PROPERTIES OF A SYSTEM OF CURRENCY BASKETS

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This paper interprets the resulting regime when a number of countries tie their currencies to unilaterally designed baskets of other currencies, as a noncooperative exchange rate system. Consistency and stability of such a system of currency baskets are investigated through an application of the Frobenius-Perron Theorem on semipositive square matrices. It is established that devaluations preserve (while sufficiently large revaluations undermine) these properties. This asymmetry is caused by the use of arithmetic baskets which do not preserve effective weights when exchange rates differ from their base settings.

1. Introduction

Since the collapse of the fixed exchange rate regime in the early 1970s, countries have been faced with the question of whether and, if so, to what, to peg their currencies. Several countries have chosen to tie their currencies to unilaterally designed baskets of other currencies; European examples include Austria, Finland, Norway and Sweden. Instead of raising the question of a country's optimal peg,¹ this paper takes the individual baskets as given and interprets the resulting regime — when a number of countries tie their currencies to such baskets, and moreover, the baskets are interdependent — as a noncooperative exchange rate system.

Properties of such a system of currency baskets are investigated in section 2 through an application of the Frobenius–Perron Theorem on semipositive square matrices, providing a condition for the necessary coordination between baskets. By describing the effects of devaluations and revaluations, section 3 illuminates some undesirable features of this kind of exchange rate system associated with the use of arithmetic baskets. Section 4 contains concluding remarks, while the Frobenius–Perron Theorem as well as necessary notation are included in a mathematical appendix.

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¹For a survey of the literature on optimal peg, see Williamson (1982).
2. Consistency and stability

When a country (country \( i \)) ties its currency to a basket of \( n-1 \) other currencies, it selects a positive base exchange rate vector \( e_0 = (e_{i1}, \ldots, e_{in}) \), \( e_{ii} = 1 \), in terms of its own currency, and assigns non-negative\(^2\) nominal weights \( w_{ij} \) to country \( j \)'s currency (\( \sum_j w_{ij} = 1 \) and \( w_{ii} = 0 \)). Country \( i \) adjusts the supply of its currency so that the observed positive current exchange rate vector \( e = (e_{i1}, \ldots, e_{in}) \), \( e_{ii} = 1 \), in terms of its own currency, satisfies \( \sum_j w_{ij} e_{ij} / e_{ii} = 1 \).\(^3\) We assume that the country's central bank is perfectly able so to peg its currency, provided that a certain consistency requirement (to be discussed below) is fulfilled.\(^4\)

If countries 1,\ldots,\( m \), where \( m < n \), adopt this mechanism, the resulting linear system of \( m \) equations is called a system of currency baskets. Countries 1,\ldots,\( m \) will be referred to as the \( m \) member countries; countries \( m+1,\ldots, n \) as the \( n-m \) nonmember countries. Let \( x = (x_1, \ldots, x_n) \) be a relative exchange rate vector, with \( \sum_j x_j = 1 \). Assuming perfect arbitrage between the \( m \) currency markets, every member country observes the same \( x \); this unique \( x \) is equal to \( e_i / \sum_j e_{ij} \) for any \( i \). The system of currency baskets can thus be represented by

\[
A_1 x_1 + A_{H} x_{H} = x_H
\]

where \( A_1 = [a_{ij}] (i, j = 1, \ldots, m) \) and \( A_{H} = [a_{ij}] (i = 1, \ldots, m; j = m+1, \ldots, n) \) are both semipositive matrices whose elements are defined by \( a_{ij} = w_{ij} / e_{ii} \), and where \( x_1 = (x_1, \ldots, x_m) \) and \( x_{H} = (x_{m+1}, \ldots, x_n) \). We will require the coefficient matrix \( A_1 \) to be indecomposable so that no subsystem is self-contained (see the appendix). We define a system of currency baskets to be consistent if, for any given relative exchange rates between nonmember currencies, there exists a unique positive vector \( x^* \) satisfying (1) and \( \sum_j x_j^* = 1 \). The following proposition gives a sufficient and necessary condition for consistency.

**Proposition 1.** A system of currency baskets characterized by the indecomposable coefficient matrix \( A_1 \) is consistent if and only if the Frobenius-Perron root of \( A_{H} \), \( \lambda(A_{H}) \), is smaller than one.

