Journal of Environmental Economics and Management **41,** 252–268 (2001)
doi:10.1006/jeem.2000.1137, available online at http://www.idealibrary.com on IDEAL®

# Justifying Sustainability[1]

Geir B. Asheim

*University of Oslo, Department of Economics, P.O. Box 1095 Blindern, N-0317 Oslo, Norway*
E-mail: g.b.asheim@econ.uio.no

Wolfgang Buchholz

*University of Regensburg, Department of Economics, D-93040 Regensburg, Germany*
E-mail: wolfgang.buchholz@wiwi.uni-regensburg.de

and

Bertil Tungodden

*Norwegian School of Economics and Business Administration and the Norwegian Research Centre in Organization and Management, Bergen, Norway*
E-mail: bertil.tungodden@nhh.no

Received March 2, 1999; revised November 9, 1999; published online September 7, 2000

In the framework of ethical social choice theory, sustainability is justified by efficiency and equity as ethical axioms. These axioms correspond to the Suppes–Sen grading principle. In technologies that are productive in a certain sense, the set of Suppes–Sen maximal utility paths is shown to equal the set of non-decreasing and efficient paths. Since any such path is sustainable, efficiency and equity can thus be used to deem any unsustainable path as ethically unacceptable. This finding is contrasted with results that seem to indicate that an infinite number of generations cannot be treated equally. © 2001 Academic Press

*Key Words:* sustainability; ethical preferences; Suppes–Sen grading principle.

## 1. INTRODUCTION

Motivated by a concern about environmental deterioration and natural resource depletion, sustainability is by now one of the key concepts in environmental discussion and, at least partly, in environmental policy. It was a major topic in the Brundtland Report [50] and it has become a main objective of many international organizations like the UN where, after the 1992 United Nations Earth Summit in Rio, it was put on Agenda 21. Sustainability has also found its firm place as a "leitmotif" in the programs of political parties and green political movements. The increasing importance of sustainability as a guideline of environmental policy is also reflected in environmental and resource economics; see, e.g., Pezzey [29] where sustainable paths are confronted with standard optimal solutions as described in the traditional theory of economic growth.

[1] We thank three anonymous referees, Kenneth Arrow, Aanund Hylland, and seminar participants in Bergen, Davis, Heidelberg, Oslo, Paris, Santiago, Ulvön, and Zurich for helpful comments. Figure 1 was suggested by Minh Ha-Duong. Financial support from the Research Council of Norway (Ruhrgas grant) is gratefully acknowledged.



0095-0696/01 $35.00

An ethical concern is at the heart of the interest in a sustainable environmental policy (cf. Toman *et al*. [44, pp. 140–142] and Sandler [35, p. 64]). In particular, sustainability implies that environmental and natural resources have to be shared with future generations. These resources are seen as a common heritage of mankind to which every generation should have the same right of access. Following Sidgwick [39], Pigou [30], and Ramsey [31] there is also a long tradition in economics for considering the unfavorable treatment of future generations as ethically unacceptable. Not much work has, however, been done on the relationship between an ethical postulate of equal treatment of all generations, on the one hand, and sustainability, on the other. The present paper seeks to provide such a contribution by giving a justification for sustainability in the framework of ethical social choice theory.[2] Our main result is that within a relevant class of technologies only sustainable behavior is ethically justifiable provided that the social preferences satisfy two focal normative axioms, equal treatment being one, efficiency being the other.

There is a technical literature on intergenerational social preferences that contains rather negative results concerning the possibility of treating generations equally. This literature includes Koopmans [24], Diamond [16], Svensson [42], Epstein [17, 18], and Lauwers [26], and it essentially presents the finding that complete social preferences that treat an *infinite* number of generations equally need not admit optimal solutions. This negative conclusion appears not to have been much noticed by environmental and resource economists, Dasgupta and Heal [13, pp. 277–281] being a remarkable exception. Still, it represents a challenge to everyone concerned with sustainability: *Is the quest for the equal treatment of an infinite number of generations* (*implicitly*) *assumed in the recent literature on sustainability a vain one since earlier technical contributions have shown that such equal treatment need not be possible*? This paper resolves this apparent conflict by looking directly at the possibility of having intergenerational preferences that are *effective* (in the sense of having a non-empty set of maximal paths) in a *relevant class* of technologies.[3]

