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Net National Product as an Indicator of Sustainability

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I. Introduction

Net National Product (NNP) can potentially serve several objectives, among others to measure value added and to be an indicator of welfare. In the aftermath of the World Commission on Environment and Development, however, it also seems important to investigate whether the concept of NNP can serve as an indicator of sustainability. My point of departure, following Hicks (1946, Ch. 14), is therefore to require that NNP should measure what can be consumed in the present period without reducing future consumption possibilities[1] and, in line with this, to argue that the NNP should equal the maximum per capita consumption level that can be sustained.

If sufficiently many facilitating assumptions are made, then a concept of NNP that serves as an exact indicator of sustainability can easily be constructed. For instance, for a closed economy with a constant population, a stationary technology, and with only one capital good, it follows that the present does not reduce future consumption possibilities if and only if it does not decrease the stock of the single capital good. Hence, under the assumption that the one capital good k is identical to the one consumption good x, NNP defined as $y_t := x_t + \dot{k}_t$ measures the maximal sustainable consumption at time t, because then $x_t \leqq y_t$ is equivalent to $\dot{k}_t \geqq 0$. The purpose of this note is to establish that *with multiple capital goods*, it is not in general possible to construct an exact indicator of sustainability on the

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[1] "[I]t would seen that we ought to define a man's income as the maximum value which he can consume during a week, and still expect to be as well off at the end of the week as he was in the beginning" (Hicks, 1946, p. 172). See Scott (1990) for a recent discussion of Hicksian income, with a reply and a comment in the same issue of *Journal of Economic Literature* by Robert Eisner and David Bradford.

basis of current price information as usually suggested. This conclusion holds even if (1) the remaining facilitating assumptions are retained and (2) the existence of an intertemporal competitive equilibrium is assumed; in particular, price information for all goods (including natural capital goods) is assumed to be available in an economy without market imperfections of any kind.

The background for the interest in the problem of sustainability is that human economic activity leads to the depletion of natural capital. It therefore becomes an important question whether our accumulation of (real and human) man-made capital is sufficient to make up for the decreased availability of natural capital. Hence, *sustainability is interesting only within models of heterogeneous capital.* Furthermore, even with a constant population and a stationary technology, an efficient path giving rise to constant consumption (or constant utility) will not be a stationary path. In contrast, the stocks of natural capital will tend to be depleted, while the stocks of man-made capital will tend to be accumulated. As argued by Bliss (1975), there will not in general exist one constant rate of interest along a non-stationary path in a heterogeneous capital model. Rather, each capital good will have its own rate of interest that will vary over time. Moreover, maximizing the sum of utilities disocunted at a constant rate will not in general give rise to a constant utility path.[2]

In contrast to Weitzman (1976), Solow (1986), Hartwick (1990) and Mäler (1991), who lay the foundation for a concept of NNP that is adjusted for the depletion of natural and environmental resources, I find it more appropriate for the discussion of NNP as an indicator of sustainability to build on a framework which allows for a non-constant rate of utility discounting. Section II presents such a framework, which is then used in Section III to discuss the seminal analyses of Weitzman (1976) and Hartwick (1977). In Section IV it is shown through a counter example that the concept of NNP which — on the basis of these contributions — has been suggested in order to take into account the depletion of natural capital is not in general an indicator of sustainability. The interpretation of this result is that — even in an economy without market imperfections of any kind — the competitive prices at a given time cannot, with multiple capital goods, provide information on whether consumption exceeds the sustainable level since, in an intertemporal competitive equilibrium, the relative prices of the multiple capital goods will depend on the entire equilibrium path.

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[2] In fact, implementing an efficient constant consumption path in the model analyzed by Solow (1974) and Hartwick (1977) corresponds to maximizing the sum of utilities discounted at a positive and *decreasing* rate; see the model of Section IV.

