Capital gains and net national product in open economies

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Abstract

It is shown within a given framework that, with the world economy following an egalitarian path, the aggregate capital gains being positive is equivalent to the interest rate tending to decrease. This result is of importance for the concept of net national product in open economies. In particular, with positive aggregate capital gains, an open economy cannot sustain consuming the return on its capital stocks, since some part of the return must be used to augment the country’s national wealth. It is established that a country’s share of world-wide sustainable consumption equals its share of worldwide wealth.

Keywords: Net national product; Open economies; Capital gains; Natural resources

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1. Introduction

In the aftermath of the World Commission on Environment and Development (WCED), there has been forthcoming a line of theoretical contributions\textsuperscript{1} suggesting how to correct the notion of net national product.

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\textsuperscript{1} See for example Hartwick (1990) and Mäler (1991).
(NNP) so that depletion of natural and environmental resources are taken properly into account. Most, if not all, of these contributions are based on Weitzman (1976). Weitzman shows that NNP can serve as an indicator of welfare in a closed economy with a constant population and with no exogenous technological progress. If \( x_t \) is consumption (being an indicator of the quality of life), \( k_t \) is a vector of capital stocks, and \( Q_t \) are competitive prices of the capital stocks in terms of current consumption, then this welfare indicator is given by \( x_t + Q_t k_t \). Hence, NNP includes current consumption and the value of net investments; capital gains, \( Q_t k_t \), are not included.

In line with the emphasis of the WCED on the concept of a sustainable development, it would seem, however, desirable that the concept of NNP could serve as an indicator of sustainability. This would amount to, following Hicks (1946, chapter 14), requiring that the NNP should measure what can be consumed in the present period without reducing future consumption possibilities.\(^2\) In other words, the NNP should equal the maximum consumption level that can be sustained. In Asheim (1994), I argue that even with the facilitating assumptions of a closed economy, a constant population, and no exogenous technological progress, NNP defined as \( x_t + Q_t k_t \) is not in general an exact indicator of sustainability, except in the uninteresting case with only one capital good.

At the least, however, one would like the NNP to equal consumption if consumption happens to be constant and at any time equal the maximum level that can be sustained. That NNP := \( x_t + Q_t k_t \) has this property in a closed economy with a constant population and with no exogenous technological progress, follows from the Hartwick rule (Hartwick, 1977; Dixit et al., 1980) since \( Q_t k_t = 0 \) at any time along such an egalitarian path. Hence, even though NNP := \( x_t + Q_t k_t \) does not in general indicate the maximum sustainable consumption level, it does equal the maximum sustainable level along an efficient consumption path that happens to be egalitarian.

If the Weitzman–Hartwick concept of NNP is being used in an open economy, then it implies that an economy living solely by harvesting non-renewable resources has NNP equal to zero: \(^3\) \( x_t = Q_t (-\dot{k}_t) \), where \((-\dot{k}_t)\) is the vector of extraction. Dasgupta (1990, footnote 24) claims that NNP in such an economy is equal to zero, but admits that this is paradoxical. The reason why this is paradoxical is that, with increasing resource prices on the

\(^2\) "It would seem that we ought to define a man’s income as the maximum value which he can consume during a week, and still expect to be as well off at the end of the week as he was in the beginning" (Hicks, 1946, p. 172).

\(^3\) This holds if the rents that arise by extracting the non-renewable resources are solely Hotelling rents. As implied by the analysis of Hartwick (1991), any Ricardian rents that arise will contribute to a positive NNP.
world market, the economy's 'technology' (when taking into account its terms-of-trade) will not be constant. The economy may therefore be able to sustain a positive stationary level of consumption, and if NNP is to serve as an indicator of sustainability may have a positive NNP. Hence, capital gains cannot be excluded when the closed world economy is split into the open economies that the separate countries represent.

At an opposite extreme, each country could include capital gains fully by letting NNP be given by \( x_t + Q_tk_t + \dot{Q}_tk_t \). This means that in an economy living solely by harvesting non-renewable resources (i.e. \( x_t = Q_t(-k_t) \)), NNP would exactly equal the capital gains, \( Q_tk_t \). If the NNP is being consumed (i.e. \( x_t = x_t + Q_tk_t + \dot{Q}_tk_t \)), it follows that the national wealth is constant ((\( d/dt \)[\( Q_tk_t \]) = \( Q_tk_t + \dot{Q}_tk_t = 0 \)). This points to a more general result: If the goal is to keep the national wealth non-decreasing, then a concept of NNP which includes capital gains would indicate the maximum allowable level of consumption.

The above yields a paradox. If each country wants to keep its national wealth constant, consumption equals a measure of NNP that includes capital gains. However, if we add all countries together to form a closed world economy and assume that the purpose is to keep the level of consumption constant, then consumption equals a measure of NNP that does not include capital gains.

