison to earlier ones, a practice which is ethical indefensible and arises merely from the weakness of imagination.” These positions also invalidate unequal treatment of generations (Collard 1996).

This section presents different ways in which undiscounted utilitarianism, as suggested by Ramsey (1928), can be adapted to the setting with an infinite but countable number of generations. All these variants of utilitarian SWRs satisfy both the strong Pareto and finite anonymity axioms and thereby extend the Suppes-Sen grading principle.

This section also covers variants of maximin, the principle of maximizing the well-being of the worst-off generation, which also satisfies the finite anonymity axiom and is thus an alternative way of treating generations equally. Maximin is often identified with Rawls’s (1971) difference principle, although Rawls applies this principle to an index of primary goods (which cannot necessarily be identified with well-being) and does not recommend its use in the intergenerational setting. Solow (1974) is, in his own words, “plus Rawlsien que le Rawls” by applying the maximin principle for finding optimal intergenerational distributions.

I follow Sen (1970) by considering maximin in its lexicographic form, referred to as leximin. This makes the principle compatible with the strong Pareto axiom and thus an extension of the Suppes-Sen grading principle that differs from utilitarianism.

Even though both utilitarian and leximin social preferences treat generations equally, I illustrate how they lead to quite different (and perhaps undesirable) consequences in a class of simple present-future conflicts.

4.1. Utilitarianism and Leximin

Some additional notation is convenient for defining an infinite-dimensional version \( \succeq^U \) of the utilitarian SWR: Write, for all \( x \in X \) and \( T \in \mathbb{N} \),

\[
x \succeq^U y \quad \text{if and only if} \quad \sum_{t \in T} x_t - y_t \geq 0
\]
\[ \sigma(\mathbf{x}_T) := \sum_{t=1}^{T} x_t. \]

For all \( \mathbf{x}, \mathbf{y} \in \mathbb{X} \),
\[ \mathbf{x} \succeq^U \mathbf{y} \iff \text{there exists } T \in \mathbb{N} \text{ such that } \sigma(\mathbf{x}_T) \geq \sigma(\mathbf{y}_T) \text{ and } T+1\mathbf{x} \succeq T+1\mathbf{y}. \]

Note that this SWR is incomplete as it compares two streams only if their tails beyond some finite time coincide or Pareto-dominate each other.

To characterize the SWR \( \succeq^U \), adapt the translation scale invariance axiom for finite-population social choice theory to the present infinite setting. Joint with the finite anonymity, this axiom implies indifference among intergenerational transfers of well-being.

**Axiom PTSI (partial translation scale invariance):** For all \( \mathbf{x}, \mathbf{y} \in \mathbb{X} \) such that, for some \( T \in \mathbb{N}, x_t = y_t \) for all \( t > T \), if \( \mathbf{x} \succeq \mathbf{y} \) and \( \alpha \in \mathbb{R}_{\geq 0}^{[0]} \) satisfies that \( \mathbf{x} + \alpha \in \mathbb{X} \) and \( \mathbf{y} + \alpha \in \mathbb{X} \), then \( \mathbf{x} + \alpha \succeq \mathbf{y} + \alpha \).

**Proposition 3 (Basu & Mitra 2007b, theorem 1):** The utilitarian SWR \( \succeq^U \) is a subrelation to an SWR \( \succeq \) if and only if \( \succeq \) satisfies axioms SP, FA, and PTSI.

To define an analog infinite-dimensional version \( \succeq^L \) of the lexicimin SWR, consider the following notation: For all \( \mathbf{x} \in \mathbb{X} \) and \( T \in \mathbb{N} \),
\[ (x_{(1)}, \ldots, x_{(T)}) \text{ denotes the rank ordered permutation of } \mathbf{x}_T \]
such that \( x_{(1)} \leq \ldots \leq x_{(T)} \), ties being broken arbitrarily. For all \( \mathbf{x}, \mathbf{y} \in \mathbb{X}, \mathbf{x}_T \succ_T \mathbf{y}_T \) if and only if there exists \( t \in \{1, \ldots, T\} \) such that \( x_i = y_i \) for all \( i \in \{1, \ldots, t-1\} \) and \( x_t > y_t \) and \( \mathbf{x}_T \sim_T \mathbf{y}_T \) if and only if \( x_i = y_i \) for all \( i \in \{1, \ldots, T\} \). For all \( \mathbf{x}, \mathbf{y} \in \mathbb{X} \),
\[ \mathbf{x} \succeq^L \mathbf{y} \iff \text{there exists } T \in \mathbb{N} \text{ such that } \mathbf{x}_T \succeq^L \mathbf{y}_T \text{ and } T+1\mathbf{x} \succeq T+1\mathbf{y}. \]

As in the finite-dimensional case, lexicimin can be characterized by means of Hammond’s (1976) equity axiom combined with axioms SP and FA.

**Axiom HE (Hammond equity):** For all \( \mathbf{x}, \mathbf{y} \in \mathbb{X} \), if there exist \( i, j \in \mathbb{N} \) such that \( y_i > x_i > x_j > y_j \) and \( x_i = y_i \) for all \( t \neq i, j \), then \( \mathbf{x} \succeq \mathbf{y} \).

**Proposition 4 (Bossert et al. 2007, theorem 2):** The lexicimin SWR \( \succeq^L \) is a subrelation to an SWR \( \succeq \) if and only if \( \succeq \) satisfies axioms SP, FA, and HE.

The basic utilitarian and lexicimin SWRs, as presented above, satisfy the following axioms, which have all played an important role in axiomatic analyses of intergenerational equity since Koopmans’ (1960) seminal contribution.