II. The General Framework

The general analysis of a constant population economy with heterogeneous capital builds on Dixit et al. (1980). The indicator of the quality of life $u(\boldsymbol{x})$ depends on the net output vector $\boldsymbol{x}$, where $u(\cdot)$ is a stationary and differentiable utility function. Let $(\boldsymbol{x}, \boldsymbol{k}, \dot{\boldsymbol{k}})$ be feasible if and only if $(\boldsymbol{x}, \boldsymbol{k}, \dot{\boldsymbol{k}}) \in Y$, where Y is a smooth and convex set and where $\boldsymbol{k}$ is the vector of non-negative capital stocks. The set Y is assumed to satisfy free disposal of investment flows; i.e., if $(\boldsymbol{x}, \boldsymbol{k}, \dot{\boldsymbol{k}}) \in Y$ and $\dot{\boldsymbol{k}}' \leqq \dot{\boldsymbol{k}}$, then $(\boldsymbol{x}, \boldsymbol{k}, \dot{\boldsymbol{k}}') \in Y$. The components of $\boldsymbol{k}$ that represent non-renewable natural resources can only be depleted, and the corresponding components of $\dot{\boldsymbol{k}}$ must be non-positive. Stocks that directly influence the quality of life (e.g. environmental resources) are taken into account by letting these components of $\boldsymbol{k}$ correspond to components of $\boldsymbol{x}$. For environmental resources, the assumption of free disposal of investment flows means that these positive valued resources can be freely destroyed; hence, negatively valued waste products can be freely generated, not freely disposed of. Accumulated knowledge from learning and research activities may be included as components of $\boldsymbol{k}$, thus allowing for *endogenous* technological progress; see Weitzman (1976).

A feasible path $(\boldsymbol{x}_t^*, \boldsymbol{k}_t^*, \dot{\boldsymbol{k}}_t^*)_{t=0}^{\infty}$ will be called *competitive* at present value prices $(\boldsymbol{p}_t, \boldsymbol{q}_t)_{t=0}^{\infty}$ and utility discount factors $\lambda_t > 0$ if and only if

(i) for each t, $(\boldsymbol{x}_t^*, \boldsymbol{k}_t^*, \dot{\boldsymbol{k}}_t^*)$ maximizes instantaneous profit $\boldsymbol{p}_t\boldsymbol{x} + \boldsymbol{q}_t\dot{\boldsymbol{k}} + \dot{\boldsymbol{q}}_t\boldsymbol{k}$ subject to $(\boldsymbol{x}, \boldsymbol{k}, \dot{\boldsymbol{k}}) \in Y$,
(ii) for each t, $\boldsymbol{x}_t^*$ maximizes $\lambda_t u(\boldsymbol{x}) - \boldsymbol{p}_t\boldsymbol{x}$ over all $\boldsymbol{x}$.

The imputed rents to the assets are equal to $-\dot{\boldsymbol{q}}_t$, measuring the marginal productivity of the capital stocks. For capital goods that are unproductive as stocks (e.g. non-renewable natural resources), the corresponding components of $-\dot{\boldsymbol{q}}_t$ are zero (i.e., the Hotelling rule). The corresponding components of $\boldsymbol{q}_t$ equal the profits or rents that arise when such resources are depleted. Note that (i) combined with the assumption of free disposal of investment flows implies that the vector $\boldsymbol{q}_t$ is non-negative.

A competitive path $(\boldsymbol{x}_t^*, \boldsymbol{k}_t^*, \dot{\boldsymbol{k}}_t^*)_{t=0}^{\infty}$ will be called *regular* at present value prices $(\boldsymbol{p}_t, \boldsymbol{q}_t)_{t=0}^{\infty}$ and utility discount factors $\lambda_t > 0$ if and only if

(a) $\int_0^{\infty} \lambda_t u(\boldsymbol{x}_t^*)\, dt$ exists and is finite
(b) $\boldsymbol{q}_t\boldsymbol{k}_t^* \to 0$ to $t \to \infty$.