It is the purpose of this paper to resolve this paradox. In Section 3, it is established as Propositions 1 (and 2) that if the aggregate capital gains are positive along an efficient (and egalitarian) path, then the interest rate tends to decrease. This means that it is necessary for each country to accumulate national wealth in order to keep the consumption constant. In particular, an economy living solely by harvesting non-renewable resources can consume only a part of the capital gains if it wants to sustain its consumption level; the rest must be used to accumulate national wealth in order to compensate for the decreasing rate of return. As the central result of the paper, Proposition 3 presents a measure - based on current prices (and price changes) - of any country's sustainable consumption in a world economy that implements an egalitarian path by having each country keep consumption constant. This result is illustrated in Section 4 by a constant population, stationary technology model of capital accumulation and resource depletion analyzed by Solow (1974). It is an interesting feature of Solow's (1974)
model that along an egalitarian path, world-wide wealth is increasing, reflecting positive aggregate capital gains as well as a decreasing interest rate. Using this model, a numerical example is provided in Section 5 by considering a two-country world. The formal analysis – being based on Dixit et al. (1980) and Asheim (1986) – as well as all proofs are relegated to an appendix. Section 2 provides introductory intuition, while Section 6 concludes by relating the present results to the analyses of other contributions.

2. Intuition

The following provides an intuition for the general result (of Propositions 1 and 2) that the existence of positive aggregate capital gains is equivalent to the interest rate tending to decrease. Assume that output $y$ depends on manmade capital $k_c$ and a resource flow $\dot{k}_r$, and can be split into consumption $x$ and investment in manmade capital $\dot{k}_c : y = x + \dot{k}_c = F(k_c, -\dot{k}_r)$. Assume that $F$ exhibits constant-returns-to-scale (CRS), and that the (non-renewable) resource is unproductive as a stock. Let the consumption (or rather, composite) good serve as numeraire. In a competitive equilibrium, the resource price $Q_r$ equals the marginal productivity of the resource flow,

$$F_2(k_c, -\dot{k}_r) = Q_r,$$

while the interest rate $i$ measures the marginal productivity of manmade capital and equals, with no profitable arbitrage opportunities, the growth rate of the resource price,

$$F_1(k_c, -\dot{k}_r) = i = \frac{\dot{Q}_r}{Q_r}.$$

With the consumption good as numeraire, aggregate capital gains equal $\dot{Q}_r k_r$. It follows that, for aggregate capital gains to be positive, it is sufficient that the marginal productivity of manmade capital and the value of the resource stock both be positive.

Let $c(i, Q_r)$ denote the minimum cost of producing one unit of output, $c(i, Q_r) := \min\{ik_c + Q_r(-\dot{k}_r) | F(k_c, -\dot{k}_r) = 1\}$. Given the CRS, and with the consumption good as numeraire, it follows that $c(i, Q_r) = 1$ in a competitive equilibrium. This is a zero-profit condition for the competitive firms of the CRS economy. The equation $c(i, Q_r) = 1$ defines a factor price contour, which – by a standard result of duality theory – is strictly decreasing given that the cost minimizing pair of capital stock and resource flow is strictly positive. Hence, an increasing $Q_r$ is equivalent to a decreasing $i_r$.

It follows that, in the particular model considered, a competitive equilibrium entails both positive aggregate capital gains and a decreasing interest
rate. Within a similar framework, this conclusion is supported by van Geldrop and Withagen (1993). While retaining the assumption of CRS, the equivalence of positive aggregate capital gains and a decreasing interest rate will be shown under general technological assumptions in Propositions 1 and 2.

The simple model of the present section serves as a two-fold warning:
- For the analysis of a competitive equilibrium in a resource economy, one should not carelessly assume a constant interest rate since this assumption may well be inconsistent with the competitiveness of the economy. The formal analysis of the present paper is therefore performed in a framework allowing for a non-constant interest rate;
- an open economy that trades in resources should be analyzed in a general equilibrium setting. As a consequence, the present paper discusses the concept of NNP in open economies using general equilibrium prices.

3. Results

The general framework in which the analysis is embedded is presented formally in the appendix, where it is assumed – in addition to a constant population and no exogenous technological progress – that the technology of the closed world economy shows CRS. This is appropriate here since it enables:

(1) consumption to be split into the productive contributions of the factors of production, and

(2) the world economy to be split into open economies (countries) with subpopulations of constant size.

Each of these countries obtains a fraction of the total consumption that equals the productive contributions of the factors of production it owns. The world economy is assumed to be competitive. For each country’s national wealth to comprise the full productive capabilities of its factors of production, it is necessary to treat all factors of production, also labor, as capital goods, the market prices of which correspond to the present value of future earnings. That is, labor corresponds to a vector of human capital, which may include accumulated knowledge from learning and research activities.

