Adjusting Green NNP to Measure Sustainability

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Abstract
Weitzman provides a foundation for NNP as the stationary equivalent of a wealth-maximizing path when there is a constant interest rate and no exogenous technological progress. Here, the implications of Weitzman’s foundation are explored in a case encountered in resource models, i.e., the case of non-constant interest rates. In a setting that allows for exogenous technological progress, an expression for NNP is obtained that adjusts Green NNP for anticipated capital gains and interest rate effects to produce a measure that indicates sustainability. This result is important when measuring the relative sustainability of resource rich and resource poor countries.

I. Introduction
How should net national product (NNP) be adjusted for the depletion of natural and environmental resources? A prerequisite for examining this problem is to decide on an underlying notion of income. At the personal level, Hicks (1946) defines income as the consumption that, if kept constant, would yield the same present value as a person’s actual future receipts. Since market prices are not influenced by any single person, this notion of income equals a person’s maximal sustainable consumption. At the national level, national income can likewise be defined as the consumption that, if kept constant, would yield the same present value as the actual future consumption path. Unless the nation is a small open economy faced with given international prices, this notion of national income will exceed

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sustainable consumption; see Weitzman (1976, pp. 159–60). Hence, it is the hypothetical stationary equivalent of future consumption.

Weitzman (1976, p. 160) shows that the conventional measure of NNP — consumption + value of net investments in capital stocks — equals this notion of national income under the assumptions of a constant interest rate and no exogenous technical progress. Weitzman (1976, p. 157) emphasizes that stocks of knowledge accumulated through learning and research activities as well as stocks of natural and environmental resources must be contained in the vector of capital stocks for this result to obtain. Moreover, competitive prices for such stocks must be assumed to be available. Hence, Weitzman requires that both endogenous technological progress as well as resource depletion be taken into account by the conventional measure. Such an expanded conventional measure has been suggested by Hartwick (1990), Mäler (1991) and others, and is usually referred to as Green NNP. Thus, Weitzman (1976) provides a Hicksian foundation for the concept of Green NNP under the given assumptions.

Weitzman (1995) shows that Green NNP is a poor measure of the hypothetical stationary equivalent if there is exogenous technological progress, while the example of Asheim (1994) can be used to show that Green NNP does not necessarily equal the hypothetical stationary equivalent if there is not a constant interest rate. Faced with this discrepancy, one possibility is to rely on an alternative foundation for Green NNP; e.g. Green NNP may serve as a measure of welfare changes resulting from small policy changes; see Mäler (1991) and Dasgupta (1996). The route pursued here is instead to apply Weitzman’s (1976) foundation by asking how NNP would need to be defined in terms of current prices and quantities in order to equal the hypothetical stationary equivalent of future consumption.

In certain resource models, ethically acceptable outcomes seem to entail that the assumption of a constant interest rate may be violated not only in terms of consumption, but even in terms of utility. It is therefore a major challenge to extend Weitzman’s foundation to the case where interest rates are not constant. By offering such an analysis in a setting that allows for exogenous technological progress, the present analysis holds also (i) for an economy where accumulated knowledge cannot be represented by augmented capital stocks, and (ii) for open economies whose “technology” is changing exogenously due to changing terms of trade.

The cases of exogenously accumulating knowledge and/or exogenously changing terms of trade are treated in a number of contributions; see the references listed in Section III. In contrast to most of these contributions, but following Asheim (1986, 1996), I here assume constant returns to scale. This amounts to an assumption that all flows of future earnings can be treated as currently existing capital. Exogenous technological progress
contributes to the appreciation of capital; i.e., capital gains. In the deterministic setting of this study, there is no windfall profit or loss, cf. e.g. Hicks (1946, p. 178); rather, capital gains are fully anticipated. An example is the anticipated capital gains on in situ resources when the future development of the resource price is known.

If interest rates are not constant, then there is a term structure of interest rates. In order to generalize Weitzman’s (1976) analysis, it turns out that the infinitely long-term interest rate is significant. In particular, I establish here that:

\[
\text{NNP satisfying the Weitzman foundation} = (\text{long-term interest rate}) \cdot (\text{current wealth}).
\]

Under the assumption of constant returns to scale, but allowing for exogenous technological progress, this in turn is shown to imply that NNP should include capital gains:

\[
\text{NNP satisfying the Weitzman foundation} = \text{consumption} + \text{value of net investments} + \text{anticipated capital gains} + (\text{rate of change of long-term interest rate}) \cdot (\text{current wealth}).
\]

NNP must be adjusted for interest rate changes because, without a constant interest rate, the present value of a constant flow of future earnings will vary. This part of the capital gains should not be included when NNP is based on the Weitzman foundation. Kemp and Long (1995) and Hartwick and Long (1995) also analyze the case of non-constant interest rates, but their analyses do not appear to be based on the Weitzman foundation.