Condition (b) entails that the value of the capital stocks along a regular path equals the present value of the rents that arise from the future productivity and depletion of the stocks: $\boldsymbol{q}_t\boldsymbol{k}_t^* = \int_t^{\infty} [(-\dot{\boldsymbol{q}}_s)\boldsymbol{k}_s^* + \boldsymbol{q}_s(-\dot{\boldsymbol{k}}_s^*)]\, ds$. A regular path is efficient and maximizes $\int_0^{\infty} \lambda_t u(\boldsymbol{x}_t)\, dt$ over all feasible paths $(\boldsymbol{x}_t, \boldsymbol{k}_t, \dot{\boldsymbol{k}}_t)_{t=0}^{\infty}$ with given initial stocks $\boldsymbol{k}$ since, for each

t, $\lambda_t > 0$, $\boldsymbol{q}_t \boldsymbol{k}_t \geqq 0$, and, by (i) and (ii), $\lambda_t[u(\boldsymbol{x}_t - u(\boldsymbol{x}_t^*)] \leqq \mathrm{d}[\boldsymbol{q}_t \boldsymbol{k}_t^* - \boldsymbol{q}_t \boldsymbol{k}_t]/\mathrm{d}t$; i.e., $\int_0^T \lambda_t[u(\boldsymbol{x}_t) - u(\boldsymbol{x}_t^*)]\,\mathrm{d}t \leqq \boldsymbol{q}_T \boldsymbol{k}_T^*$. Hence, a regular path provides price information for the net output vector and capital stocks and can be realized as a competitive equilibrium if the intergenerational altruism of each generation t is represented by $\int_t^\infty \lambda_s u(\boldsymbol{x}_s)\,\mathrm{d}s$.

With the price information provided by a regular path, is it possible to construct a concept of NNP that can serve as an indicator of sustainability? Let $(v_t^*)_{t=0}^\infty$ denote the NNP in terms of utility along the regular path $(\boldsymbol{x}_t^*, \boldsymbol{k}_t^*, \dot{\boldsymbol{k}}_t^*)_{t=0}^\infty$. Generalizing the analysis for the case with one capital good to multiple capital goods, a natural candidate for a concept of NNP in utility terms is $v_t^* := u(\boldsymbol{x}_t^*) + \boldsymbol{q}_t \dot{\boldsymbol{k}}_t^*/\lambda_t$. If net output is a scalar x, the NNP may alternatively be defined in terms of the single consumption good. For this case, let $(y_t^*)_{t=0}^\infty$ denote the NNP in consumption terms along the regular path $(x_t^*, \boldsymbol{k}_t^*, \dot{\boldsymbol{k}}_t^*)_{t=0}^\infty$. A natural candidate for this "dollar-value" NNP, see Hartwick (1990), is $y_t^* := x_t^* + \boldsymbol{q}_t \dot{\boldsymbol{k}}_t^*/p_t$. The question of whether v_t^* (or y_t^*) is an indicator of sustainability becomes a question of whether $\boldsymbol{q}_t \dot{\boldsymbol{k}}_t^* \geqq 0$ is equivalent to $u(\boldsymbol{x}_t^*)$ (or x_t^*) not exceeding the maximum sustainable utility (consumption) level. Two alternative foundations for the use of $v_t^* := u(\boldsymbol{x}_t^*) + \boldsymbol{q}_t \dot{\boldsymbol{k}}_t^*/\lambda_t$ (or $y_t^* := x_t^* + \boldsymbol{q}_t \dot{\boldsymbol{k}}_t^*/p_t$) can be found in Weitzman (1976) and Hartwick (1977). Can these contributions be used to defend v_t^* (or y_t^*) as an indicator of sustainability? Is capital management sustainable if and only if the market value of net investments is non-negative?

III. The Contributions of Weitzman (1976) and Hartwick (1977)

The following is a generalized version of the result reported in Weitzman (1976).