**Proof.** If \( \lambda(A_{H}) < 1 \), then \( x_1 = (I - A_{H})^{-1} A_{H} x_{H} > 0 \) by (1) and lemma 5 in the appendix; i.e. the relative exchange rates between nonmember currencies uniquely determine all relative exchange rates. \( \sum_j x_j^* = 1 \) insures a unique \( x^* \).\(^5\)

\(^1\) For mathematical convenience we choose to limit ourselves to non-negative weights, even though negative weights to some currencies is a theoretical possibility that appears to have policy relevance; see Williamson (1982, p. 43).

\(^2\) This kind of arithmetic index is currently used to indicate the external value of the Finnish markka, the Norwegian krone and the Swedish krona.

\(^3\) In practice, most basket peggers intervene only when the basket deviates by some percentage from 1.
If \( x^* \) exists, then \((I - A_j)x_j^* = A_{1j}x_j^* \geq 0\) by (1), and lemma 4 in the appendix implies \( \lambda(A_i) < 1 \).

**Remark.** The condition \( \lambda(A_i) < 1 \) is mathematically equivalent to the productiveness condition that applies to a Leontief system with input coefficient matrix \( A_i \).

The condition on \( A_i \) that secures the consistency of a system of currency baskets also implies that the new solution vector \( x^* \), in the wake of altered relative exchange rates between nonmember countries, can be reached through an iterative process, without any coordination between member countries other than a common choice of a nonmember currency as numeraire.\(^5\) Let \( x^t = (x_1^t, x_2^t) \) be the relative exchange rate vector at time \( t \), with \( \sum_j x_j^t = 1 \). Furthermore, let \( y^t = (y_1^t, y_{11}^t) \) be the exchange rate vector at time \( t \) in terms of the common numeraire, with \( y^t = x^t / x_n^t \) (\( n \) is the chosen numeraire currency), and where \( y_{11}^t \) is constant for any given relative exchange rates between nonmember currencies. Assume that every member country determines its desired exchange rate in terms of this particular numeraire at time \( t \) on the basis of observed exchange rates at time \( t-1 \), i.e.

\[
A_{ij}y_j^{t-1} + A_{1j}y_{11}^t = y_i^t.
\]

Then we may define a system of currency baskets to be stable if the sequence \( (y_1^0, y_1^1, \ldots, y_1^{t-1}, y_1^t, \ldots) \) determined by (2) approaches the unique positive vector \( y^* \), with \( y^* = (y_1^*, y_{11}^*) \) satisfying (1) and \( \sum_j y_j^* = 1 / x^* \), for any initial vector \( y_1^0 \).

**Proposition 2.** Any consistent system of currency baskets is stable.

*Proof.* Note that \( y_1^* = (I - A_j)^{-1} A_{1j}y_{11}^t \), by (1), and \( y_i^t = (\sum_{j=1}^{t-1} A_j) A_{1j}y_{11}^t + A_{1j}y_1^0 \), by (2). The proposition follows from lemma 5 in the appendix.

The last proposition of this section shows that restrictions on base exchange rates provide a sufficient condition for the consistency and stability of a system of currency baskets. Base exchange rates among the \( m \) member countries are called internally consistent if there exists a positive vector \( x_0^* = (x_1^0, \ldots, x_m^0) \) satisfying \( x_j^0 = e_j^0 / \sum_{k=1}^m e_k^0, j = 1, \ldots, m \), for any \( i \).

**Proposition 3.** A system of currency baskets is consistent and stable if the base exchange rates among its members are internally consistent.

\(^5\)An obvious real-world candidate is the U.S. $. As the argument below goes through also when a composite of nonmember currencies is used as numeraire, the 1981 definition of the SDR is a relevant alternative.
Proof.

\[ \sum_{j=1}^{m} a_{ij} x_j^0 = \sum_{j=1}^{m} \left( \left( \frac{w_{ij}}{e_{ij}^0} \right) e_{ij}^0 \right) / \sum_{k=1}^{m} e_{ik}^0 = \left( 1 - \sum_{j=m+1}^{n} w_{ij} \right) x_i^0, \]

recalling that \( \sum_{j=1}^{m} w_{ij} = 1 \) and \( e_{ij}^0 = 1 \). Since \( A_n \geq 0 \), there exists some \( i \) for which \( \sum_{j=m+1}^{n} w_{ij} > 0 \). Hence, \( (I - A) x_i^0 \geq 0 \), and the proposition follows from lemma 4 in the appendix and propositions 1 and 2.