**Axiom SEP (separable present):** For all \( \mathbf{x}, \mathbf{y} \in \mathbb{X} \) and all \( T \in \mathbb{N} \), if \( (\mathbf{x}_T, T+1\mathbf{x}) \succeq (\mathbf{y}_T, T+1\mathbf{y}) \), then \( (\mathbf{x}_T, T+1\mathbf{y}) \succeq (\mathbf{y}_T, T+1\mathbf{y}) \).

**Axiom SEF (separable future):** For all \( \mathbf{x}, \mathbf{y} \in \mathbb{X} \) and all \( T \in \mathbb{N} \), if \( (\mathbf{x}_T, T+1\mathbf{x}) \succeq (\mathbf{y}_T, T+1\mathbf{y}) \), then \( (\mathbf{x}_T, T+1\mathbf{y}) \succeq (\mathbf{y}_T, T+1\mathbf{y}) \).

**Axiom ST (stationarity):** For some \( \mathbf{z} \in \mathbb{X} \) and all \( \mathbf{x}, \mathbf{y} \in \mathbb{X} \), \( (z_1, 2\mathbf{x}) \succeq (z_1, 2\mathbf{y}) \) if and only if \( 2\mathbf{x} \succeq 2\mathbf{y} \).

The stationarity axiom is a central feature of Koopmans’ (1960) framework. Combined with the axiom of separable future, it ensures time consistency even if social preferences are time invariant.
An obvious problem with the utilitarian and leximin SWRs, as presented in this subsection, is their incompleteness. In particular, if the tails of two streams beyond some finite time do not coincide or Pareto-dominate each other, then the streams are noncomparable. The next section investigates how to achieve more comparability, keeping in mind that the Lauwers-Zame impossibility result (see Section 3.2) rules out completeness of any explicitly defined SWR.

4.2. Extending Finite-Dimensional Preferences

In the context of finite-population social choice theory, both utilitarianism and leximin are ordering extensions of the Suppes-Sen grading principle. Hence, the problem of finding infinite-dimensional extensions of \( \succeq^U \) and \( \succeq^L \) can be posed as a problem of extending finite horizontal utilitarianism and leximin to the infinite-dimensional setting.

For any SWR \( \succeq \), consider the following notation:

\[
1x_T \succeq_T^z 1y_T \iff (1x_{T+1}z) \succeq (1y_{T+1}z).
\]

If \( \succeq \) satisfies axiom SEP, then \( \succeq_T^z \) is independent of \( 1z \).

It follows from Proposition 3 that if \( \succeq \) satisfies axioms SP, FA, and PTSI, then, for all \( T \in \mathbb{N} \), \( \succeq_T^z \) is the usual finite-dimensional utilitarian SWO on \([0, 1]^T\). Likewise, it follows from Proposition 4 that if \( \succeq \) satisfies axioms SP, FA, and HE, then, for all \( T \in \mathbb{N} \), \( \succeq_T^z \) is the usual finite-dimensional leximin SWO on \([0, 1]^T\).

Following Atsumi (1965) and von Weizsäcker (1965), we can define overtaking and catching-up versions of \( \succeq^U \) and \( \succeq^L \):

\[
1x \succeq^U_0 1y \iff \text{there exists } T' \in \mathbb{N} \text{ such that } \sigma(1x_{T'}) > \sigma(1y_{T'}) \text{ for all } T \geq T',
\]

\[
1x \sim^U_0 1y \iff \text{there exists } T' \in \mathbb{N} \text{ such that } \sigma(1x_{T'}) = \sigma(1y_{T'}) \text{ for all } T \geq T'.
\]

\[
1x \succeq^L_0 1y \iff \text{there exists } T' \in \mathbb{N} \text{ such that } 1x_{T'} \succeq^L_{T'} 1y_{T'} \text{ for all } T \geq T',
\]

\[
1x \sim^L_0 1y \iff \text{there exists } T' \in \mathbb{N} \text{ such that } 1x_{T'} \sim^L_{T'} 1y_{T'} \text{ for all } T \geq T'.
\]

It follows from the results of Asheim & Tungodden (2004), Basu & Mitra (2007b), and Asheim & Banerjee (2010) that the SWRs \( \succeq^U_0 \), \( \succeq^L_0 \), \( \succeq^U \), and \( \succeq^L \) can be characterized by adding one of the following two axioms.

**Axiom WLP (weak limiting preference):** For all \( 1x, 1y \in X \), if, for all \( 1z \in X \),

\[
1x_{T+1}z \succeq 1y_{T+1}z \text{ for all } T \in \mathbb{N}, \text{ then } 1x \sim 1y.
\]

**Axiom SLP (strong limiting preference):** For all \( 1x, 1y \in X \), if, for all \( 1z \in X \),

\[
1x_{T+1}z \succeq 1y_{T+1}z \text{ for all } T \in \mathbb{N} \text{ and, for all } T' \in \mathbb{N}, \text{ there exists } T \geq T' \text{ such that } 1x_{T'} \sim 1y_{T'}, \text{ then } 1x \sim 1y.
\]

**Proposition 5:** The utilitarian overtaking SWR \( \succeq^U_0 \) is a subrelation to an SWR \( \succeq \) if and only if \( \succeq \) satisfies axioms SP, FA, PTSI, and WLP.

**Proposition 6:** The leximin overtaking SWR \( \succeq^L_0 \) is a subrelation to an SWR \( \succeq \) if and only if \( \succeq \) satisfies axioms SP, FA, HE, and WLP.
Proposition 7: The utilitarian catching-up SWR $\succeq^U_{c}$ is a subrelation to an SWR $\succeq$ if and only if $\succeq$ satisfies axioms SP, FA, PTSI, and SLP.