Proposition 1. *If a path $(\boldsymbol{x}_t^*, \boldsymbol{k}_t^*, \dot{\boldsymbol{k}}_t^*)_{t=0}^\infty$ is regular at prices $(\boldsymbol{p}_t, \boldsymbol{q}_t)_{t=0}^\infty$ and utility discount factors $\lambda_t = \mathrm{e}^{-rt}$, $r > 0$, then for each t, $(u(\boldsymbol{x}_t^*) + \boldsymbol{q}_t \dot{\boldsymbol{k}}_t^*/\lambda_t) \cdot \int_t^\infty \lambda_s\,\mathrm{d}s = \int_t^\infty \lambda_s u(\boldsymbol{x}_s^*)\,\mathrm{d}s$.*

Proof: By the Lemma below, $\mathrm{d}(\lambda_t u(\boldsymbol{x}_t^*))/\mathrm{d}t = \dot{\lambda}_t u(\boldsymbol{x}_t^*) - \mathrm{d}(\boldsymbol{q}_t \dot{\boldsymbol{k}}_t^*)/\mathrm{d}t$ or $\mathrm{d}(\lambda_t(u(\boldsymbol{x}_t^*) + \boldsymbol{q}_t \dot{\boldsymbol{k}}_t^*)/\mathrm{d}t = \dot{\lambda}_t u(\boldsymbol{x}_t^*)$. Hence, $\lambda_t u(\boldsymbol{x}_t^*) + \boldsymbol{q}_t \dot{\boldsymbol{k}}_t^* - (\lambda_T u(\boldsymbol{x}_T^*) + \boldsymbol{q}_T \dot{\boldsymbol{k}}_T^*) = \int_t^T (-\dot{\lambda}_s) u(\boldsymbol{x}_s^*)\,\mathrm{d}s = r\int_t^T \lambda_s u(\boldsymbol{x}_s^*)\,\mathrm{d}s$ with $r = -\dot{\lambda}_t/\lambda_t > 0$. Assuming that the growth rate of each capital stock and price is bounded above, it follows from (a) and (b) that $\lambda_T u(\boldsymbol{x}_T^*) + \boldsymbol{q}_T \dot{\boldsymbol{k}}_T^* \to 0$ as $T \to \infty$.[3] □

Lemma. *If a path $(\boldsymbol{x}_t^*, \boldsymbol{k}_t^*, \dot{\boldsymbol{k}}_t^*)_{t=0}^\infty$ is competitive at prices $(\boldsymbol{p}_t, \boldsymbol{q}_t)_{t=0}^\infty$ and utility discount factors $\lambda_t > 0$, then for each t, $\lambda_t\,\mathrm{d}u(\boldsymbol{x}_t^*)/\mathrm{d}t = \boldsymbol{p}_t \dot{\boldsymbol{x}}_t^* = -\mathrm{d}(\boldsymbol{q}_t \dot{\boldsymbol{k}}_t^*)/\mathrm{d}t$.*

Proof: The result follows from Dixit et al. (1980, proof of Theorem 1). □

[3] Proof of this claim is available on request from the author.

Hence, if the utility discount rate $-\dot{\lambda}_t/\lambda_t$ is constant and equal to r, the present value of future utility equals the present value for receiving the utility flow $u(\boldsymbol{x}_t^*)+\boldsymbol{q}_t\dot{\boldsymbol{k}}_t^*/\lambda_t$ for all $s \geqq t$. However, if $(\boldsymbol{x}_s^*)_{s=t}^{\infty}$ uniquely maximizes $\int_t^{\infty} \lambda_s u(\boldsymbol{x}_s)\,\mathrm{d}s$ over all feasible paths and does not yield constant utility, then Proposition 1 implies that it is *not* feasible to sustain a constant utility level equal to $u(\boldsymbol{x}_t^*)+\boldsymbol{q}_t\dot{\boldsymbol{k}}_t^*/\lambda_t$. Also, $\boldsymbol{q}_t\dot{\boldsymbol{k}}_t^* \geqq 0$ does not imply that $u(\boldsymbol{x}_t^*)$ is sustainable. This result, which undermines a claim made by Mäler (1991, p. 11) and Hulten (1992, p. 17), will be formally established in Section IV.