Remark. Internal consistency of base exchange rates is sufficient to establish proposition 3, but not sufficient to insure \( x_i^0 / x_j^0 = x_i^* / x_j^* \).

Internal consistency clearly holds if member countries choose base exchange rates equal to observed exchange rates at the same point in time. Some thought should make it clear that adoption of baskets at different points in time still produces a consistent and stable system even though base exchange rates become internally inconsistent.\(^6\) However, as will be shown in the next section, subsequent devaluations or revaluations also make base exchange rates internally inconsistent and may undermine the consistency and stability of the system.

3. Devaluations and revaluations

Country \( i \) devalues (revalues) by proportionally increasing (decreasing) all elements of \( e_i^0 \), save \( e_{ii}^0 (= 1) \),\(^7\) i.e. it proportionally decreases (increases) its row of coefficients in \( A = (A_n, A_n) \). Hence, a devaluation (revaluation) is achieved by premultiplying \( A \) by a diagonal matrix \( D \), where the devaluing (revaluing) country's diagonal element is positive and smaller (greater) than one, and where all other diagonal elements are equal to one. If any one country devalues (revalues), the other \( m-1 \) member countries will have their currencies appreciated (depreciated) in terms of the devaluing (revaluing) country's currency. This effect will, however, be cancelled out by a depreciation (appreciation) of their currencies in terms of the \( n-m \) nonmember currencies. The other member countries will in effect pass on the consequences of a devaluation or revaluation to the nonmember countries. It follows that a devaluation (revaluation) by any one country is not equivalent to a revaluation (devaluation) by the \( m-1 \) other member countries. Hence,

\(^6\)Provided that \( e_i^0 = e_i \) when a nonmember country \( i \) adopts a basket and becomes a member, the solution vector \( x* \) will not change.

\(^7\)It might be argued that also \( e_i^0 \) be changed proportionally so that internal consistency of base exchange rates is preserved. However, this undermines the proof of proposition 3.

\(^8\)Norway applies this procedure [see Norges Bank (1982)]. Sweden and Finland adjust the target for \( \sum w_i e_{ij} / e_{ij} \) (currently being 1.32 in Sweden and in the range 1.219-1.275 in Finland). The effect on the coefficient matrix \( A_i \) is identical.
the effects on the consistency and stability of the system of a devaluation by a member country is not symmetric to the effects of a simultaneous revaluation by the other member countries. This asymmetry is captured by the following proposition.

**Proposition 4.** Any devaluation preserves the consistency and stability of a system of currency baskets. A revaluation may undermine the consistency and stability of such a system.

**Proof.** $D \leq I$ implies $DA_1 \leq A_1$. Hence, $\lambda(DA_1) < \lambda(A_1) < 1$ by lemma 3 in the appendix, where $\lambda(DA_1)$ is the Frobenius-Perron root of $DA_1$. The first part of the proposition also follows trivially from lemma 4 in the appendix. The second part of the proposition is most easily shown by assuming that all $m$ countries revalue simultaneously. If no diagonal element of $D$ is smaller than $\lambda(A_1)^{-1}$, it follows from lemma 3 in the appendix that $\lambda(DA_1) \geq 1$ since $DA_1 \geq A_1/\lambda(A_1)$.

**Remark.** Returning to the Leontief system analogy, a revaluation is mathematically equivalent to increased input coefficients. This clearly reduces the productiveness of a Leontief system.

Proposition 4 is not meant to indicate that inconsistency and unstability of a system of currency baskets is an empirical possibility. For the interdependence between currencies in existing noncooperative exchange rate systems is much too weak. Rather, it illuminates an undesirable feature associated with the use of arithmetic baskets when current exchange rates differ greatly from their base settings, namely that effective weights $(a_{ij}x^*_j/x^*_i)$ will deviate from their nominal values $(w_{ij})$. When one member country devalues, all member currencies depreciate in terms of nonmember currencies. Hence, the effective weights on nonmember currencies will increase and, on balance, the effective weights on other member currencies will decrease since effective weights sum to one. Should another devaluation occur, the member countries will therefore, on balance, insufficiently depreciate their currencies in terms of nonmember currencies. Inversely, revaluations, on balance, increase the effective weights on other member currencies and induce member countries to overreact, excessively appreciating their currencies in terms of nonmember currencies. This excessive response may theoretically undermine the consistency and stability of a noncooperative exchange rate system.