Let \((x_t^*, k_t^*, \dot{k}_t^*)_{t=0}^{\infty}\) denote a competitive equilibrium of the world economy with capital prices \((Q_t)_{t=0}^{\infty}\), where the consumption good serves as numeraire. The CRS imply that consumption exactly suffices to pay capital}

\[\text{In line with usual meaning of NNP, it is the ownership of factors that matters, not where they are employed.}\]
owners the marginal products of their capital stocks plus the profits (resource rents) that arise stocks are depleted: \( x_t^* = R_t k_t^* + Q_t (-k_t^*) \), where \( R_t \) is a vector measuring the marginal productivities of the capital goods as stocks. Consequently, in aggregate, the Weitzman–Hartwick concept of NNP exactly suffices to pay the capital owners the marginal products of their capital stocks: \( x_t^* + Q_t k_t^* = R_t k_t^* \). From the arbitrage equation, \( i_t Q_t = R_t + Q_t \), where \( i_t \) denotes the instantaneous consumption interest rate, it follows that the instantaneous return on the capital stocks equals the sum of marginal products of the stocks plus the aggregate capital gains: \( i_t Q_t k_t^* = R_t k_t^* + Q_t k_t^* \). Hence, with \( Q_t k_t^* > 0 \), \( i_t Q_t k_t^* > R_t k_t^* \). This observation reflects that, with positive aggregate capital gains, the instantaneous interest rate tends to decrease in the precise sense given below.

**Proposition 1.** Within the given framework, if the competitive world economy implements an efficient path \((x^*, k^*, i_*^*)\)_{t=0} with capital prices \((Q_t)_{t=0}^\infty\), then \( Q_t k_t^* > 0 \) is equivalent to \( i_t Q_t k_t^* = \int_0^\infty \frac{s}{p_t} dx_t^* ds/p_t > \int_0^\infty i_t p_t x_t^* ds/p_t \) (\(=R_t k_t^*\)), where \((p_t)_{t=0}^\infty\) is the (implicit) consumption discount factor.

If the world economy at any time consumes the maximum sustainable level, it will be assumed that such a maximin path is efficient. Let \((x^m, k^m, i^m_t)\) denote the efficient maximin path. The Hartwick rule implies that \( x^m = x^m + Q_t k^m_t = R_t k^m_t \), the CRS imply that \( R_t k^m_t = [R_t k^m_t/(R_t k^m_t + Q_t k^m_t)] \cdot i_t Q_t k^m_t \). Hence, the maximum sustainable consumption level \( x^m = [R_t k^m_t/(R_t k^m_t + Q_t k^m_t)] \cdot i_t Q_t k^m_t \) falls short of the instantaneous return on the capital stocks \( i_t Q_t k^m_t \) if and only if the aggregate capital gains \( Q_t k^m_t \) are positive. As the following specialization of Proposition 1 shows, this observation also reflects the result that interest rates tend to decrease whenever aggregate capital gains are positive.

**Proposition 2.** Within the given framework, if the competitive world economy implements an efficient maximin path \((x^m, k^m, i^m_t)\)_{t=0} with capital prices \((Q_t)_{t=0}^\infty\), then the instantaneous consumption interest rate \( i_t \) exceeds the

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7 Note that the assumption of CRS entails that the profits that arise when non-renewable resources are depleted are solely Hotelling rents; any Ricardian rents that arise if the marginal cost of extraction exceeds the average cost accrue to other factors of production.

8 Furthermore, the price of a perpetual consumption annuity is assumed to be positive and finite along such a path. This corresponds to what in the appendix is called a regular maximin path, following the terminology of Burmeister and Hammond (1977).
ininitely long term consumption interest rate\(^9\) if and only if the aggregate capital gains \(\dot{Q}\dot{k}^n\) are positive.

Since, with the world economy implementing an efficient maximin path, the aggregate capital gains being positive is equivalent to the interest rate tending to decrease, a country \(j\) choosing an efficient maximin path cannot allow itself fully to consume the instantaneous return on its capital stocks, \(i\dot{Q},k_i\dot{t} = R,k_i\dot{t} + \dot{Q},k_i\dot{t}\), if the aggregate capital gains are positive. This is stated as the next proposition.

**Proposition 3.** Within the given framework, a worldwide efficient maximin path can be implemented in a decentralized economy where all countries choose efficient maximin paths, if and only if the consumption of each country \(j\) is given by:

\[
\text{For } a.e. \quad t, \quad x^j = R,k_i^n \cdot \left(\frac{Q,k_i^n}{Q,k_i^n}\right) \\
= \left[\frac{R,k_i^n}{(R,k_i^n + \dot{Q},k_i^n)}\right] \cdot (R,k_i^n + \dot{Q},k_i^n),
\]

where \(x^j\) and \(k_i^j\) are country \(j\)’s positive and constant consumption and semi-positive vector of capital stocks.

Proposition 3 supports the intuition that Hicksian income from a country’s ownership of capital depends on the value of its capital only; portfolio changes not influencing the value of its capital do not matter.\(^10\) In fact, since \(x^m = R,k_i^m\), Proposition 3 shows that a country’s share of worldwide sustainable consumption equals its share of worldwide wealth. Moreover, for given aggregate capital gains there is no reason for each separate country to differentiate between the return on capital caused by the productivity of its stocks and the return caused by capital gains.