The result originally stated by Weitzman (1976) follows as a corollary.

Corollary. *(Weitzman, 1976) Suppose $u(x)=x$ with net output x being a scalar. If a path $(x_t^*, \boldsymbol{k}_t^*, \dot{\boldsymbol{k}}_t^*)_{t=0}^{\infty}$ is regular at prices $(p_t, \boldsymbol{q}_t)_{t=0}^{\infty}$ and utility discount factors $\lambda_t = e^{-rt}$, $r>0$, then for each t, $\lambda_t = p_t$ and $(x_t^* + \boldsymbol{q}_t\dot{\boldsymbol{k}}_t^*/p_t)\cdot \int_t^{\infty} p_s\,\mathrm{d}s = \int_t^{\infty} p_s x_s^*\,\mathrm{d}s$.*

Hence, if the own interest rate of consumption good is constant, the present value of future consumption equals the present value of consuming $x_t^* + \boldsymbol{q}_t\dot{\boldsymbol{k}}_t^*/p_t$ for all $s \geqq t$. Weitzman (1976) did not make the incorrect claim that this implies that it is feasible to sustain a consumption level equal to $x_t^* + \boldsymbol{q}_t\dot{\boldsymbol{k}}_t^*/p_t$.

Moreover, note Svensson's (1986) observation that Proposition 1 (or its corollary) cannot be established without the assumption of a positive and constant utility discount rate (or consumption interest rate); in the model of Solow (1974) and Hartwick (1977), this assumption is inconsistent with sustainable development. This alone undermines the relevance of this result for the study of NNP and sustainability.

Hartwick (1977) finds that, in a closed economy with a constant population and a stationary technology steering along an efficient path with $\boldsymbol{q}_t\dot{\boldsymbol{k}}_t^* = 0$ *for all* t, the utility level is constant and equal to the maximal sustainable level.

Proposition 2. *(Hartwick 1977) If a path $(\boldsymbol{x}_t^*, \boldsymbol{k}_t^*, \dot{\boldsymbol{k}}_t^*)_{t=0}^{\infty}$ is regular at prices $(\boldsymbol{p}_t, \boldsymbol{q}_t)_{t=0}^{\infty}$ and utility discount factors $\lambda_t > 0$, and for each t, $\boldsymbol{q}_t\dot{\boldsymbol{k}}_t^* = 0$, then $u(\boldsymbol{x}_t^*)$ is constant.*

Proof: The Lemma implies that along a competitive path, $u(\boldsymbol{x}_t^*)$ is constant if and only if $\boldsymbol{q}_t\dot{\boldsymbol{k}}_t^*$ is constant. Having $\boldsymbol{q}_t\dot{\boldsymbol{k}}_t^* = 0$ for all t is therefore sufficient. □

Dixit et al. (1980) show the converse under weak assumptions. If the utility level is constant along an efficient path, then $\boldsymbol{q}_t\dot{\boldsymbol{k}}_t^* = 0$ *for all* t. Hence, $\boldsymbol{q}_t\dot{\boldsymbol{k}}_t^* = 0$ *for all* t becomes equivalent to $u(\boldsymbol{x}_t^*)$ being equal to the maximum sustainable utility level *for all* t. This supports $v_t^* := u(\boldsymbol{x}_t^*) + \boldsymbol{q}_t\dot{\boldsymbol{k}}_t^*/\lambda_t$ as an indicator of sustainability.