The paper is organized as follows: After describing the proposed axiomatic basis for intergenerational ethical preferences in Section 2, we introduce the technological framework of the analysis and define the concept of sustainability in Section 3. Throughout the paper we apply two different productivity assumptions, *immediate productivity* and *eventual productivity*, and we show that both assumptions apply in many important classes of technologies. In Section 4 we develop a justification for sustainability by showing that an axiom of equal treatment (*equity*) combined with the strong Pareto axiom (*efficiency*) is sufficient to rule out non-sustainable

[2] Most of the extensive social choice literature on the evaluation of infinite utility paths does not deal with the issue of sustainability. Exceptions are Asheim [2] and the prominent contributions by Chichilnisky [10, 11], as well as an informal treatment by Buchholz [7] who suggested the idea that the present paper develops.

[3] The concern for effectiveness was emphasized by Koopmans [24, Postulate 5] and Epstein [17, 18, p. 723], while the importance of limiting attention to a particular class of technologies was illustrated by, e.g., Asheim [2]. A problem with the social choice approach to sustainability suggested by Chichilnisky [10, 11] is that it is not effective even in relevant technologies (provided that the term in her maximand reflecting the sustainable utility level is not made redundant by a decreasing discount rate). For a specific discussion of this problem, see [3]; for a general investigation into the applicability of Chichilnisky's criterion, see [23].

intergenerational utility paths as maximal paths and thus as optimal solutions for any social preferences satisfying these axioms, as long as immediate productivity holds. This result depends only on the assumption that the utility of any generation is (at least) ordinally measurable and level comparable to the utility levels of other generations. In this respect it strengthens earlier results (cf. [2]), where the more demanding assumption of full cardinal unit comparability had to be assumed. Finally, we show that social preferences satisfying efficiency and equity are effective (i.e., yield a non-empty set of maximal paths) under the provision that the technology satisfies eventual productivity. As demonstrated in Section 5, eventual productivity is even sufficient to ensure the existence of *complete* social preferences obeying both efficiency and equity and resulting in a unique and sustainable solution to the problem of intergenerational justice. Thus, this paper yields results in a positive spirit as compared to much of the literature on intergenerational social preferences: An infinite number of generations can be treated equally, and such treatment justifies sustainability.

We acknowledge that some may not subscribe to equal treatment as an ethical axiom in the intergenerational context (cf. [1]). Here we do not directly address the normative issue of whether this axiom should be endorsed. Rather, we establish the *feasibility* of imposing such an axiom in the context of an infinite number of generations and investigate its *implications* for sustainability, one conclusion being that this axiom should not be identified with undiscounted utilitarianism.

## 2. ETHICAL PREFERENCES

In deriving criteria for intergenerational distributive justice we adopt a purely consequentialistic approach which completely abstains from judging, e.g., the intentions and procedures lying behind each generation's actions. Then the problem of giving an ethical basis for sustainability is reduced to making comparisons between feasible intergenerational distributions. There are many possible ways of solving conflicts of interests between generations in this framework. Here we will look for "a political conception of justice that we hope can gain the support of an overlapping consensus of reasonable ... doctrines," "a political conception the principles and values of which all citizens can endorse" [33, p. 10].

Formally, there is an infinite number of generations $t = 1, 2 \ldots$ . The utility level of generation $t$ is given by $u_t$ which should be interpreted as the utility level of a representative member of this generation. Thus we do not discuss the issue of *intra*generational distribution. We assume that $u_t$ measures the instantaneous well-being that generation $t$ derives from its current situation. The term "instantaneous well-being" signifies that $u_t$ does not include altruism or envy toward other generations. We take instantaneous well-being as a starting point, because we consider it important to separate the definition and analysis of sustainability from the forces (e.g., altruism toward future generations) that can motivate generations to act in accordance with the requirement of sustainability; see also Rawls [32, Sect. 22]. Moreover, we assume that the utilities need not be more than ordinally measurable and level comparable.[4] Hence, it is sufficient to require that the utility

[4] Sen [37] is the basic reference on measurability and comparability assumptions in social choice theory. See Blackorby *et al.* [5] for an instructive survey.

levels of each generation can be ranked on an ordinal scale and that these levels can be compared between generations.