The paper is organized as follows. In Section II, a problem of wealth maximization is used as the point of departure when defining a concept of NNP that satisfies the Weitzman foundation. This definition is used to derive an expression for NNP in Section III; two special cases where the expression coincides with Green NNP are also investigated. The implications of this expression in open economies are illustrated in Section IV by a numerical example of a two-country world, and by discussing the empirical relevance of the results for the measurement of sustainability in open economies. The results are also compared to Sefton and Weale (1996) by showing how their interesting analysis would change if their concept of NNP were based on the Weitzman foundation. Some of the formal analysis is contained in an Appendix. Throughout it is assumed that there exists an intertemporal competitive equilibrium in a constant population economy. The issues of how to define NNP when efficiency prices are not available, or when population is not constant, are not addressed.
II. The Weitzman Foundation in Terms of Consumption and Utility

Weitzman (1976) considers an economy maximizing at time $t$ 

$$P_l \mu (s) x(s) \mu t(s) x(s) ds \quad (1)$$

over all feasible consumption paths. Here, $x(s)$ denotes consumption at time $s$, and $r$ is the positive consumption discount rate. Expression (1) measures current wealth. Let $(x^*(s))_{t=0}^\infty$ be a consumption path maximizing (1) over all feasible consumption paths. Let $y(t)$ denote the consumption NNP at time $t$. To satisfy the Weitzman foundation of NNP, $y(t)$ has to be the hypothetical stationary equivalent; i.e. the level of consumption that, if sustained indefinitely, would yield the same wealth as the wealth-maximizing path:

$$P_l \mu (s) y(t) ds = \int_0^\infty e^{-(s-t)} x^*(s) ds,$$

or

$$y(t) = r \int_0^\infty e^{-(s-t)} x^*(s) ds.$$

Kemp and Long (1982) generalize Weitzman's analysis to the case of a concave utility function. They consider an economy maximizing at time $t$

$$\int_0^\infty e^{-(s-t)} u(x(s)) ds \quad (2)$$

over all feasible consumption paths, where $u(\cdot)$ is a time-invariant, strictly increasing, concave and differentiable utility function, and $\varrho$ is the positive utility discount rate. Expression (2) measures current discounted utilitarian welfare. Let $(x^*(s))_{t=0}^\infty$ maximize (2) over all feasible consumption paths. Let $v(t)$ denote the utility NNP at time $t$. To satisfy the Weitzman foundation of NNP, $v(t)$ has to be the level of utility that, if sustained indefinitely, would yield the same welfare as the welfare-maximizing path:

$$P_l \mu (s) v(t) ds = \int_0^\infty e^{-(s-t)} u(x^*(s)) ds,$$

or

$$v(t) = \varrho \int_0^\infty e^{-(s-t)} u(x^*(s)) ds.$$

Expression (2) is a controversial welfare criterion since, in certain resource models, it gives rise to ethically unacceptable implications. For

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2 Kemp and Long (1982) also generalize Weitzman’s analysis to multiple consumption goods and exogenous technological progress. To focus attention on the central issue of this paper, multiple consumption goods will not be considered here. The single consumption good is assumed to be an indicator of instantaneous well-being derived from the situation in which people live. Hence, it includes more than material consumption.

instance, in the Dasgupta–Heal–Solow model of capital accumulation and resource depletion, the discounted utilitarian criterion yields a maximizing path in which consumption converges to zero even though paths with positive and non-decreasing consumption are feasible; see Dasgupta and Heal (1974), Solow (1974) and Section IV below. As an alternative, Solow (1974) explores the implications in the Dasgupta–Heal–Solow model of the Rawlsian maximum principle on utility. In the same model I have followed Calvo’s (1978) suggestion that the maximum principle be used on discounted utilitarian welfare in Asheim (1988), as well as imposed Dalton’s (1920) principle of transfers as a side constraint when (2) is maximized in Asheim (1991). Each of these alternative criteria gives rise to an efficient path that can be supported by utility discount factors. The economy acts as if it maximizes

\[ \int_0^\infty \lambda(s) u(x(s)) \, ds \]  