In the context of a competitive economy, Hartwick's rule states that a competitive equilibrium leads to a completely egalitarian utility path if and only if, at all times, the values of depleted natural capital measured in competitive prices equals the reinvestment in man-made capital. However, Hartwick's rule does *not* claim that a competitive economy which, *for the moment*, at market value reinvests depleted natural capital in man-made capital, manages its stocks of natural and man-made capital in a sustainable manner. For it is conceivable that such reinvestment is achieved *because* the competitive prices of natural capital are low. This, in turn, can be caused by the economy not being managed in a sustainable manner. If future generations are poorer than we are, they will be unable to "bid' highly through the intertemporal competitive equilibrium for the depletable natural capital we manage, leading to low prices of such capital today. Although Hartwick's rule implies that $\mathbf{q}_t\dot{\mathbf{k}}_t^* = 0$ *at any time t* if the economy follows an efficient and egalitarian utility path, it cannot be concluded that $u(\mathbf{x}_t^*)$ is sustainable if $\mathbf{q}_t\dot{\mathbf{k}}_t^* = 0$ *at some time t*. This is shown formally in the next section. Hence, *Hartwick's rule characterizes a sustainable development; it is not a prescriptive rule for a sustainable development.*

IV. A Counter example

Consider a model in which the flow of natural capital (a non-renewable resource), $-\dot{k}_n$, is combined with a stock of man-made capital, k_m, in order to produce a consumption good, x. The model, that fits into the framework of Section II, is described by the following technology and utility function:

$$x + \dot{k}_m \leqq (k_m)^a(-\dot{k}_n)^b, \qquad b < a < a + b < 1, \tag{1}$$

$$u(x_t) = -x_t^{-(\eta-1)}, \tag{2}$$

where $\eta > 1$ denotes the elasticity of marginal utility.

For positive initial stocks, an efficient path with positive and constant consumption is feasible; see Solow (1974). Still, for any positive discount rate r, the unique path $(x_t^*, \mathbf{k}_t^*, \dot{\mathbf{k}}_t^*)_{t=0}^{\infty}$ maximizing $\int_0^\infty e^{-rt}u(x_t)\,dt$ over all feasible paths forces consumption to eventually approach zero; cf. Dasgupta and Heal (1974; 1979, pp. 292–303). Furthermore, $(x_t^*)_{t=0}^{\infty}$ is single peaked, and, as illustrated by Dasgupta and Heal (1979, Diagram 10.3), for r "large", $(x_t^*)_{t=0}^{\infty}$ starts out with a consumption flow that exceed the maximum sustainable level, while for r "small", the initial consumption flow falls short of this level. By a continuity argument, there exists a rate of discount r' such that x_0^* equals the maximum sustainable level given the initial stocks. Since $(x_t^*)_{t=0}^{\infty}$ is efficient, consumption must be increasing in an initial phase, before the eventual phase sets in with consumption decreasing and asymptotically approaching zero. For later reference, let t'' denote the time at which consumption reaches its peak.