Proposition 3 implies that if an efficient maximin path is implemented and the aggregate capital gains are positive, then some part of the instantaneous

\(^9\) If the (implicit) consumption discount factor decreases exponentially as a function of time, then there is one constant consumption interest rate. Otherwise, there is a term structure of consumption interest rates. The very short term interest rate is \(i\), the instantaneous consumption interest rate. The other extreme is the infinitely long term consumption interest rate. The inverse of the latter equals the price of a perpetual consumption annuity in terms of current consumption. It can be shown that the infinitely long term interest rate is strictly decreasing if and only if the instantaneous interest rate exceeds the infinitely long term interest rate.

\(^10\) In contrast, if the Weitzman–Hartwick concept of NNP is being used by a country living solely by harvesting non-renewable resources, then its NNP changes from zero to a positive value if its in situ resources are sold in return for ownership in foreign productive capital stocks.
return on a country's capital stocks must be used to augment the country's national wealth.

**Proposition 4.** Within the given framework, if a worldwide maximin path is implemented in a decentralized economy by having all countries choose efficient maximin paths, then each country $j$'s national wealth $Q_j k_i^1$ is increasing if and only if the aggregate capital gains $Q_j k_i^m$ are positive.

This result reflects the fact that, with positive aggregate capital gains, the interest rate tends to decrease; hence, this diminishing rate of return must be compensated by accumulating national wealth. It follows, however, from Proposition 3 that, since $\frac{R_i k_i^m}{(R_j k_j^m + Q_j k_j^m)} > 0$, even a country that is endowed solely with resources that are unproductive as stocks can sustain a positive level of consumption.

### 4. Illustration

In order to show that the results of Section 3 are not vacuous, a much used model of manmade capital accumulation and natural capital (resource) depletion—in which the aggregate capital gains are positive along an efficient maximin path—is presented as an illustration. Consider a model in which a flow of a non-renewable resource $-k_r$ is combined with constant human capital $k_h$ and manmade capital $k_c$ in order to produce a consumption good $x$. By letting its technology be described by $y := x + k_c \leq (k_h)^{1-a-b} (k_c)^a (1-\gamma)^b$, $b < a < a + b < 1$, this model fits into the given framework. As the analysis of Solow (1974) shows, a positive and constant consumption can be sustained indefinitely by letting accumulated manmade capital substitute for a diminishing resource extraction. Let $k_i^m = (k_h^m, k_c^m, k_r^m)$ denote the capital vector along such an efficient maximin path, with $x^m$ being the corresponding consumption level, where variables without subscript $t$ are constant. The investment in manmade capital turns out to be constant along this path, implying that total output $y^m$ is constant and is split between consumption $x^m = (1-b)y^m$ and investment in manmade capital $k_c^m = by^m$.

The world economy can be divided into open economies (countries) by letting each country have at its disposal the same CRS technology as the aggregate economy. Assume no labor mobility, implying that each country $j$ must employ its human capital $k_h^j$ domestically. Its asset management problem is to choose paths for $k_c^1$ and $k_i^1$ for given initial holdings. Competitive world markets with perfect capital mobility and free trade in the resource and the consumption good ensure overall productive efficiency.
Owing to identical CRS technologies, such efficiency requires that \( k_t^I/k_m^I \)
equal the relative size of the domestic economy where \( k^m_t k^I_t/k_m^I \) of manmade capital and \(-k^I_t k^I_t/k_m^I \) of resource flow are combined with \( k_t^I \) in order to produce \( y^m k_t^I/k_m^I \). In this multi-country world, \( k_t^I - k^m_t k^I_t/k_m^I \) is \( j \)'s financial assets on an international capital market and \(-k^I_t + k^m_t k^I_t/k_m^I \) is \( j \)'s resource exports.

What is the consumption of each country \( j \) when a worldwide maximin path is implemented by having all countries choose efficient maximin paths? By combining (A2), (5), (6), and (7) of Asheim (1986), it is obtained that the consumption of country \( j \) owning the vector \( k^I_t = (k^I_t, k^I_t, k^I_t) \) at time \( t \) is given by

\[
x^I = R_h k^I_t + \frac{a-b}{a} \cdot R_c t k^I_t + Q_r(-k^I_t) k^I_t \frac{k^m_t}{k^I_t}.
\]

Hence, country \( j \), on an efficient maximin path, will be consuming the marginal product of its human capital and resource rents in proportion to its stock of the resource, but only a fraction \((a-b)/a\) of the marginal product of its manmade capital. Since the CRS imply \( y^I = x^I + k^I_t = R_h k^I_t + R_r k^I_t + Q_r(-k^I_t) \), it follows from (*) that \( k^I_t = (b/a) \cdot R_c t k^I_t + Q_r(-k^I_t) + k^m_t k^I_t/k_m^I \). Thus, each country ends up reinvesting resource rents in proportion to its ownership of the total stock of manmade capital, provided that countries extract the resource in proportion to their stocks. In particular, since \((b/a) \cdot R_c t k^m_t = by^m = Q_r(-k^I_t)\), a country owning the whole stock of manmade capital would be investing in manmade capital at a level equal to the sum of worldwide resource rents. Alternatively, a country owning the whole resource stock and no manmade capital would be using all resource rents for consumption. These conclusions will be verified by the simple two-country example of Section 5.