over all feasible consumption paths, where \( \lambda(s) \) is the positive utility discount factor applicable at time \( s \). The path \( (\lambda(s))_{s=1}^{\infty} \) yields the present value prices at which marginal utility at one time can be exchanged for marginal utility at some other time. Hence, \( (\lambda(s))_{s=1}^{\infty} \) determines utility interest rates. If \( \lambda(s) = \lambda(0) e^{-\mu s} \), there is one constant utility interest rate: \( \rho = -\mu \lambda(t)/\lambda(t) = \lambda(t)/\int_0^t \lambda(s) \, ds \) for all \( t \). However, if (2) is not the welfare criterion, \( \lambda(s) \) will not, in general, be an exponential function. Then there is a term structure of utility interest rates: the instantaneous (very short-term) utility interest rate is \( \phi_0(t) = -\dot{\lambda}(t)/\lambda(t) \). The infinitely long-term utility interest rate is \( \phi_\infty(t) = \lambda(t)/\int_0^\infty \lambda(s) \, ds \) (provided that \( \phi_0(t) \) is positive and does not fall “too fast” so that \( \int_0^\infty \lambda(s) \, ds \) still exists). Note that, along a maximizing path, \( 1/\phi_\infty(t) \) is the price at time \( t \), in terms of utility, of a utility annuity from time \( t \) on. Note also that \( \phi_\infty(t)/\phi_\infty(t) = \phi_\infty(t) - \phi_0(t) \). Hence, \( \phi_\infty(t) \) is decreasing if and only if \( \phi_0(t) > \phi_\infty(t) \).

Let \( (x^*(s))_{s=1}^{\infty} \) maximize (3) over all feasible consumption paths. In particular, \( \int_0^\infty \lambda(s) u(x^*(s)) \, ds \) exists. Let \( v(t) \) still denote utility NNP. To satisfy the Weitzman foundation, \( v(t) \) must be the stationary equivalent of \( (u(x^*(s)))_{s=1}^{\infty} \): \( \int_0^\infty \lambda(s) v(t) \, ds = \int_0^\infty \lambda(s) u(x^*(s)) \, ds \), or,

\[ v(t) = \frac{\int_0^\infty \lambda(s) u(x^*(s)) \, ds}{\int_0^\infty \lambda(s) \, ds} = \frac{\phi_\infty(t)}{\lambda(t)} u(x^*(s)) \, ds. \]  

Since \( i(t) \) can be shown to equal \( (\lambda(t)/\int_0^\infty \lambda(s) \, ds) (v(t) - u(x^*(t))) \), \( i(t) \) is a solution to

\[ i(t) = \phi_\infty(t) \cdot (v(t) - u(x^*(t))). \]
The differential equation (5) can be rewritten as 
\[ v(t) - u(x^*(t)) = \dot{v}(t) \left( \frac{1}{u(x^*(t))} \right) \]
This yields the following interpretation. The difference between the stock of utility annuities at time \( t \) and the actual utility level at time \( t \) equals the rate at which utility annuities can be accumulated times the price of such annuities.

It is natural to seek a justification for \( v(t) \) in terms of sustainable income; see Hicks (1946, Ch. 14, "Income No. 3") and Nordhaus’ (1995) discussion of Fisher (1906). A s noted in the introduction, a constant utility flow equal to \( v(t) \) is only attainable if the actual utility path \( (u(x^*(s)))_{s=t} \) could be changed to a constant utility path without changing the supporting prices. In particular, if \( (x^*(s))_{s=t} \) is the unique path maximizing (3) and \( (x^*(s))_{s=t} \) does not yield constant consumption, then a constant utility flow equal to \( v(t) \) is not attainable. Hence, the Weitzman foundation gives an upper bound for the level of utility that is actually sustainable. Likewise, \( \mu \frac{\dot{v}(t)}{v(t)} \) is an upper bound for sustainable consumption. With a justification in terms of sustainable income, it is desirable to obtain a measure that is as close as possible to what is actually sustainable. It is shown below that the Weitzman foundation in terms of consumption yields a more accurate measure. Therefore, let us turn again to consumption NNP.

Under regularity conditions (see the Appendix) there exists a path of present value prices \( (p(s))_{s=t} \) such that maximizing (3) is equivalent to the maximization of

\[ \int_t^\infty p(s) x(s) \, ds \]  
(6)

over all feasible consumption paths. Expression (6) divided through by \( p(t) \) measures current wealth. It must hold for all \( s \) that \( x^*(s) \) maximizes \( \lambda(s) u(x(s)) - p(s) x(s) \); hence, if \( x^*(s) \) is interior, then \( \lambda(s) u'(x^*(s)) = p(s) \). In analogy with \( \lambda(s) \), \( p(s) \) determines a term structure of consumption interest rates. The instantaneous (very short-term) consumption interest rate is \( r_0(t) = -\frac{\dot{p}(t)}{p(t)} \). The infinitely long-term consumption interest rate is \( r_l(t) = p(t)/\mu \int_t^\infty p(s) \, ds \). In analogy, \( 1/r_l(t) \) is the price at time \( t \), in terms of consumption, of a consumption annuity from time \( t \) on. Note also that \( r_l(t) r_0(t) = r_0(t) - r_l(t) \). Hence, \( r_l(t) \) is decreasing if and only if \( r_0(t) > r_l(t) \).