Consider the path $(x_t^*, \mathbf{k}_t^*, \dot{\mathbf{k}}_t^*)_{t=0}^{\infty}$ maximizing $\int_0^{\infty} e^{-r't} u(x_t)\,dt$ over all feasible paths. By standard arguments there exist supporting present value prices $(p_t, \mathbf{q}_t)_{t=0}^{\infty}$ such that $(x_t^*, \mathbf{k}_t^*, \dot{\mathbf{k}}_t^*)_{t=0}^{\infty}$ is regular at these prices and utility discount factors $\lambda_t = e^{-r't}$. Hence, $(x_t^*, \mathbf{k}_t^*, \dot{\mathbf{k}}_t^*)_{t=0}^{\infty}$ can be realised as a competitive equilibrium if the intergenerational altruism of each generation t is represented by $\int_t^{\infty} \lambda_s u(x_s)\,ds$. What can be said about the behavior of $\mathbf{q}_t \dot{\mathbf{k}}_t^*$ along this path? Note that by construction, consuming x_0^* for all $t \geqq 0$ is feasible and efficient. Hence, since $(x_t^*)_{t=0}^{\infty}$ uniquely maximizes $\int_0^{\infty} \lambda_t u(x_t)\,dt$ over all consumption paths, it follows that $\int_0^{\infty} \lambda_t u(x_t^*)\,dt > \int_0^{\infty} \lambda_t u(x_0^*)\,dt$. Moreover, Proposition 1 applies and can be used to establish that $(u(x_0^*) + \mathbf{q}_0 \dot{\mathbf{k}}_0^*/\lambda_0)\int_0^{\infty} \lambda_t\,dt > \int_0^{\infty} \lambda_t u(x_0^*)\,dt$. Hence, $\mathbf{q}_0 \dot{\mathbf{k}}_0^* > 0$ even though consumption at time 0 equals the maximum sustainable level. For $s > t \geqq t''$, $u(x_s^*) < u(x_t^*)$. Hence, by Proposition 1 it follows that $(u(x_t^*) + \mathbf{q}_t \dot{\mathbf{k}}_t^*/\lambda_t)\int_t^{\infty} \lambda_s\,ds = \int_t^{\infty} \lambda_s u(x_s^*)\,ds < \int_t^{\infty} \lambda_s u(x_t^*)\,ds$ and $\mathbf{q}_t \dot{\mathbf{k}}_t^* < 0$ for all $t \geqq t''$. Since x_t^* is increasing for $t \in [0, t'']$, by the Lemma, $\mathbf{q}_t \dot{\mathbf{k}}_t^*$ is differentiable and decreasing for $t \in [0, t'']$. Consequently, there exists a unique $t' \in (0, t'')$ such that $\mathbf{q}_{t'} \dot{\mathbf{k}}_{t'}^* = 0$. From the facts that (a) consuming x_0^* for all $t \geqq 0$ is feasible and efficient, and (b) x_t^* is increasing for $t \in [0, t']$, it follows that for each $t \in (0, t']$, x_t^* exceeds the maximum sustainable consumption level given $\mathbf{k}_t^*$.

These findings are illustrated in Figure 1 and can be summarised as follows.

Proposition 3. *For the model described by (1) and (2), there exist positive initial stocks* $\mathbf{k}$, *a utility discount rate* $r' > 0$, *and some* $t' > 0$ *such that the path* $(x_t^*, \mathbf{k}_t^*, \dot{\mathbf{k}}_t^*)_{t=0}^{\infty}$ *is regular at prices* $(p_t, \mathbf{q}_t)_{t=0}^{\infty}$ *and utility discount factors* $\lambda_t = e^{-r't}$ *and satisfies:*

- *For* $t = 0$, $\mathbf{q}_t \dot{\mathbf{k}}_t^* > 0$ and x_t^* *equals the maximum sustainable consumption level.*
- *For* $t \in (0, t')$, $\mathbf{q}_t \dot{\mathbf{k}}_t^* > 0$ *and* x_t^* *exceeds the maximum sustainable consumption level.*
- *For* $t = t'$, $\mathbf{q}_t \dot{\mathbf{k}}_t^* = 0$ *and* x_t^* *exceeds the maximum sustainable consumption level.*

For a discussion of this result, consider $t = t'$ for which $\mathbf{q}_t \dot{\mathbf{k}}_t^* = 0$, but with x_t^* exceeding the maximum sustainable consumption level. By the structure of $(x_t^*)_{t=0}^{\infty}$ there exists some time $t''' > t'$ such that $x_s^* > x_{t'}^*$ for $s \in (t', t''')$ and $x_s^* < x_{t'}^*$ for $s \in (t''', \infty)$. By Proposition 1, $\int_{t'}^{t'''} e^{-r's}[u(x_s^*) - u(x_{t'}^*)]\,ds = \int_{t'''}^{\infty} e^{-r's}[u(x_{t'}^*) - u(x_s^*)]\,ds$. Hence, if utility could have been invested with a rate of return r', then $x_{t'}^*$ would have been sustainable. However, the utility