In order to compare different intergenerational utility paths, some binary relation $R$ over paths ${}_1\mathbf{u} = (u_1, u_2, \dots)$ starting in period 1 is needed. Any such binary relation $R$ is throughout assumed to be reflexive and transitive,[5] but $R$ may be complete or incomplete. If ${}_1\mathbf{v} R {}_1\mathbf{u}$ and ${}_1\mathbf{u} R {}_1\mathbf{v}$ hold simultaneously, then there is indifference between ${}_1\mathbf{v}$ and ${}_1\mathbf{u}$, which is denoted by ${}_1\mathbf{v} I {}_1\mathbf{u}$. If, however, ${}_1\mathbf{v} R {}_1\mathbf{u}$ but *not* ${}_1\mathbf{u} R {}_1\mathbf{v}$ holds, then there is (strict) preference for ${}_1\mathbf{v}$ over ${}_1\mathbf{u}$, which is, as usual, denoted by ${}_1\mathbf{v} P {}_1\mathbf{u}$. Thus $I$ gives the symmetric and $P$ the asymmetric part of the social preferences $R$.

In this paper the social preferences $R$ will be used to determine solutions that are *ethically acceptable*. Such an approach might be questioned (cf. [29, pp. 450–460], since any norm stems from subjective value judgements that cannot be scientifically substantiated. Nevertheless, there may exist some basic norms whose ethical appeal seems rather uncontroversial and which can thus be used as axioms for characterizing ethical preferences. Anyone disagreeing with the conclusions that can be drawn from these ethical preferences will then have to argue against the basic norms. Such an axiomatic method makes an ethical debate about normative prescription more transparent by reducing it to an evaluation of the underlying axioms.

The least controversial ethical axiom on $R$ is that any social preferences must deem one utility path superior to another if at least one generation is better off and no generation is worse off.

*Efficiency (of R) Axiom.* If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths with $v_t \geq u_t$ for all $t$ and $v_s > u_s$ for some $s$, then ${}_1\mathbf{v} P {}_1\mathbf{u}$.

We call this axiom *efficiency* as it implies that any maximal path is efficient. It is also called *strong Pareto* or *strong sensitivity*. The axiom ensures that the social preferences are sensitive to utility increases of any one generation.

The other basic ethical axiom on $R$ imposes equal treatment of all generations by requiring that any social preferences must leave the social valuation of a utility path unchanged when the utility levels of any two generations along the path are permuted.

*Equity (of R) Axiom.* If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths with $u_{s'} = v_{s''}$ and $u_{s''} = v_{s'}$ for some $s'$, $s''$ and $u_t = v_t$ for all $t \neq s'$, $s''$, then ${}_1\mathbf{v} I {}_1\mathbf{u}$.

The *equity* axiom is sometimes also called *weak anonymity* or *intergenerational neutrality*. It can be considered a basic fairness norm as it ensures that everyone counts the same in social evaluation.[6] In the intergenerational context the equity axiom implies that it is not justifiable to discriminate against some generation only because it appears at a later stage on the time axis. Also in the intergenerational

[5] *Reflexivity* means that ${}_1\mathbf{u} R {}_1\mathbf{u}$ for any ${}_1\mathbf{u}$. *Transitivity* means that ${}_1\mathbf{u} R {}_1\mathbf{w}$ if ${}_1\mathbf{u} R {}_1\mathbf{v}$ and ${}_1\mathbf{v} R {}_1\mathbf{w}$.

[6] Invoking impartiality in this way is the cornerstone of ethical social choice theory reaching far beyond intergenerational comparisons (see, e.g., [15; 22; 28; 34, p. 32; 37, Chap. 5]). In a setting with an infinite number of time periods, the *equity* axiom was first introduced by Diamond [16]. Later it has been used in many contributions to formalize distributional concerns between an infinite number of generations (see, e.g., [42], where the term *ethical preferences* is associated with the efficiency and equity axioms).

context equity seems to fall within the category of principles that many endorse, at least in a world of certainty (as we assume here). This motivates a clarification of what this prevalent normative view implies for the acceptability of intergenerational utility paths in various technological environments.