Proposition 3 above sheds new light on these results. In the model considered, the instantaneous interest rate equals the marginal productivity of manmade capital; hence, \( i_t = ay^m/k^m_t \). Furthermore, the interest rate is decreasing along the efficient maximin path since total output \( y^m \) is constant and manmade capital is accumulated: \( k^m_t = k^m_t + by^m t \). Consequently, by Proposition 2, the term \([R(k^m_t)/(R^m_t + Q^m_t)]\) of Proposition 3 is smaller than 1; in fact, it equals \((a-b)/a\). Hence, if country \( j \) owns the vector \( k^I_t = (k^I_t, k^I_t, k^I_t) \) at time \( t \) in a world economy implementing a worldwide maximin path by having all countries choose efficient maximin paths, then

\[\text{By the proof of Proposition 3, the term in question is } \frac{(a-b)}{a}\frac{y^m}{k^m_t}.\]
with calculations (e.g. based on (4)–(10) of Asheim, 1986) showing that
\[ R_h = (1 - a - b)y^m/k_h, \quad \dot{Q}_h = [b/(a - b)](1 - a - b)y^m/k_h \] (i.e. \((a - b)/a\).
\[ (R_h + \dot{Q}_h) = (1 - a - b)y^m/k_h = R_h, \quad R_{ct} = ay^m/k_{ct}, \quad \dot{Q}_{ct} = 0, \quad R_{rt} = 0, \quad \text{and} \quad \dot{Q}_{rt} = [a/(a - b)] \cdot by^m/k_{rt}. \]
Although this conclusion derived from Proposition 2 confirms the results given by (*), new interpretations can be made:

(1) the owners of human capital can consume exactly the marginal product of their stocks since the capital gains exactly make up for the declining interest rate;

(2) the owners of manmade capital enjoy no capital gains and can consequently consume only a fraction \((a - b)/a\) of the marginal product of their stocks;

(3) the resource has no marginal productivity as a stock; the owners of the resource can still consume a fraction \((a - b)/a\) of the capital gains.

The last observation reiterates a point made earlier, viz. that a country endowed solely with a resource that is unproductive as a stock (e.g. oil) can sustain a positive consumption level indefinitely by consuming a fraction of the capital gains at each date.

5. A two-country example

Consider a two-country numerical example where human capital is distributed evenly \((k_h^1 = k_h^2 = \frac{1}{2}k_h^m)\), while only country 1 is endowed with the resource \((k_{rt}^1 = k_{rt}^m, k_{rt}^2 = 0)\). By choosing \(a = 0.20\) and \(b = 0.16\), it follows that \((a - b)/a = \frac{1}{3}\). Furthermore, \([a/(a - b)] \cdot (R_h + \dot{Q}_h) = R_h = 0.64y^m/k_h^m, (R_{ct} + \dot{Q}_{ct}) = R_{ct} = 0.20y^m/k_{ct}^m, (R_{rt} + \dot{Q}_{rt}) = R_{rt} = 0.80y^m/k_{rt}^m, \text{and} \ Q_{rt} = 0.16y^m/(-k_{rt}^m).\) Table 1 can now be constructed.

Note that maximin consumption varies to a small degree with ownership of manmade capital since (**)) implies that the owners of manmade capital can consume only a fraction \((a - b)/a = \frac{1}{3}\) of the marginal product of their stocks. Note also that a country owning the whole resource stock and no manmade capital (country 1 of Case 3) is increasing its financial debt along a maximin path.

6. Concluding remarks

National accounting seeks to measure income based on current prices (and price changes). Such an undertaking can easily be dismissed by the fact that all prices (in particular, prices for natural and environmental resources)
Table 1

<table>
<thead>
<tr>
<th>Case</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
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<tbody>
<tr>
<td>( k_{11} = k_{22}, k_{21} = 0 )</td>
<td>( k_{11} = k_{22} = \frac{1}{2} k_{11} )</td>
<td>( k_{11} = 0, k_{22} = k_{11} )</td>
</tr>
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</table>