The deviation between effective and nominal weights can be eliminated by using geometric indices to indicate the external value of member currencies.

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9This may explain why Finland devalued its currency following the Swedish devaluation on 8 October 1982, even though the Swedish krona is represented in the Finnish basket.
Country $i$ then adjusts the supply of its currency so that $e_i = (e_{i1}, \ldots, e_{in})$ satisfies $\sum_j w_{ij} \ln (e_{ij}/e_{ij}^0) = 0$. There may be several reasons why the arithmetic basket is still preferred empirically: it is more easily calculated and, being a fixed combination of currencies, it has most of the properties that individual currencies have.  

Finally, it should be noted that the case in which there is only one nonmember currency ($m = n - 1$) is a roundabout description of a fixed exchange rate system, since all relative exchange rates are uniquely determined given the $m$ baskets. The system consisting of all $n$ countries may then be considered a cooperative exchange rate system, where the $n$th country 'coordinates' agreed upon devaluations (revaluations) by the other $n-1$ countries. Proposition 4 points to the fact that a simultaneous finite revaluation by the $n-1$ countries may lead to an infinite appreciation of their currencies in terms of the $n$th currency. ‘Coordination’ by the $n$th country thus becomes impossible since the exchange rate of its currency is driven down to zero. If we want the size of devaluations (revaluations) — as conveyed through the devaluation (revaluation) matrix $D$ — to correspond to the resulting change in relative exchange rates, this can be achieved by not only premultiplying $A = (A_i, A_{ii})$ by $D$ but also postmultiplying $DA_i$ by $D^{-1}$, since effective weights can be shown to be preserved through such an operation.  

4. Concluding remarks

We have shown that a system of currency baskets is consistent and stable under a wide set of circumstances, including the situation where base exchange rates among the member countries are internally consistent. Moreover, the consistency and stability are preserved whenever countries within such a noncooperative exchange rate system devalue, while sufficiently large revaluations undermine these properties. This asymmetry is caused by the use of arithmetic baskets which do not preserve effective weights when exchange rates differ from their base settings. Finally, the condition securing consistency and stability of a system of currency baskets is mathematically equivalent to the productiveness condition that applies to Leontief systems.

Appendix

A vector $x$ or a matrix $A$ is positive ($> 0$) if all its elements are positive, non-negative ($\geq 0$) if all its elements are non-negative, and semipositive ($\geq 0$) if all its elements are non-negative.

\footnote{For the latter point, see Polak's (1979) arguments in favour of an arithmetic basket for the SDR.}

\footnote{This is in some sense what the EMS intends to do through adjusting the composition of the eecu.}
its elements are non-negative, but not all zero. Notation for inequalities between vectors or matrices is analogous. A semipositive square matrix \( A \) is called **indecomposable** (or **irreducible**) if there exists no permutation of like rows and columns for which \( A \) may be written in the form

\[
\begin{bmatrix}
A_{11} & A_{12} \\
0 & A_{22}
\end{bmatrix}
\]

with square matrices \( A_1 \) and \( A_{22} \) on the diagonal. Let \( A \) be an indecomposable matrix. Then:

**Lemma 1.** The matrix \( A \) has a dominant characteristic root \( \lambda(A) \) which is simple, real, and positive.

**Lemma 2.** The characteristic vector \( x^* \) associated with \( \lambda(A) \) is positive.

**Lemma 3.** \( \lambda(A) \) increases when any element of \( A \) increases.

**Lemma 4.** There exists a positive vector \( x \) such that \((I - A)x \geq 0\) if and only if \( \lambda(A) < 1 \).

**Lemma 5.** \((I - A)^{-1}\) exists and equals \( \sum_{i=0}^{\infty} A^i > 0 \) if and only if \( \lambda(A) < 1 \).

Lemmas 1, 2, and 3 are a restatement of the **Frobenius-Perron Theorem** which is proved in numerous references, e.g. Debreu and Herstein (1953). \( \lambda(A) \) is often called the **Frobenius-Perron root**. Lemmas 4 and 5 are well established in the economic literature and are referred to as conditions securing **productiveness** (or **workability** and **profitability**) of a Leontief system with input coefficient matrix \( A \) [see, for example, Nikaido (1968, pp. 87-108)].

**References**