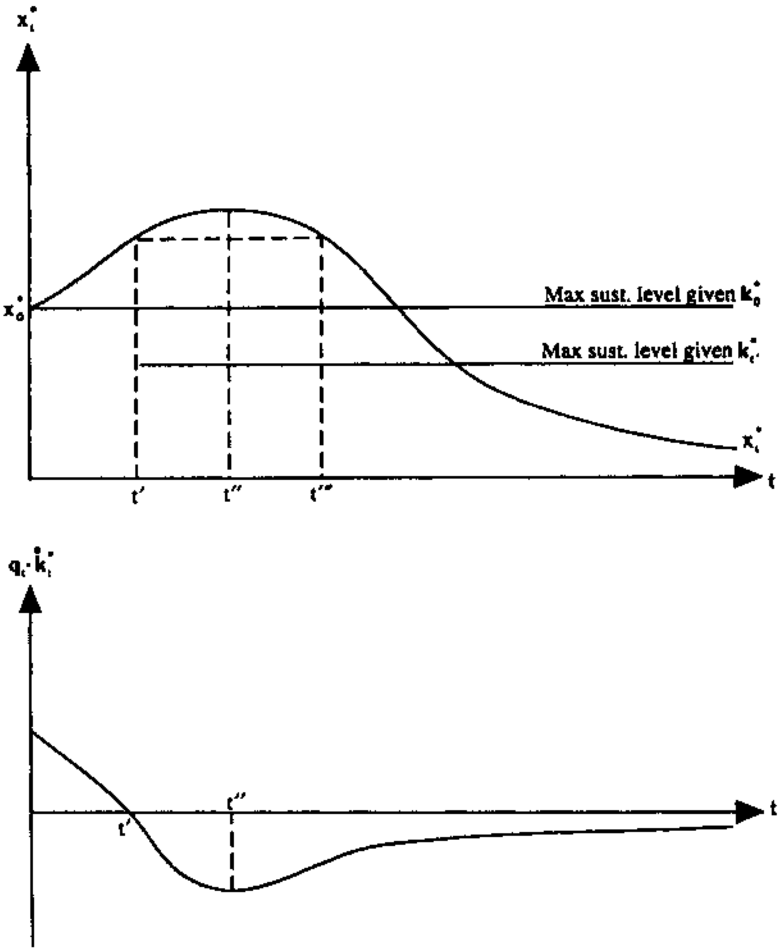

Fig. 1.

flow is skewed towards the present for the precise reason that such an investment would give rise to a lower rate of return. This again hinges on the decreasing productivity of the technology as man-made capital is accumulated and the flow of resource extraction dwindles. The above discussion explains why the generalized Weitzman result (Proposition 1) cannot be used to support the interpretation, made by e.g. Mäler (1991, p. 11) and Hulten (1992, p. 17), of $u(\boldsymbol{x}_t^*) + \boldsymbol{q}_t \dot{\boldsymbol{k}}_t^* / \lambda_t$ as the maximum sustainable level of utility flow.

The behavior of the constructed regular path at $t = t'$ also serves to reject the claim that $\boldsymbol{q}_t \dot{\boldsymbol{k}}_t^* = 0$ *at a given time t* implies that $u(x_t^*)$ equals the maximum sustainable level. In relation to Hartwick's rule ($\boldsymbol{q}_t \dot{\boldsymbol{k}}_t^* = 0$ *for all t* implies that $u(x_t^*)$ equals the maximum sustainable utility level), this can be interpreted as follows. The relative price of man-made in terms of neutral capital in an intertemporal competitive equilibrium depends on the entire future equilibrium path. In the present model, a path that distributes utility in favor of generations in the near future increrases this relative price, leading to a higher valuation of the investment in man-made capital relative to the depletion of natural capital. Thus, the insufficient altruism (relative to the requirement of sustainability) that the present generation extends to future generations increases to present NNP above the maximal sustainable level of utility flow.

Hence, it would seen impossible to develop the concept of NNP into an indicator of sustainability, even if prices for the valuation of natural and environmental resources were readily available through a perfect intertemporal competitive equilibrium.

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