Proposition 8: The leximin catching-up SWR $\succeq^L_{c}$ is a subrelation to an SWR $\succeq$ if and only if $\succeq$ satisfies axioms SP, FA, HE, and SLP.

Because axiom SLP implies axiom WLP, Propositions 3–8 imply that $\succeq^U_{c}$ extends $\succeq^U_{o}$, which in turn extends $\succeq^U$ (and likewise for the leximin SWRs). Brock (1970) provides an alternative characterization of $\succeq^U_{o}$.

Following Lauwers (1997b) and Fleurbaey & Michel (2003), we can define fixed-step versions of $\succeq^U_{o}$, $\succeq^L_{o}$, $\succeq^U_{c}$, and $\succeq^L_{c}$:

1. $x \succ^U_{o} y$ if there exists $k \in \mathbb{N}$ such that $\sigma(x_{kT}) > \sigma(y_{kT})$ for all $T \in \mathbb{N}$,
2. $x \sim^U_{o} y$ if there exists $k \in \mathbb{N}$ such that $\sigma(x_{kT}) = \sigma(y_{kT})$ for all $T \in \mathbb{N}$.

3. $x \succ^L_{o} y$ if there exists $k \in \mathbb{N}$ such that $x_{kT} \succ^L_{c} y_{kT}$ for all $T \in \mathbb{N}$,
4. $x \sim^L_{o} y$ if there exists $k \in \mathbb{N}$ such that $x_{kT} \sim^L_{c} y_{kT}$ for all $T \in \mathbb{N}$.

5. $x \succ^U_{c} y$ if there exists $k \in \mathbb{N}$ such that $\sigma(x_{kT}) \geq \sigma(y_{kT})$ for all $T \in \mathbb{N}$,
6. $x \sim^L_{c} y$ if there exists $k \in \mathbb{N}$ such that $x_{kT} \sim^L_{c} y_{kT}$ for all $T \in \mathbb{N}$.

It follows from the results of Kamaga & Kojima (2009a) and Asheim & Banerjee (2010) that the SWRs $\succeq^U_{o}$, $\succeq^L_{o}$, $\succeq^U_{c}$, and $\succeq^L_{c}$ can be characterized by applying one or both of the following two axioms.

Axiom SSLP (strong fixed-step limiting preference): For all $x, y \in X$, if, for all $z \in X$, there exists $k \in \mathbb{N}$ such that $x_{kT} \succeq^L_{o} y_{kT}$ for all $T \in \mathbb{N}$ and, for all $k, T' \in \mathbb{N}$, there exists $T \geq T'$ such that $x_{kT} \approx^L_{c} y_{kT}$ then $x \succeq^L_{c} y$.

Axiom SLI (fixed-step limiting indifference): For all $x, y \in X$, if, for all $z \in X$, there exists $k \in \mathbb{N}$ such that $x_{kT} \sim^L_{o} y_{kT}$ for all $T \in \mathbb{N}$, then $x \sim^L_{o} y$.

Proposition 9: The utilitarian fixed-step overtaking SWR $\succeq^U_{o}$ is a subrelation to an SWR $\succeq$ if and only if $\succeq$ satisfies axioms SP, FA, PTSI, WLP, and SLI.

Proposition 10: The leximin fixed-step overtaking SWR $\succeq^L_{o}$ is a subrelation to an SWR $\succeq$ if and only if $\succeq$ satisfies axioms SP, FA, HE, WLP, and SLI.

Proposition 11: The utilitarian fixed-step catching-up SWR $\succeq^U_{c}$ is a subrelation to an SWR $\succeq$ if and only if $\succeq$ satisfies axioms SP, FA, PTSI, SSLP, and SLI.

Proposition 12: The leximin fixed-step catching-up SWR $\succeq^L_{c}$ is a subrelation to an SWR $\succeq$ if and only if $\succeq$ satisfies axioms SP, FA, HE, SSLP, and SLI.

Because axiom SSLP implies axiom WLP, Propositions 5–6 and Propositions 9–12 imply that $\succeq^L_{c}$ extends $\succeq^L_{o}$, which in turn extends $\succeq^U_{o}$ (and likewise for the corresponding leximin SWRs). Because axiom SSLP neither implies nor is implied by axiom SLP, $\succeq^U_{c}$ is neither an extension of nor a subrelation to $\succeq^U_{o}$.

The axioms of finite anonymity and fixed-step limiting indifference imply the axiom of fixed-step anonymity (see Lauwers 1997b, Mitra & Basu 2007). Fixed-step anonymity means that the social evaluation of a well-being stream remains unchanged when the well-being levels along the stream are permuted by a fixed-step permutation: For all $x, y \in X$, if there exists $k \in \mathbb{N}$ such that $x_{kT}$ is a finite permutation of $y_{kT}$ for all $T \in \mathbb{N}$, then $x \sim^L_{o} y$. 

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This axiom is motivated by the claim that the finite anonymity axiom is too weak to ensure impartiality, combined with the fact that indifference to all permutations is incompatible with the strong Pareto axiom (Van Liedekerke & Lauwers 1997). As analyzed by Mitra & Basu (2007), the fixed-step anonymity axiom extends the finite anonymity axiom while remaining compatible with the axiom of strong Pareto.

Banerjee (2006) considers the utilitarian SWR obtained by adding the axiom of fixed-step anonymity to axioms of strong Pareto and partial translation scale invariance, whereas Kamaga & Kojima (2009b) consider the leximin SWR obtained by adding the axiom of fixed-step anonymity to axioms of strong Pareto and Hammond equity. It follows from Propositions 3 and 9 that the former extends $\succsim^U$, but is a subrelation to $\succsim^U_{_{250}}$, whereas it follows from Propositions 4 and 10 that the latter extends $\succsim^L$, but is a subrelation to $\succsim^L_{_{250}}$.