Let \( y(t) \) still denote consumption NNP. To satisfy the Weitzman foundation, \( y(t) \) must be the stationary equivalent of \( (x^*(s))_{s=t} \):

\[ \int_t^\infty p(s) y(s) \, ds = \int_t^\infty p(s) x^*(s) \, ds \]

or

\[ y(t) = \frac{\int_t^\infty p(s) x^*(s) \, ds}{\int_t^\infty p(s) \, ds} = r_0(t) \frac{\int_t^\infty p(s)}{p(t)} x^*(s) \, ds \]  
(7)
Hence, consumption NNP is the infinitely long-term consumption interest rate times current wealth. Since \[ \dot{y}(t) = \left( \frac{p(t)}{t} \right) \int_{t}^{\infty} p(s) \, ds \] \( y(t) - x^*(t) \), \( y(t) \) is a solution to \( \dot{y}(t) = r_{\infty}(t) (y(t) - x^*(t)) \). \hspace{1cm} (8)

This differential equation has the same interpretation as (5).

\( y(t) \) is also an upper bound for sustainable income. This can be shown by repeating the above argument for \( \nu(t) \). Therefore, to establish that \( y(t) \) is a more accurate measure of sustainable income than \( \nu(t) \), it is sufficient to show that \( \nu(t) \geq u(y(t)) \), or equivalently, \( u^{-1}(\nu(t)) \geq y(t) \). By (4) and (7) and the property that \( \lambda(s) u'(x^*(s)) = p(s) \) for all \( s \), it follows that \( \int_{t}^{\infty} \lambda(s) (\nu(t) - u(y(t))) \, ds = \int_{t}^{\infty} \lambda(s) [u(x^*(s)) - u(y(t)) + u'(x^*(s)) (y(t) - x^*(s))] \, ds \), where the term in brackets is non-negative by the concavity of \( u \). Therefore, since \( \lambda(s) > 0 \) for all \( s \), \( \nu(t) \geq u(y(t)) \), with strict inequality if \( (x^*(s))_{s \in T} \) is not constant and \( u \) is strictly concave.

For a small open economy, \( (p(s))_{s \in T} \) is exogenously determined by the international capital market, and \( (x^*(s))_{s \in T} \) can be changed into a constant consumption path without changing the supporting prices. Hence, as argued by Brekke (1996, Ch. 4), \( y(t) \) is an exact indicator of sustainability for such an economy.

### III. Expressions for Consumption NNP

Through (7), consumption NNP is expressed as the infinitely long-term consumption interest rate times the discounted value of future consumption, the latter term having been interpreted as wealth. This is not the form that national accountants find useful, since wealth — especially as given by (7) — is not easily measured; see e.g. Usher (1994). Therefore, combine (7) and (8) to yield

\[ y(t) = x^*(t) + \frac{\dot{y}(t)}{r_{\infty}(s)} \]

\[ = x^*(t) + \frac{\dot{r_{\infty}(t)}}{r_{\infty}(t)} \int_{t}^{\infty} \frac{p(s)}{p(t)} x^*(s) \, ds. \] \hspace{1cm} (9)

Hence, consumption NNP equals consumption plus the growth of current wealth plus the rate of change in the infinitely long-term consumption interest rate times current wealth. In this section I investigate cases where expression (9) can be evaluated using current prices and quantities only.

For this purpose, consider a constant population economy as described by Dixit et al. (1980) and reproduced in the Appendix, but where the present analysis allows for exogenous technological progress. Here, \( k^*(t) \)
is the vector of capital stocks at time \( t \), and \( Q(t) \) are the competitive prices of the capital stocks in terms of current consumption. Within this framework, three different cases will be analyzed. In the first two cases, Green NNP — i.e., consumption plus the value of net investments: \( x^*(t) + Q(t)k^*(t) \) — is shown to satisfy the Weitzman foundation. In both cases, the assumption of no exogenous technological progress is combined with either a constant consumption interest rate or constant consumption.

Case 1: No exogenous technological progress, and a constant consumption interest rate \( \frac{r(s)}{p(t)} = r_0 \) for all \( s \). This is the case considered by Weitzman (1976). Since \( \dot{r}_0(t) = 0 \), the third term of the r.h.s. of (9) is equal to zero. Also, since \( p(s+\Delta)/p(t+\Delta) = e^{-r_0(t)\Delta} \) is constant as a function of \( \Delta \), the second term of the r.h.s. of (9) equals \( \int\frac{r(s)}{p(t)} \dot{x}(s) \, ds \). Lemma 1 in the Appendix implies that this latter term equals \( Q(t)k^*(t) \) in the absence of exogenous technological progress; hence, by (9), \( y(t) = x^*(t) + Q(t)k^*(t) \). This amounts to an alternative demonstration of Weitzman’s (1976) original result: with no exogenous technological progress and a constant consumption interest rate, Green NNP satisfies the Weitzman foundation.