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<thead>
<tr>
<th>Count 1</th>
<th>Count 2</th>
<th>Count 1</th>
<th>Count 2</th>
<th>Count 1</th>
<th>Count 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_j / y_m )</td>
<td>0.68</td>
<td>0.32</td>
<td>0.58</td>
<td>0.42</td>
<td>0.48</td>
</tr>
<tr>
<td>( x_j / y_m )</td>
<td>0.52</td>
<td>0.32</td>
<td>0.50</td>
<td>0.34</td>
<td>0.48</td>
</tr>
<tr>
<td>( I / y_m )</td>
<td>0.16</td>
<td>0</td>
<td>0.08</td>
<td>0.08</td>
<td>0</td>
</tr>
<tr>
<td>Domestic output / ( y_m )</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Financial assets</td>
<td>( \frac{1}{2} k_{11} )</td>
<td>( -\frac{1}{2} k_{11} )</td>
<td>0</td>
<td>0</td>
<td>( -\frac{1}{2} k_{11} )</td>
</tr>
<tr>
<td>Financial investments / ( y_m )</td>
<td>0.08</td>
<td>-0.08</td>
<td>0</td>
<td>0</td>
<td>-0.08</td>
</tr>
<tr>
<td>Resource exports</td>
<td>( \frac{1}{2} (-k_{11}) )</td>
<td>( -\frac{1}{2} (-k_{11}) )</td>
<td>( \frac{1}{2} (-k_{11}) )</td>
<td>( -\frac{1}{2} (-k_{11}) )</td>
<td>( \frac{1}{2} (-k_{11}) )</td>
</tr>
<tr>
<td>Cons. g. exports / ( y_m )</td>
<td>-0.10</td>
<td>0.10</td>
<td>-0.08</td>
<td>0.08</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Line (1) follows from CRS since \( y_j / y_m = 0.64 k_{11} k_{11} + 0.20 k_{11} k_{11} + 0.16 (-k_{11}) / (-k_{11}) \). Line (2) follows from \( x_j / y_m = 0.64 k_{11} k_{11} + 0.20 k_{11} k_{11} + 0.80 k_{11} / k_{11} \). Line (3) is obtained since (1) = (2) + (3). Line (4) follows since productive efficiency requires that \( k_{11} / k_{11} = \frac{1}{2} \) for \( j = 1, 2 \) equal the relative size of the domestic economy.

For the same reason, half of the stock of manmade capital must be employed in each country (implying the financial assets of line (5)), half of the investment in manmade capital must occur in each country (implying that the financial investment of line (6) follows from line (3)), and half of the extracted resource must be used in each country (implying the resource exports of line (7)). Line (8) follows from the identity (4) = (2) + ((3) - (6)) + (8), or alternatively, (6) = (8) + \( Q_{11}(7) + R_{11}(5) \).

The question is answered through Proposition 3 which establishes the following.

(1) If there are no aggregate capital gains, it is appropriate for each country to fully include its own individual capital gains (arising for example

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\[ ^{12} \text{This does not rule out that practical steps for adjusting national accounts can be suggested; see for example, El Serafy (1989).} \]
from changing terms-of-trade) in the measure of sustainable consumption. There is no reason for each separate country to differentiate between the return on capital caused by the productivity of its stocks and the return on capital caused by capital gains.

(2) The latter observation holds even if there are aggregate capitals gains. However, then, the fraction of the return on capital that can be used for consumption along an egalitarian path must be adjusted for the aggregate capital gains.

(3) In any case, a country's share of world-wide sustainable consumption equals its share of world-wide wealth.

These results seem to be novel. An antecedent is Asheim (1986), which, however, is not a contribution on national accounting, since sustainable consumption along an egalitarian path is not characterized by current prices (and price changes) only. Weitzman (1976), Solow (1986), Hartwick (1990) and Måler (1991), who lay a foundation for a Weitzman–Hartwick concept of NNP that is adjusted for the depletion of natural and environmental resources, do not discuss the problems associated with applying this concept of NNP in open economies which are confronted with changing terms-of-trade. Vellinga and Withagen (1992) analyze problems associated with defining NNP in an open economy with changing terms-of-trade. Looking at improving terms-of-trade as a kind of exogenous (or anticipated) technological progress also makes the analysis of Aronsson and Löfgren (1993) relevant. These contributions do not, however, present general equilibrium analyses, in contrast to the present paper. Van Geldrop and Withagen (1993) present a general equilibrium analysis of a multi-country world, but they are not concerned with national accounting. The paper that is most closely related to the present one is Sefton and Weale (1994) who discusses the national accounting of open economies in a general equilibrium setting with non-constant interest rates. However, their main concern goes beyond the scope adopted here.