### 4.3. Resolving Present-Future Conflicts

The previous two subsections cover axiomatizations of different variants of the utilitarian and leximin SWRs, adapted to the evaluation of infinite well-being streams. All these variants satisfy the finite anonymity axiom, a condition often considered to be a weak condition that ensures equal treatment of generations.

In line with Rawls’ reflective equilibrium (see Section 2.3), the current subsection tests the performance of the utilitarian and leximin SWRs in choice situations. I do so in a particularly simple environment, in which there is conflict between the present generation and the equally well-off future generations.

Consider an egalitarian stream $z$, where every generation’s well-being equals $z \in (0,1)$. Consider an alternative stream $x$, where generation 1 makes a sacrifice, leading to a uniform gain for all future generations; i.e., $x_1 \in [0, z)$ and $x_t = \bar{x} \in (z, 1]$ for all $t \geq 2$. Should generation 1 make such a sacrifice, when the evaluation is made having the interests of all generations in mind? The utilitarian and leximin SWRs answer this question in opposite ways.

According to the utilitarian SWR (in any of the variants reviewed), the sacrifice should always be made, also when $z - x_1$ is large and $\bar{x} - z$ is small. From a utilitarian point of view, any sacrifice, however large, by the current generation should be made to ensure the uniform gain, however small, for the infinite number of future generations.

This observation illustrates the argument of Rawls (1971, p. 287) that “the utilitarian doctrine may direct us to demand heavy sacrifices of the poorer generations for the sake of greater advantages for the later ones that are far better off.” Following such arguments, Rawls (1971, p. 297) reluctantly points out that “[t]his consequence can be to some degree corrected by discounting the welfare of those living in the future,” and Arrow (1999, p. 16) concludes “that the strong ethical requirement that all generations be treated alike, itself reasonable, contradicts a very strong intuition that it is not morally acceptable to demand excessively high savings rates of any one generation, or even of every generation.”

According to the leximin SWR (in any of the variants reviewed), the sacrifice should never be done, even when $z - x_1$ is small and $\bar{x} - z$ is large. From a leximin point of view, no sacrifice, however small, by the current generation should be made to ensure the uniform gain, however large, for the infinite number of future generations. In particular, as pointed out by Solow (1974, p. 41), the principle of maximizing the well-being of the worst-off generation may perpetuate poverty.
Are any of these conclusions consistent with commonly held ethical intuitions? Most of us will probably claim that the answer depends on the circumstances. We may hold the position that generation 1 should make the sacrifice for the benefit of the infinite number of future and better-off generations if its sacrifice is small relative to their uniform gain, but not if its sacrifice is relatively large. Owing to the infinite number of generations, the utilitarian and egalitarian criteria yield extreme and opposite conclusions, neither of which may be defensible.

5. EQUITABLE AND COMPLETE PREFERENCES

The previous section considers incomplete SWRs satisfying the axioms of strong Pareto, finite anonymity, separable present, separable future, and stationarity. The Diamond-Basu-Mitra impossibility result implies that there exists no numerically representable SWR satisfying both the strong Pareto and finite anonymity axioms, whereas the Lauwers-Zame impossibility result entails that we cannot explicitly define any complete SWR satisfying these two axioms. This section explores numerically representable (and, thus, complete) SWRs obtained by dropping the finite anonymity axiom.

Completeness combined with continuity in the supnorm topology and monotonicity is sufficient to obtain numerical representability. Hence, if an SWO satisfies the following two axioms, then it is numerically represented by an SWF.

**Axiom C (continuity):** For all $x, y \in X$, if the sequence of streams $(x^n)_{n \in \mathbb{N}}$ satisfies $\lim_{n \to \infty} \sup_t |x^n_t - x_t| = 0$ with, for each $n \in \mathbb{N}$, $x^n \in X$ and $x^n \succeq y$ ($x^n \preceq y$, respectively), then $x \succeq y$ ($x \preceq y$, respectively).

**Axiom M (monotonicity):** For all $x, y \in X$, if $x > y$, then $x \succeq y$.

What are the consequences of letting a numerically representable SWO combine sensitivity for the present with the remaining axioms of separable present, separable future, and stationarity? This section first reproduces Koopmans’ (1960) well-known result, establishing that this set of axioms characterizes the discounted utilitarian (DU) SWO. However, no equity axiom is included in this set, and discounting future generations’ well-being may lead to undesirable consequences in simple present-future conflicts.

The remainder of this section considers numerically representable SWOs that combine sensitivity for the interests of the present with respect for the interests of the future. It follows from Koopmans’ (1960) result that at least one axiom of the three axioms listed above (separable present, separable future, and stationarity) has to go if an equity axiom is to be imposed. Two different approaches are reviewed. In one approach, Chichilnisky (1996) drops the axiom of stationarity and instead imposes that the present has no dictorial role. In another, Asheim et al. (2010) drop the axiom of separate present and instead impose that the present has no say in intergenerational distributional conflicts if its well-being is higher than that of the future.

5.1. Discounted Utilitarianism and Present-Future Conflicts

Both in the theory of economic growth and in the practical evaluation of economic policy with long-term effects (e.g., climate policies), it is common to apply the DU SWO. Discounted utilitarianism means that one infinite stream of well-beings is deemed better
than another if and only if it generates a higher sum of utilities discounted by a constant discount factor $\delta \in (0, 1)$.

To present an axiomatization for the DU SWO, the following weak sensitivity axiom is useful. Note that the axiom of strong Pareto implies both the axiom of monotonicity and the axiom of weak sensitivity.