Case 2: No exogenous technological progress, and constant consumption \( \frac{r(s)}{x^*(s)} = r^* \) for all \( s \). By (7) it follows that \( y(t) = x^* \), while the converse of Hartwick’s (1977) rule, cf. Dixit et al. (1980), reproduced as Lemma 2 in the Appendix, implies that \( Q(t)k^*(t) = 0 \) in the absence of exogenous technological progress. Hence, \( y(t) = x^*(t) + Q(t)k^*(t) \). Even though the second and third terms of (9) cancel out in this case, there are important resource models in which each of the terms differs from zero. A justly well-known example is the Dasgupta–Heal–Solow model, where Solow’s (1974) analysis shows that the consumption interest rates are decreasing along the constant consumption path.

The above assumption of no exogenous technological progress is more appropriate for a closed than for an open economy. The “technology” of an open economy trading in a competitive world economy has to include its trade opportunities. Therefore, the assumption of no exogenous technological progress will be violated if its terms of trade are changing. In resource models, Hotelling’s rule implies that a resource-exorting economy will enjoy improving terms of trade, which in the present context correspond to exogenous technological progress. If such an economy follows a constant consumption path, its consumption level will not equal Green NNP; see Asheim (1986, 1996), Sefton and Weale (1996), Hartwick (1995), Vincent et al. (1995) and Section IV below. Changing terms of...
trade are also analyzed by Kemp and Long (1995). Even for a closed economy, the assumption of no exogenous technological progress is strong. This has been emphasized and/or analyzed by Aronsson and Löfgren (1993, 1995), Hartwick and Long (1995), Kemp and Long (1982), Nordhaus (1995), Weitzman (1997), Withagen (1996) and others. Therefore, let us turn to the case of exogenous technological progress.

If the analysis allows for exogenous technological progress, can the consumption NNP satisfying the Weitzman foundation be expressed in terms of current prices and quantities, and hence be an operational measure of NNP? It can under the alternative assumption that the technology exhibits constant returns to scale (CRS) in the capital stocks. This assumption is in the spirit of Lindahl (1933, pp. 401–2) and implies that all factors of production, including labor, are dealt with as capital that is evaluated by the present value of future earnings. It amounts to assuming that all flows of future earnings can be treated as currently existing capital. CRS means that in the hypothetical case where all capital stocks were a given percentage larger, consumption and investments could be increased by the same percentage. This clearly allows for stocks in fixed supply — like “raw labor” and land — that cannot actually be accumulated. The CRS assumption is not restrictive since the existence of an intertemporal competitive equilibrium entails that returns to scale are nonincreasing. Hence, CRS can be obtained by adding an additional fixed capital stock with which returns to scale become constant.

Case 3: Constant returns to scale. As shown in Lemma 3 in the Appendix, if the assumption of CRS is imposed, then, for all \( s \), \( x^*(s) + Q(s)k^*(s) + \dot{Q}(s)k^*(s) = r_s(s)Q(s)k^*(s) \) or, equivalently, \( p(s)x^*(s) + (\frac{d}{dt})[p(s)Q(s)k^*(s)] = 0 \). This yields \( p(t)Q(t)k^*(t) = \int p(s)x^*(s)ds \) or, equivalently, \( Q(t)k^*(t) = \int p(s) \frac{x^*(s)}{p(t)} ds \).

Hence, the current value of the capital stocks equals current wealth. It now follows from (9) that

\[
y(t) = x^*(t) + Q(t)k^*(t) + \dot{Q}(t)k^*(t) + (\dot{r}_x(t)\dot{r}_x(t))Q(t)k^*(t). \tag{10}
\]

The first two terms constitute Green NNP. The hypothetical stationary equivalent of future consumption adjusts this measure for anticipated capital gains \( Q(t)k^*(t) \) and the rate of change in the infinitely long-term consumption interest rate.

Since \( y(t) = x^*(t) + Q(t)k^*(t) \) under the assumptions of Case 1, (10) provides an alternative demonstration of the result shown in Asheim.
(1996, Proposition 1), namely that in an economy with CRS and no exogenous technological progress, a constant consumption interest rate implies that there are no capital gains. It is now apparent that Weitzman’s original result is somewhat misleading. In the case of no exogenous technological progress and a constant consumption interest rate, Weitzman (1976) shows that NNP should exclude capital gains in order to satisfy the Weitzman foundation. However, as argued above, there are no capital gains in this special case. On the other hand, (10) implies that anticipated capital gains should be fully included in the presence of exogenous technological progress, provided that there is a constant consumption interest rate. Therefore, in the case of a constant consumption interest rate considered by Weitzman (1976), a concept of NNP that includes anticipated capital gains satisfies his foundation independently of whether there is exogenous technological progress.