Note that both efficiency and equity are compatible with $u_t$ being only an ordinal measure that is level comparable to the utility level of any other generation. This in turn means that these ethical axioms do *not* entail that a certain *decrease* of utility for one generation will be compensated by the same *increase* of utility for another generation since *changes* in utility need not be comparable. In other words, equity does not imply undiscounted utilitarianism.

Obviously, the efficiency and equity axioms are not sufficient to determine a complete binary relation. It is of interest to consider the incomplete binary relation $R^*$ that is generated by efficiency and equity, i.e., which is obtained when only these two axioms are assumed. Formally, we seek a reflexive and transitive binary relation $R^*$ that satisfies efficiency and equity and has the property of being a subrelation[7] to any reflexive and transitive binary relation $R$ satisfying efficiency and equity. It turns out that such a binary relation $R^*$ exists and coincides with the well-known *Suppes–Sen grading principle* $R_S$ (cf. [37, 41] and e.g. [27], and [42] in the intergenerational context). The binary relation $R_S$ deems two paths to be indifferent if one is obtained from the other through a finite permutation, where a permutation $\pi$, i.e., a bijective mapping of $\{1, 2, \ldots\}$ onto itself, is called *finite* whenever there is a $T$ such that $\pi(t) = t$ for any $t \geq T$.

DEFINITION 1. For any two utility paths ${}_1\mathbf{u} = (u_1, u_2, \ldots)$ and ${}_1\mathbf{v} = (v_1, v_2, \ldots)$, the relation ${}_1\mathbf{v} R_s {}_1\mathbf{u}$ holds if there is a finite permutation $\pi$ of $\{1, 2, \ldots\}$ which has $v_{\pi(t)} \geq u_t$ for all $t$.

Let $P_S$ denote the asymmetric part of $R_S$. Say that ${}_1\mathbf{v}$ *Suppes–Sen dominates* ${}_1\mathbf{u}$ if ${}_1\mathbf{v} P_S {}_1\mathbf{u}$. By Definition 1, a utility path Suppes–Sen dominates an alternative path if a finite permutation of the former Pareto dominates the latter. The following proposition states that the Suppes–Sen relation is indeed generated by efficiency and equity:

PROPOSITION 1. *$R_S$ satisfies efficiency and equity and $R_S$ is a subrelation to any reflexive and transitive binary relation satisfying efficiency and equity.*

Thus, in the intergenerational context the Suppes–Sen grading principle can be given an ethical foundation in terms of two focal normative postulates for social preferences. The proof (which is straightforward and hence deleted) is based on the observation that if a reflexive and transitive binary relation satisfies equity, then two utility paths are indifferent if the one is obtained from the other by moving around the utility levels of a *finite* number of generations.[8]

In Section 4, we will justify sustainability by the use of the Suppes–Sen grading principle. Since this justification is concerned with non-decreasing paths, the

[7] $R^*$ is said to be a *subrelation* to $R$ if ${}_1\mathbf{v} R^* {}_1\mathbf{u}$ implies ${}_1\mathbf{v} R {}_1\mathbf{u}$ and ${}_1\mathbf{v} P^* {}_1\mathbf{u}$ implies ${}_1\mathbf{v} P {}_1\mathbf{u}$, with $P^*$ denoting the asymmetric part of $R^*$.

[8] Van Liedekerke and Lauwers [48] argue that moving around only a *finite* number of utility levels is *not* sufficient to ensure the impartial treatment of an *infinite* number of generations. But if equity is strengthened to allow for the permutation of the utility levels of an *infinite* number of generations, then a reflexive and transitive binary relation cannot simultaneously satisfy efficiency. See Vallentyne [46] for a defence of the finite version of equity.

following characterization of the Suppes–Sen grading principle turns out to be useful, where, for any path ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and any time $T$, ${}_1\mathbf{u}_T = (u_1, \dots, u_T)$ denotes the truncation of ${}_1\mathbf{u}$ at $T$ and ${}_1\tilde{\mathbf{u}}_T$ denotes a permutation of ${}_1\mathbf{u}_T$ having the property that ${}_1\tilde{\mathbf{u}}_T$ is non-decreasing.[9]