**Axiom WS (weak sensitivity):** There exist $1x, 1y, 1z \in X$ such that $(x_1, 2z) \succ (y_1, 2z)$.

Then the following result is obtained.

**Proposition 13:** The following two statements are equivalent.

1. The SWO $\succeq$ satisfies axioms C, M, WS, SEP, SEF, and ST.
2. There exists an SWF $W : X \to \mathbb{R}$ numerically representing $\succeq$ and satisfying, for some nondecreasing and continuous utility function, $U : [0, 1] \to \mathbb{R}$, with $U(0) < U(1)$ and utility discount factor, $\delta \in (0, 1)$,

$$W(x) = (1 - \delta)U(x_1) + \delta W(x) = (1 - \delta)\sum_{t=1}^{\infty} \delta^{t-1} U(x_t)$$

for all $1x \in X$.

Strengthening axioms M and WS to axiom SP in statement 1 is equivalent to replacing nondecreasing by increasing in statement 2.

In the present setting, in which well-being for every generation is a scalar in the unit interval, this characterization of the DU SWO is quite close to Koopmans’ (1960) original axiomatization. For alternative sets of axioms leading to the DU SWO, I refer the reader to Lauwers (1997c), Bleichrodt et al. (2008), and Asheim et al. (2010).

In line with Rawls’ reflective equilibrium (see Section 2.3), the remaining part of this subsection tests the performance of the DU SWO in choice situations in which there is conflict between the present generation and the equally well-off future generations. As in Section 4.3, alternative streams are compared with an egalitarian stream $\text{con}_z$, where every generation’s well-being equals $z \in (0, 1)$.

If in an alternative stream, $1x$, generation 1 makes a sacrifice, leading to a uniform gain for all future generations, i.e., $x_1 \in [0, z)$ and $x_t = x \in (z, 1]$ for all $t \geq 2$, then the class of DU SWOs leads to the appealing conclusion that the ranking depends on the gain/sacrifice ratio. Any DU SWO is consistent with the position that generation 1 should make a sacrifice increasing the well-being of the infinite number of future generations if its sacrifice is small relative to their uniform gain, but not if its sacrifice is relatively large.

If in an alternative stream, $1y$, all future generations make a uniform sacrifice, leading to a uniform gain for generation 1, i.e., $y_1 \in (z, 1]$ and $y_t = y \in [0, z)$ for all $t \geq 2$, then again the class of DU SWOs leads to the conclusion that the ranking depends on the gain/sacrifice ratio. Any DU SWO is consistent with the position that the infinite number of future generations should make a uniform sacrifice increasing the well-being of the present generation if their uniform sacrifice is small relative to its gain, but not if their sacrifice is relatively large.

However, in the latter case, the conclusion might not be deemed appealing. If the infinite number of future generations makes a uniform sacrifice increasing the well-being of the present generation, then, compared with the egalitarian stream, inequality is
increased, and the undiscounted sum of utilities is reduced (independently of the cardinal scale chosen). Hence, both from an egalitarian and utilitarian perspective, $\con x$ is socially preferred to $1y$, independently of the gain/sacrifice ratio. In particular, for all SWRs $\succeq$ considered in Section 4, $\con x >_1 y$.

Hence, even though a trade-off in present-future conflicts in which the present generation makes a sacrifice for the infinite number of better-off future generations might be ethically appealing, this conclusion need not extend to situations in which the infinite number of future generations makes a sacrifice for the benefit of the better-off present generation. Thus, it might be a drawback that the class of DU SWOs does not differentiate between these two kinds of present-future conflicts.

### 5.2. No Dictatorship of the Present and the Future

As observed by Chichilnisky (1996), the DU SWO is a dictatorship of present, in the following sense.

**Axiom DP (dictatorship of the present):** For all $1x, 1y, 1z, 1v \in X$, if $1x >_1 y$, then there exists $T' \in \mathbb{N}$ such that $(1x_{T'}, T_{+1}z) > (1y_{T'}, T_{+1}v)$ for all $T \geq T'$.

Hence, if one stream is preferred to another, then what happens after some finite time does not matter for the strict ranking. To take into account the interests of the generations in the infinite future, Chichilnisky (1996) suggests the following axiom, ruling out a dictatorial role for the present.

**Axiom NDP (no dictatorship of the present):** Axiom DP does not hold.

Chichilnisky (1996) considers also the symmetric axiom, requiring that no dictatorial role be given to the future.

**Axiom DF (dictatorship of the future):** For all $1x, 1y, 1z, 1v \in X$, if $1x >_1 y$, then there exists $T' \in \mathbb{N}$ such that $(1z_{T'}, T_{+1}x) > (1v_{T'}, T_{+1}y)$ for all $T \geq T'$.

**Axiom NDF (no dictatorship of the future):** Axiom DF does not hold.

Hence, when axiom of no dictatorship of the present is satisfied, it is not only what happens before some finite time that matters, whereas when axiom of no dictatorship of the future is satisfied, it is not only what happens beyond some finite time that matters.

The axiom of no dictatorship of the present is substantive, as it rules out DU SWOs. However, as the following argument shows, the axiom of no dictatorship of the future is implied by the weak sensitivity axiom and thereby also by the strong Pareto axiom. By axiom WS, there exist $1x, 1y \in X$ with $2x = 2y$ such that $1x >_1 y$. Let $1z, 1v \in X$ be given by $1z = 1v = 1x$. Then, for all $T \in \mathbb{N}$, $(1z_{T'}, T_{+1}x) = 1x = (1x_{T'}, T_{+1}y) = (1v_{T'}, T_{+1}y)$, implying by reflexivity that $(1z_{T'}, T_{+1}x) \sim (1v_{T'}, T_{+1}y)$. This contradicts axiom DF.