IV. Open Economies: A Numerical Example and Empirical Evidence

Consider a closed economy with no exogenous technological progress and constant consumption (Case 2 above). Then Green NNP correctly ignores non-zero aggregate capital gains, since the last two terms of (10) cancel out. Let this closed economy be split into open economies with constant subpopulations, each having access to the same CRS technology. Allow for free trade and sufficient factor mobility to ensure overall productive efficiency. If terms of trade are not constant, then the “technology” of each open economy is not stationary. This in turn means that the corresponding terms of (10) — for each open economy — do not cancel out. In the context of resource models, an open economy with relatively large stocks of in situ resources will have large capital gains in relation to its wealth, while an open economy with relatively small such stocks will in comparison have small capital gains in relation to its wealth. The other adjustment term, however, is proportional to wealth. Hence, even when Green NNP needs no adjustment at the aggregate level to satisfy the Weitzman foundation, the necessary adjustments at the disaggregate level will influence the distribution of NNP between different countries. A numerical example is provided prior to discussing the empirical relevance of this observation.

The numerical example is based on the Dasgupta-Heal-Solow model of man-made capital accumulation and resource depletion, in which a flow of a non-renewable resource, $-k_n$, is combined with constant human capital, $k_H$, and man-made capital, $k_C$, in order to produce a consumption good, $x$. With its technology described by $x + k_C \leq (k_n)^{1-a-b}(k_C)^a(-k_n)^b$, $b < a < a + b < 1$, this model fits into the framework of Section III. Choose
a = 0.20 and b = 0.16. Solow (1974) shows that positive and constant consumption can be sustained indefinitely by letting accumulated man-made capital substitute for a diminishing resource extraction. Let \( k^*(t) = (k_1^*, k_2^*(t), k_3^*(t)) \) denote the capital vector along such an efficient maximin path, with \( x^* \) as the corresponding consumption level, where variables without time dependence are constant. The investment in man-made capital is constant along this path, implying that total output is constant, with a fraction \( 1 - b \) going to consumption and a fraction \( b \) going to investment in man-made capital. By normalizing total output to 1, \( x_1^* = 1 - b = 0.84 \) and \( k_2^* = b = 0.16 \). If the constant consumption path is implemented as a competitive equilibrium, interest rates are decreasing \( (r_0(t) = a/k_1^*(t), \ r_1(t) = (a-b)/k_1^*(t) = 0.04/k_1^*(t)) \) and \( \dot{r}_1(t)/r_1(t) = a/b/k_1^*(t) = 0.16/k_1^*(t) \), competitive capital prices and price changes are given by:

\[
\begin{align*}
Q_h(t) &= \frac{1-a-b}{a-b} \left( \frac{k_1^*(t)}{k_1^*} \right) = 16 \left( \frac{k_1^*(t)}{k_1^*} \right), \\
\dot{Q}_h(t) &= \frac{1-a-b}{a-b} \left( \frac{b}{k_1^*} \right) = 2.56 \left( \frac{k_1^*(t)}{k_1^*} \right), \\
Q_c &= 1, \ 
\dot{Q}_c = 0, \\
Q_r(t) &= \frac{b}{a-b} \left( \frac{k_2^*(t)}{k_1^*(t)} \right) = 4 \left( \frac{k_2^*(t)}{k_1^*(t)} \right), \\
\dot{Q}_r(t) &= \frac{b}{a-b} \left( \frac{a}{k_2^*(t)} \right) = 0.80 \left( \frac{k_2^*(t)}{k_2^*} \right),
\end{align*}
\]

while resource extraction \( -\dot{k}_2^* \) equals \( b/Q_r(t) = 0.16/Q_r(t) \).

Let this competitive world economy be split into two countries, between which human capital is distributed evenly \( (k_1^1 = k_2^1 = \frac{1}{2}k_1^*) \), while only country 1 is endowed with the resource \( (k_1^1(t) = k_2^*(t), k_3^1(t) = 0) \), and only country 2 owns man-made capital \( (k_1^2 = 0, k_2^2(t) = k_2^*(t)) \). Productive efficiency is ensured by using half of the capital stock and resource flow in each country. Assume that each country implements a constant consumption path. As I show in Asheim (1996; Table 1, Case 3), each country keeps consumption constant if \( \dot{k}_1^1(t) = \dot{k}_2^*(t), k_1^1(t) = 0, k_2^1(t) = 0, \) and \( k_2^2 = k_2^* \), with \( x_1 = 0.48 \) and \( x_2 = 0.36 \). Green NNP then becomes

\[
\begin{align*}
x_1^* + Q(t) \dot{k}_1^1(t) &= x_1^* + Q_1(t) \dot{k}_1^1(t) = 0.48 - 0.16 = 0.32 \\
x_2^* + Q(t) \dot{k}_1^2(t) &= x_2^* + \dot{k}_2^*(t) = 0.36 + 0.16 = 0.52.
\end{align*}
\]

Hence, for each country, Green NNP differs at any time from the constant consumption level: 0.32 v. 0.48 for country 1; 0.52 v. 0.36 for country 2.