The present paper defines NNP in open economies as the maximum sustainable consumption and measures this notion using the prices that exist if consumption at any time equals the maximum sustainable level. It does not address the question of how NNP can be measured in open economies that do hold consumption constant. As argued in the introduction, observable prices will not, in this case, yield a notion of NNP that satisfies the Hicksian (1946, Chapter 14) foundation by serving as an exact indicator of sustainability. An alternative would be to analyze how Weitzman's (1976) welfare foundation of NNP can be extended to the case of open economies interacting within a world economy in a general equilibrium setting. It is towards the solution of this problem that Sefton and Weale (1994) make an interesting contribution.
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Appendix

Based on Dixit et al. (1980) and recalling Asheim (1986), consider the following constant population economy. Its stationary technology is described by a smooth convex cone $Y$ consisting of feasible triples $(x, k, \dot{k})$, where consumption $x$ (being an indicator of the quality of life) is a non-negative scalar, and where $k$ denotes a non-negative vector of capital stocks. Assume that the set $Y$ satisfies free disposal of investment flows; i.e. if $(x, k, \dot{k}) \in Y$ and $\dot{k}' \leq \dot{k}$, then $(x, k, \dot{k}') \in Y$. The components of $k$ that represents non-renewable natural resources can only be depleted, and the corresponding components of $\dot{k}$ must be non-positive. For environmental resources that positively influence the quality of life the assumption of free disposal of investment flows means that these positively valued components of $k$ can freely be destroyed; hence, this assumption implies that negatively valued waste products can freely be generated, not freely be disposed of. Since production possibilities show CRS, assume that $k$ includes human capital components being employed as labor. Such components may measure accumulated knowledge from learning and research activities. Hence, as noted by Weitzman (1976), the assumption of a stationary technology does not exclude endogenous technological progress.

A feasible path $(x^*_t, k^*_t, \dot{k}^*_t)_{t=0}^\infty$ has competitive present value prices $(p_t, q_t)_{t=0}^\infty$ if and only if

for each $t$, $(x^*_t, k^*_t, \dot{k}^*_t)$ maximizes instantaneous profit $p_t x + q_t \dot{k} + \dot{q}_t k$

subject to $(x, k, \dot{k}) \in Y$.

The imputed rents to the assets are equal to $-\dot{q}_t$, measuring the marginal productivity of the capital stocks. The arbitrage equation $i_t Q_t = R_t + Q_t$ is obtained by letting $i_t := -\dot{p}_t / p_t$ denote the instantaneous consumption interest rate and by letting $Q_t := q_t / p_t$ and $R_t := -\dot{q}_t / p_t$ denote the prices and the marginal productivities respectively of the capital goods in terms of current consumption. For capital goods that are unproductive as stocks (e.g.
non-renewable natural resources), the corresponding components of \(-\dot{q}_t\) are zero (i.e. the Hotelling rule). The corresponding components of \(q_t\), equal the profits or rents that arise when such resources are depleted. \(P\) combined with the assumption of free disposal of investment flows implies that the vector \(q_t\) is non-negative. Note that as a consequence of CRS

\[ p_t x_t^* = -d(q_t k_t^*)/dt . \]  

(A1)

A path \((x_t^*, k_t^*, \mathbf{k}_t^*)_{t=0}^\infty\) with competitive prices \((p_t, q_t)_{t=0}^\infty\) is called regular if and only if

\[ q_t k_t^* \to 0 \text{ as } t \to \infty . \]  

(N)

It is easily shown that a regular path is efficient. Following Burmeister and Hammond (1977), a regular path \((x_t^*, k_t^*, \mathbf{k}_t^*)_{t=0}^\infty\) is a regular maximin path (RMP) if and only if

\[ x_m^t = x^m > 0 \text{ (constant) for all } t . \]  

(M)

An RMP is efficient and maximizes \(\inf x_t\) over the collection of feasible paths (see Burmeister and Hammond, 1977, Theorem 2). Condition (N) along with (A1) and (M) imply that \(q_t k_t^m\) equals \(\int_t^\infty p_s \, ds \cdot x^m\), where \(0 < \int_t^\infty p_s \, ds < \infty\). Hence,

\[ x^m = q_t k_t^m / \int_t^\infty p_s \, ds , \]  

(A2)

Asheim (1986) establishes that (A2) applies on a disaggregated level:

Lemma. Within the framework of the appendix, an RMP can be implemented in a decentralized competitive economy where all agents (defined as subpopulations of constant size) choose efficient maximin paths if and only if the consumption of each agent \(j\) is given by:

\[ \text{For a.e. } t, x_j = q_t k_t^j / \int_t^\infty p_s \, ds , \]  

(S)

where \(x_j^i\) and \(k_j^i\) are agent \(j\)'s positive and constant consumption and semi-positive vector of capital stocks.

Proof. The 'if' part follows from the theorem of Asheim (1986). The 'only if' part follows from the proof of the theorem of Asheim (1986) since \((x_j^i, k_j^i, k_j^i)_{t=0}^\infty\) defined by (S) and the budget constraint \(p_t x_t^i = -\dot{q}_t k_t^i - q_t k_t^i\) is efficient and yields constant consumption. Hence, if \(x_t \neq x_t^i\) for a non-trivial subset of time, then agent \(j\) does not choose an efficient maximin path. □
Dixit et al. (1980) show that an RMP satisfies a generalized Hartwick rule \((q_k' = \text{constant})\). They also demonstrate that under weak assumptions an RMP even obeys the ordinary Hartwick rule: \(q_k'' = 0\) for all \(t\). With CRS, the ordinary Hartwick rule implies that consumption is equal to the productive contributions of the capital stocks:

\[
x'' = -\frac{q_k''}{p_t}.
\]

(A3)

To see why (A3) cannot apply on a disaggregated level, consider an agent who is endowed solely with resources that are unproductive as stocks. By (A3) this agent would not be allowed positive consumption. By (S), however, positive and constant consumption is feasible if the agent's wealth is positive.