**Proposition 14:** The following two statements are equivalent.

1. The SWO $\succeq$ satisfies axioms C, SP, SEP, SEF, and NDP.
2. There exists an SWF $W : X \to \mathbb{R}$ numerically representing $\succeq$ and satisfying, for some sequence, $\langle U_t \rangle$, where for each $t \in \mathbb{N}$, $U_t : [0, 1] \to \mathbb{R}$ is an increasing and
continuous utility function, and some asymptotic part, \( \phi : X \to \mathbb{R} \), which is an integral with respect to a purely finitely additive measure,

\[
W(1^*x) = \sum_{t=1}^{\infty} U_t(x_t) + \phi(1^*x),
\]

for all \( 1^*x \in X \).

This characterization is based on Chichilnisky (1996, theorem 2), except that her independence assumption has been replaced by the separability axioms SEP and SEF. By choosing, for each \( t \in \mathbb{N} \), \( U_t(x_t) = \delta^{-1}x_t \) for some discount factor, \( \delta \in (0, 1) \), and \( \phi(1^*x) = \lim \inf_{t \to \infty} x_t \), it follows that Proposition 14 ensures the existence of an SWO \( \succeq \) satisfying axioms C, SP, SEP, SEF, and NDP.

Chichilnisky (1996, definition 6) uses the term sustainable preference for a numerically representable SWO satisfying the axioms of strong Pareto, no dictatorship of the present, and no dictatorship of the future. Because the axiom of strong Pareto implies the axiom of no dictatorship of the future, the SWO characterized by Proposition 14 is a sustainable preference, implying that a sustainable preference exists. The asymptotic part, \( \phi(1^*x) \), ensures that a sustainable preference is sensitive to what happens in the infinite future and thereby entails that the SWO is not a dictatorship of the present.

Any sustainable preference has the interesting property of not only ruling out DU SWRs by means of the axiom of no dictatorship of the present, but also any version of utilitarian and lexicin SWRs, as reviewed in Section 4, owing to its numerical representability.

By comparing Propositions 13 and 14, it follows that an SWO \( \succeq \) satisfying axioms C, SP, SEP, SEF, and NDP does not satisfy axiom ST because the sensitivity for what happens in the infinite future, as captured by \( \phi(1^*x) \), rules out that \( \succeq \) is DU. This means that such an SWO is not time consistent if social preferences are time invariant.

When testing the class of sustainable preferences by its performance in applications, the verdict is mixed. In simple present-future conflicts, its qualitative behavior is the same as that for the class of DU SWOs: The ranking depends on the gain/sacrifice ratio, both in situations in which the present generation makes a sacrifice for the infinite number of better-off future generations and in situations in which the infinite number of future generations makes a sacrifice for the better-off present generation.

When a sustainable preference, in the class characterized by Proposition 14, is applied to models of economic growth, there is a generic nonexistence problem, as welfare is increased by delaying the response to the interests of the infinite far future, whereas welfare is decreased if delay is infinite. This nonexistence problem has spurred an interest in how to adapt Chichilnisky’s sustainable preferences to ensure applicability (e.g., see Heal 1998, Li & Löfgren 2000, Alvarez-Cuadrado & Long 2009, Figuieres & Tidball 2010).

5.3. Hammond Equity for the Future

The performance of utilitarian and lexicin SWRs, as discussed in Section 4.3, and DU SWOs, as discussed in Section 5.1, motivates the following question: Does there exist a set of axioms leading to a class of SWOs allowing for a trade-off in present-future conflicts in which the present generation makes a sacrifice for the infinite number of better-off future generations, while giving priority to the future in situations in which the infinite number of future generations makes a sacrifice for the better-off present generation?
The key is to replace the axiom of separable present by the following equity axiom, discussed by Asheim et al. (2010).

**Axiom HEF (Hammond equity for the future):** For all \( x_1, y_1, z, v \in [0, 1] \), if \( x_1 > y_1 > v > z \), then \((y_1, \text{con} v) \succeq (x_1, \text{con} z)\).

For streams in which well-being is constant from the second period on, the axiom of Hammond equity for the future captures the idea of giving priority to an infinite number of future generations in the choice between alternatives in which the future is worse off compared with the present in both alternatives. If the present is better off than the future and a sacrifice now leads to a uniform gain for all future generations, then such a transfer from the present to the future leads to a stream that is at least as desirable, as long as the present remains better off than the future.

The axiom of Hammond equity for the future involves a comparison between a sacrifice by a present generation and a uniform gain for an infinite number of future generations that are worse off. Hence, contrary to the standard Hammond equity axiom, the transfer from the better-off present to the worse-off future specified in the axiom of Hammond equity for the future leads to an infinite increase in the sum of well-being, independently of what cardinal scale is used to measure well-being. Hence, the axiom of Hammond equity for the future can be endorsed from both an egalitarian and utilitarian point of view. In particular, it is weaker and more compelling than the standard Hammond equity axiom.

Introducing the axiom of Hammond equity for the future into a numerically representable SWO that combines sensitivity for interests of the present brings us to boundary of what is compatible. In particular, it is not compatible with the axioms of continuity and strong Pareto (Asheim et al. 2010, proposition 4). This motivates weakening of the latter axioms.

**Axiom RC (restricted continuity):** For all \( x, y \in X \), if \( x \) satisfies \( x_t = z \) for all \( t \geq 2 \) and the sequence of streams \( (x^n_{t})_{n \in \mathbb{N}} \) satisfies \( \lim_{n \to \infty} \sup_{t} |x^n_{t} - x_t| = 0 \) with, for each \( n \in \mathbb{N} \), \( x^n \in X \) and \( x^n \succeq y \) \((x^n \preceq y\), respectively\), then \( x \succeq y \) \((x \preceq y\), respectively\).