Let us now turn to the adjustments for capital gains and the rate of change in the infinitely long-term consumption interest rate, as specified by (10). No adjustment is needed for human capital since, for each \( j = 1, 2 \),

\[
\dot{Q}_H(t) = \dot{Q}_H(t) k_1^H(t) + \left( \frac{\dot{r}_l(t)}{r_l(t)} \right) Q_l(t) k_1^H(t) = 1.28 - (0.16/k_1^H(t)) \cdot 8k_1^H(t) = 0.
\]

Since country 1 has no stock of man-made capital,

\[
\dot{Q}(t) = \dot{Q}(t) k_1^C(t) + \left( \frac{\dot{r}_l(t)}{r_l(t)} \right) Q_l(t) k_1^C(t) = 0.80 - (0.16/k_1^C(t)) 4k_1^C(t) = 0.16,
\]

while, since country 2 has no resource stock,

\[
\dot{Q}(t) = \dot{Q}(t) k_2^C(t) + \left( \frac{\dot{r}_l(t)}{r_l(t)} \right) Q_l(t) k_2^C(t) = 0 - (0.16/k_2^C(t)) k_2^C(t) = 0.16.
\]

Therefore, it follows from (10) that \( y^1 = 0.32 + 0.16 = 0.48 \) and \( y^2 = 0.52 - 0.16 = 0.36 \). Hence, for each country, the NNP based on the Weitzman foundation is at all times equal to the constant consumption level.

Hence, Green NNP underestimates the sustainability of resource rich countries (like country 1 in the example) and overestimates the sustainability of resource poor countries (like country 2). Vincent et al. (1995) note this problem and point to a study by Pearce and Atkinson (1993), who calculated Green NNP for a number of countries. Japan turns out to consume much less than its Green NNP and is ranked as the most sustainable of the 21 countries presented. Indonesia consumes more than its Green NNP and is regarded as an unsustainable economy. Given that Indonesia is the relatively more resource rich of the two countries, these conclusions may not survive adjustments for capital gains and interest rate effects.

Sefton and Weale (1996), hereafter SW, also investigated the concept of NNP in open economies. Instead of considering the solution to the differential equation \( \dot{y}(t) = r_0(t) \cdot (y(t) - x^*(t)) \) as in the present paper, SW determined consumption NNP (or "income") as the solution to the differential equation \( \dot{Y}(t) = r_0(t) (Y(t) - x^*(t)) \). This yields

\[
Y(t) = \int_{s}^{t} \left( -\dot{p}(s) \right) \frac{x^*(s) ds}{p(t)} = \int_{s}^{t} r_0(s) \frac{p(s)}{p(t)} x^*(s) ds.
\]  

(11)

If there is one constant consumption interest rate, then \( r = r_0(s) = r_0(t) \) for all \( s \); hence, \( y(t) = Y(t) \). Also if \( x^*(s) \) is a constant consumption path, so that \( x^*(s) = x^* \) for all \( s \), then \( y(t) = Y(t) = x^* \) since
\[ r_c(t) \int_t^\infty \frac{p(s)}{p(t)} ds = \int_t^\infty r_c(s) \frac{p(s)}{p(t)} ds = 1. \]

However, in general, if there is not a constant consumption interest rate or if consumption is not constant, then \( Y(t) \) differs from NNP based on the Weitzman foundation. Still, SW’s consumption NNP has the following attractive features: in a world economy with no exogenous technological progress, it equals Green NNP. Furthermore, Green NNP is split into NNP s for the open economies in such a way that, if consumption is kept constant in an open economy, then NNP for this open economy equals its constant consumption level.

SW claim that their measure of NNP has the advantage that it can be calculated using observable market prices. In contrast, elsewhere I have measured the maximum sustainable consumption, using the prices that would exist along such an efficient constant consumption path. As seen above, however, the measure suggested by SW does not in general satisfy the Weitzman foundation, except in special cases, one of which is the constant consumption case. The same conclusion holds for the measure of the maximum sustainable consumption suggested in Asheim (1996, Proposition 3). However, in contrast to SW’s claim, it turns out that the measure provided in Asheim (1986) does satisfy the Weitzman foundation also when using observable market prices along paths where consumption is not constant. To see this, substitute \( Q(t)k^*(t) = f_t^\infty (p(s)/p(t))x^*(s) ds \) into (7) to yield \( y(t) = p(t)Q(t)k^*(t)/f_s^t p(s) ds \), the r.h.s. of which is identical to the r.h.s. of (5) in Asheim (1986). Even though this (5) is intended to measure consumption along an efficient constant consumption path, the present analysis has shown that this expression in fact measures NNP satisfying the Weitzman foundation even at prices supporting non-constant consumption paths.