These preliminaries suffice for proving Propositions 1, 2, 3, and 4.

Proof of Proposition 1. Note that \(Q_t = \frac{d(q_t/p_t)}{dt} = q_t/p_t - \left(\frac{\dot{p}_t}{p_t}\right) \cdot (q_t/p_t); \) hence, with \(Q_t > 0\), \((-\dot{p}_t/p_t) \cdot q_k'' > -q_k''/p_t, \) By (N) and (A1), \(q_k'' = \int_t^\infty p_t x_t^* ds; \) thus, the l.h.s. equals \((-\dot{p}_t/p_t) \cdot \int_t^\infty p_t x_t^* ds/p_t. \) With \(i_t = -\dot{p}_t/p_t \) (equal to the instantaneous consumption interest rate), it remains to be shown that the r.h.s. equals \(\int_t^\infty (-\dot{p}_t/p_t) p_t x_t^* ds/p_t. \) By (A1), \(-q_k'' = p_k x_t^* + q_k'k^* \). Since \(p_t x_t^* = -d(q_t/k^*)/dt \) from Dixit et al., 1980, proof of Theorem 1), it follows that \(d(p_t x_t^*)/dt = \dot{p}_t x_t^* - d(q_t/k^*)/dt, \) \(d(p_t x_t^* + q_t k^*)/dt = \dot{p}_t x_t^* \) and \(p_t x_t^* + q_t k^* = \int_t^\infty [-\dot{p}_t/p_t] x_t^* ds, \) provided \(p_t x_t^* + q_t k^* \rightarrow 0 \) as \(t \rightarrow \infty. \)

Hence, \(-q_k''/p_t = (p_t x_t^* + q_t k^*)/p_t = \int_t^\infty (-\dot{p}_t/p_t) p_t x_t^* ds/p_t, \)

Proof of Proposition 2. The proof of Proposition 1 implies that \(Q_t k'' < 0 \) is equivalent to \((-\dot{p}_t/p_t) \cdot q_k'' > -q_k''/p_t. \) Since by (A2) and (A3), \((p_t/\int_{t'}^\infty p_t ds) \cdot q_k'' = -\dot{q}_k''/p_t, \) it now follows that with \(Q_t k'' > 0, i_t = -\dot{p}_t/p_t > -\dot{q}_k''/p_t = p_t/\int_{t'}^\infty p_t ds. \) As \(\int_{t'}^\infty p_t ds/p_t \) is the price of a perpetual consumption annuity in terms of current consumption, \(p_t/\int_{t'}^\infty p_t ds \) is the infinitely long term consumption interest rate.

Proof of Proposition 3. (A2) and (A3) imply that \(p_t/\int_{t'}^\infty p_t ds = -\dot{q}_k''/q_k'' = R_t k'/Q_t k'' \); hence, by the Lemma, \(x' = (p_t/\int_{t'}^\infty p_t ds) \cdot (q_k''/p_t) = (R_t k''/Q_t k'') \cdot Q_t k''/Q_t k'' \) or \(x' = R_t k'' \cdot (Q_t k''/Q_t k''). \) From the arbitrage equation, \(i_t Q_t = R_t + \dot{Q}_t, \) it follows that \((R_t k''/Q_t k'') \cdot Q_t k'' = (R_t k''/i_t Q_t k'') \cdot i_t Q_t k'' = [R_t k''/(R_t k'' + \dot{Q}_t)] \cdot (R_t k'' + \dot{Q}_t). \)

Given (A1) and (N), a sufficient condition for this convergence is that the growth rate of each capital stock and price is bounded above. Proof of this claim is available on request from the author.
Proof of Proposition 4. From country j's budget constraint, \( x_j = (-q_i k_j - q_j k_j^*) / p_t = R_j k_j^* - Q_j k_j^* \), it follows that the time derivative of country j's wealth, \( Q_j k_j^* \), is given by: \( d(Q_j k_j^*) / dt = \dot{Q}_j k_j^* + Q_j \dot{k}_j^* = R_j k_j^* + \dot{Q}_j k_j^* - x_j \). By Proposition 3, \( R_j k_j^* + Q_j k_j^* - x_j = [Q_j k_j^* / (R_j k_j^* + Q_j k_j^*)] \cdot (R_j k_j^* + Q_j k_j^*) \); i.e. \( d(Q_j k_j^*) / dt > 0 \) is equivalent to \( \dot{Q}_j k_j^* > 0 \). □

References


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