**Axiom RD (restricted dominance):** For all \( x, z \in [0, 1] \), if \( x < z \), then \((x, \text{con} z) < (z, \text{con} z)\).

The axiom of restricted continuity combined with the axiom of monotonicity is sufficient for numerical representability, whereas the axiom of restricted dominance is sufficient for some sensitivity for the interests of the present. The axiom of strong Pareto implies the axiom of restricted dominance, which in turn implies the axiom of weak sensitivity. Hence, the axiom of restricted dominance is sufficient for the axiom of no dictatorship of the future to be satisfied. Furthermore, any SWO satisfying the axioms of restricted continuity, monotonicity, Hammond equity for the future, separable future, and stationarity also satisfies the axiom of no dictatorship of the present, the other of Chichilnisky’s (1996) main axioms (Asheim et al. 2010, proposition 5).

The next and final characterization result establishes numerical representability in terms of a recursive aggregator function, which has a value that depends on current utility and future welfare. Consider the following class of aggregator functions:

\[ \mathcal{V}(U) := \{ V: [U(0), U(1)] \to \mathbb{R}\mid V \text{ satisfies } (V.0), (V.1), (V.2), \text{ and } (V.3) \}, \]
where properties (V.0)–(V.3) are given as follows:

(V.0) \( V(u, w) \) is continuous in \((u, w)\) on \([U(0), U(1)]^2\).
(V.1) \( V(u, w) \) is nondecreasing in \(u\) for given \(w\).
(V.2) \( V(u, w) \) is increasing in \(w\) for given \(u\).
(V.3) \( V(u, w) < w \) for \(u < w\), and \( V(u, w) = w \) for \(u \geq w\).

**Proposition 15:** The following two statements are equivalent.

1. The SWO \( \succsim \) satisfies axioms RC, M, RD, HEF, SEF, and ST.
2. There exists a monotone SWF \( W : X \to \mathbb{R} \) numerically representing \( \succsim \) and satisfying, for some increasing and continuous utility function, \( U : [0, 1] \to \mathbb{R} \), and some aggregator function, \( V \in \mathcal{V}(U) \), \( W(\{x_i\}) = V(U(x_1), W(\{x_i\})) \) for all \( x \in X \), and \( W(\text{con}z) = U(z) \) for all \( z \in [0, 1] \).

Proposition 15 reproduces Asheim et al. (2010, proposition 6). Asheim et al. (2010) use the term sustainable recursive SWF for an SWF having the properties under statement 2 of this proposition, and they show that, for any increasing and continuous utility function, \( U : [0, 1] \to \mathbb{R} \), and \( V \in \mathcal{V}(U) \), there exists a sustainable recursive SWF (see Asheim et al. 2010, proposition 7).

Sustainable recursive SWFs allow for a trade-off in present-future conflicts in which the present generation makes a sacrifice for the infinite number of better-off future generations, while giving priority to the future in situations in which the infinite number of future generations makes a sacrifice for the benefit of the better-off present generation. A companion paper (Asheim & Mitra 2009) shows that sustainable recursive SWFs can be applied to models of economic growth.

Imposing the axiom of Hammond equity for the future comes at the cost of dropping the axiom of separable present. However, if one accepts the intuition that the stream \( (0, \frac{1}{2}, 1, 1, 1, \ldots) \) is socially preferred to \( (\frac{1}{3}, \frac{1}{3}, 1, 1, 1, \ldots) \), while \( (0, \frac{1}{2}, 1, 1, 1, \ldots) \) is not socially preferred to \( (\frac{1}{3}, \frac{1}{3}, 1, 1, 1, \ldots) \), then one supports dropping the axiom of separable present. It is not obvious that we should treat the conflict between the worst-off and the second worst-off generation in the first comparison in the same manner as the conflict between the worst-off and the best-off generation in the second comparison.

**6. SCOPE FOR APPLICATION**

What is the scope for applying the SWRs discussed in this review? Is axiomatic analysis of intergenerational equity at all relevant? Can any of the axiomatized SWRs be implemented? What significance does the problem of global warming have for the relevance of this literature? These issues are discussed in this concluding section.

**6.1. Relevance of Normative Theory**

Some economists argue that normative theory on intergenerational equity leads to the arbitrary judgments of “philosopher kings” (Weitzman 2007, p. 712) doing “nirvana ethics” (Sinn 2008, p. 369) and, thus, has limited relevance for economic policy. Arguments supporting such views may include the following positions: (a) The altruism of parents toward their children leads to a good outcome for the long-term development of societies. (b) The normative theory on intergenerational equity has no consequence for
intergenerational distribution, as shifting the decision from market agents to democratically controlled policy makers (rather than philosophers) would not imply a stronger representation of future generations.

One can hope and believe that it is empirically true that the altruism of parents toward their children leads to a good long-term outcome. However, the statement appears to be a tautology unless the notion of good is given a meaning that is separate from the preferences of the parents. I argue that the statement requires some notion of what a good intergenerational distribution is, given the set of feasible streams determined by initial conditions and future technological and natural constraints. What one may call nirvana ethics contributes to this.

Consider a future situation (e.g., after global warming has undermined capital productivity) in which optimization of a DU SWO with a 3% utility discount rate would gradually undermine future livelihood, whereas an increased concern for future generations would lay the foundation for renewed and sustained growth. From an impartial perspective, one might hope that the altruism of parents would respond and ensure future generations’ well-being, leading to development that is not a DU SWO optimum with a 3% utility discount rate.