**Appendix**

Following Dixit et al. (1980), but allowing for exogenous technological progress, \( (x(s), k(s), \dot{k}(s)) \) is feasible if and only if \( (x(s), k(s), \dot{k}(s)) \in F(s) \), where \( F(s) \) is a convex set of feasible triples at time \( s \), and where \( k(s) \) is a non-negative vector of capital stocks at time \( s \). For stocks like “raw labor” and land that are in fixed supply, the corresponding components of \( k \) equal 0. For other positive stocks, the set \( F(s) \) is assumed to satisfy free disposal of investment flows; i.e., for each \( s \), if

\(^3\)Hartwick and Long (1995) characterize investment behavior along a constant consumption path in the context of non-constant interest rates and exogenous technological progress. It can be shown that their analysis is related to that of SW.

(x, k, k) ∈ F(s) and k′ ≤ k, with k′ differing from k for such stock only, then (x, k, k′) ∈ F(s).

Call a feasible path (x*(s), k*(s), k*(s)) to be competitive at present value consumption and capital prices (p(s), q(s)) if and utility discount factors (λ(s)) if

(i) for each s, p(s) > 0 and (x*(s), k*(s), k*(s)) maximizes instantaneous profit

p(s)x + q(s)k + q(s)k subject to (x, k, k) ∈ F(s).

(ii) for each s, λ(s) > 0 and x*(s) maximizes λ(s)u(x) − p(s)x over all x.)

Note that (i) combined with the assumption of free disposal of investment flows imply that the vector q(s) is non-negative for stocks that are not in fixed supply.

Moreover, Q(s) = q(s)/p(s) are the current value capital prices used in Sections III and IV.

 Call a competitive path (x*(s), k*(s), k*(s)) regular at (p(s), q(s)) if and utility discount factors (λ(s)) if

(a) q(s)k*(s) → 0 as s → ∞;

(b) λ(s) exists (and λ(s) exists).

A regular path (x*(s), k*(s), k*(s)) maximizes ∫ p(s)x(s) ds and λ(s)u(x(s)) ds over all feasible paths (x(s), k(s), k(s)) with initial stocks k(t) = k*(t).

With no exogenous technological progress, the following lemmas are obtained.

**Lemma 1.** If F(s) is time invariant, and (x*(s), k*(s), k*(s)) is regular at (p(s), q(s)), then ∫ p(s)x*(s) ds = q(t)k*(t).

Proof. It follows from (i) that, for all s, p(s) + (d/ds)(q(s)k(s)) ≤ p*(s) + (d/ds)(q(s)k(s)) k*(s) k*(s) k*(s).

Hence, by (a), ∫ p(s)x*(s) ds − q(t)k*(t) ≤ ∫ p(s)x*(s) ds − q(t)k*(t). In particular, since F(s) is time invariant (i.e., no exogenous technological progress), ∫ p(s)x*(s + Δ) ds − ∫ p(s)x*(s) ds ≤ q(t)k*(t + Δ) − q(t)k*(t), which implies that ∫ p(s)[x*(s + Δ) − x*(s)] ds ≤ q(t)[k*(t + Δ) − k*(t)] / Δ if Δ > 0 and ∫ p(s)[x*(s + Δ) − x*(s)] ds ≥ q(t)[k*(t + Δ) − k*(t)] / Δ if Δ < 0. By taking limits, this establishes the result, provided that x*(s) exists for a.e. s and q(s)k*(s) → 0 as s → ∞. □

The converse of Hartwick’s (1977) rule is a straightforward consequence of Lemma 1.

**Lemma 2 (Dixit et al., 1980).** If F(s) is time invariant, (x*(s), k*(s), k*(s)) is regular at (p(s), q(s)), and x*(s) = 0 for all s, then q(t)k*(t) = 0.

Returning to the case with exogenous technological progress, but instead assuming that the technology exhibits CRS, then the following lemma can be established. Note that the assumption of CRS is equivalent to assuming that F(s) is a convex cone.

**Lemma 3.** If, for each s, F(s) is a convex cone, and (x*(s), k*(s), k*(s)) is regular at (p(s), q(s)), then q(t)k*(t) = ∫ p(s)x*(s) ds.
Proof: If $F(s)$ is a convex cone, then $(x^*(s), k^*(s), \dot{k}^*(s))$ maximizes $p(s)x + q(s)k + \dot{q}(s)\dot{k}$ subject to $(x, k, \dot{k}) \in F(s)$ only if $p(s)x^*(s) + q(s)k^*(s) + \dot{q}(s)\dot{k}^*(s) = 0$. Hence, $p(s)x^*(s) + (d/dt)(q(s)k^*(s)) = 0$ for all $s$, such that, by (a), $q(t)k^*(t) = \int p(s)x^*(s)\,ds$.

References


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