Hence, the claim that the altruism of parents toward their children leads to a good long-term outcome, combined with a notion of good that implies that a sustainable stream will be chosen whenever feasible, entails that we should not use a DU SWO with, say, a 3% utility discount rate in situations in which this undermines sustainability. The class of SWOs described in Section 5.3 provides a meaning of the notion of good that is consistent with such responsive behavior. Furthermore, the associated axiomatic analysis entails that the appropriateness of DU SWOs can be addressed effectively by discussing which of axioms of separable present and Hammond equity for the future should be respected in intergenerational evaluation.

It is of course true that the present generation, even in a democratic society, has dictatorial powers in the management of current manmade and natural assets. However, if we consider ourselves a benevolent dictator, we want to understand what a good intergenerational distribution is, given our constraints. This is in analogy to how a benevolent dictator in a static setting wants to know what a good intragenerational distribution is. As I discuss in the next subsection, comparing the consequences of our current actions to what would be an intergenerational equitable outcome may induce changed behavior by the present generation. In this way, normative theory on intergenerational distribution may affect the implemented management of manmade and natural assets.

6.2. Problem of Implementation

Why does the present generation accumulate manmade assets and conserve natural assets for the benefit of future generations? There are different mechanisms, including the following. First, the fact that generations overlap and save for their own retirement leads to accumulation of assets. Moreover, members of each generation may be motivated to bequeath assets to their children, facilitating intergenerational transfers. Finally, people may assign intrinsic value to nature, being motivated to conserve natural assets independently of their instrumental value to humans. Behavior in accordance with such motivation may be driven by norms. It may reflect social preferences and may not merely maximize individual well-being.
Axiomatic analyses of intergenerational equity and other systematic normative discussions of intergenerational distribution may promote normative reflection about intergenerational equity in society at large. If it becomes known that our actions appear not to lead to a good long-term outcome, according to some ethical norm, then people may adjust their behavior by changing what they bequeath to their children, and by giving increased intrinsic value to nature. For example, although one may argue that we may live perfectly well without polar bears, serious action to protect the habitat of polar bears requires that we do something effective about global warming, to the benefit of future generations. Hence, knowledge of the consequences of our actions compared to an accepted ethical position can change our altruism toward our children and our concern for nature. Such changes in individual preferences may help implement a desirable outcome.

Our preferences as *Homo politicus* may differ from our preferences as *Homo oeconomicus* (Arrow 1951, Harsanyi 1955, Nyborg 2000). Even though each one of us chooses to take advantage of present opportunities at the expense of future generations (e.g., because of long-term environmental effects), we may prefer collective action that makes such opportunistic behavior more costly. In this case, the present generation in a democratic society may want to introduce explicit instruments that promote intergenerational transfers (see also Marglin 1963).

In turn, the intergenerational preferences of the present generation, both at an individual and a collective level, will influence the economic evaluations (e.g., through cost-benefit analyses) that economists undertake when society faces decisions with long-term consequences, as exemplified in Section 2.1. Thus, indirectly axiomatic analyses of intergenerational equity may influence such economic evaluations. It is, however, outside the scope of this review to discuss the link between criteria of intergenerational equity and techniques for cost-benefit analysis (e.g., see Mertens & Rubinchik 2008).

6.3. Global Warming

Global warming is a test case for axiomatic analyses of intergenerational equity, as the effects of greenhouse gas emissions are external to the current emitters and lead to long-lasting and global consequences for future generations. In principle, internalization of these external effects may lead to both present and future benefits (Foley 2008). Still, it is likely that the climate policies that the present generation chooses in the first decades of this century will be important for generations that live hundreds and thousands of years from now. Hence, normative literature on intergenerational equity should offer useful structure in the discussion of such policies.

In practice, the debate on these issues, e.g., in connection with the Stern Review (Stern 2007), has been limited to a discussion of what parameters to use in the SWF numerically representing the DU SWO. In terms of axiomatic basis, some reference has been made to whether positive utility discounting is compatible with the equal treatment of generations.

The literature covered in this review shows that there is a wider choice. In particular, the finite anonymity axiom as a requirement for equal treatment may not be useful for resolving conflicts among an infinite number of generations, beyond yielding support for sustainable development under specific domain restrictions. Rather, any complete criterion of intergenerational equity should combine sensitivity for the interests of the present generation with respect for the interests of future generations by considering other equity axioms.
However, the possibility of global warming also shows the limitations of this literature. First of all, our ignorance about the future effects of greenhouse gas emissions entails that problems of intergenerational equity cannot be handled in a satisfactory manner without taking into account uncertainty. Uncertainty about future development may also suggest choosing goals for the concentration of greenhouse gases, rather than doing comparisons of well-being streams that we think might be possible (for a discussion, see, e.g., Dasgupta 2008, section 6).

The problem of global warming is a reminder of the importance of Rawls’ (1971) reflective equilibrium. In particular, our ethical intuitions concerning intergenerational equity may not be unrelated to the feasibility set with which we believe we are faced. In a simple one-sector growth model in which sustained growth is compatible with positive and constant utility discounting, the DU SWO may produce an ethically appealing optimal stream of well-being. As discussed by Asheim & Buchholz (2003, section V), however, this support for the DU SWO may not extend to the Dasgupta-Heal-Solow model (Dasgupta & Heal 1974, 1979; Solow 1974) of capital accumulation and resource depletion, in which for all positive utility discount rates, the DU optimum undermines the well-being of future generations, even though sustained development is feasible.

Our concern for global warming may lead to a change in our understanding of the feasibility set for future streams of well-being. If so, it may also change our ethical intuitions concerning how intergenerational conflicts should be resolved. This may in turn spur an interest, also among general economists, in social intergenerational preferences beyond the class of the DU SWOs.

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Shows that no SWO combining sensitivity and equal treatment can be explicitly defined.
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Errata

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