PROPOSITION 2. *For any two paths* ${}_1\mathbf{u} = (u_1, u_2, \dots)$ *and* ${}_1\mathbf{v} = (v_1, v_2, \dots)$ *the relation* ${}_1\mathbf{v} R_S\, {}_1\mathbf{u}$ *holds if and only if there is a time T such that*

(i) ${}_{T+1}\mathbf{v}$ *Pareto dominates or is identical to* ${}_{T+1}\mathbf{u}$

(ii) ${}_1\tilde{\mathbf{v}}_T$ *Pareto dominates or is identical to* ${}_1\tilde{\mathbf{u}}_T$.

*Proof.* (If) Obvious. (Only if) If ${}_1\mathbf{v} R_S\, {}_1\mathbf{u}$, there is a time $T$ at least as large as any period that is affected by the finite permutation of Definition 1. Then ${}_{T+1}\mathbf{v}$ Pareto dominates or is identical to ${}_{T+1}\mathbf{u}$; i.e., (i) holds, and a permutation of ${}_1\mathbf{v}_T$ Pareto dominates or is identical to ${}_1\mathbf{u}_T$. Suppose neither ${}_1\tilde{\mathbf{v}}_T$ neither Pareto dominates nor is identical to ${}_1\tilde{\mathbf{u}}_T$; i.e., there is a period $s \le T$ with $\tilde{v}_s < \tilde{u}_s$. Consider a finite permutation $\pi$ of $\{1, \dots, T\}$ with $\tilde{v}_{\pi(t)} \ge \tilde{u}_t$ for all $t \in \{1, \dots, T\}$, which exists by construction since a permutation of ${}_1\mathbf{v}_T$ Pareto dominates or is identical to ${}_1\mathbf{u}_T$, and hence, a permutation of ${}_1\tilde{\mathbf{v}}_T$ Pareto dominates or is identical to ${}_1\tilde{\mathbf{u}}_T$. There can be no $s' > s$ with $\pi(s') \le s$ as, ${}_1\tilde{\mathbf{u}}_T$ and ${}_1\tilde{\mathbf{v}}_T$ being non-decreasing, this would imply $\tilde{v}_{\pi(s')} \le \tilde{v}_s < \tilde{u}_s \le \tilde{u}_{s'}$, which contradicts that $\tilde{v}_{\pi(t)} \ge \tilde{u}_t$ for all $t \in \{1, \dots, T\}$. Thus $\pi$ is even a permutation of the subset of periods $\{1, \dots, s\}$. For all periods within this set, however, no $\tilde{v}_t$ exceeds $\tilde{v}_s$, as $\tilde{v}_s = \max\{\tilde{v}_t : t \le s\}$, which contradicts that $\tilde{v}_{\pi(s)} \ge \tilde{u}_s$. Hence, if a permutation of ${}_1\mathbf{v}_T$ Pareto dominates or is identical to ${}_1\mathbf{u}_T$, there is no period $s$ with $\tilde{v}_s < \tilde{u}_s$. ∎

Proposition 2 deals with the Suppes–Sen grading principle in a general setting. However, to establish a link to the concept of sustainability, we will have to consider the implications of this principle within a relevant class of technologies. This amounts to defining a *domain restriction* for $R_S$, and we now turn to this issue.

## 3. SUSTAINABLE PATHS AND PRODUCTIVE TECHNOLOGIES

In order to define sets of feasible paths we assume that the initial endowment of generation $t \ge 1$ is given by a $n$-dimensional ($n < \infty$) vector of capital stocks $k_t$ which may include different forms of man-made capital as well as different types of natural and environmental resource stocks. A generation $t$ acts by choosing a utility level $u_t$ and a vector of capital stocks $k_{t+1}$ which is bequeathed to the next generation $t+1$. For every $t$ the function $F_t$ gives the maximum utility attainable for generation $t$ if $k_t$ is inherited and $k_{t+1}$ is bequeathed; i.e., $u_t \le F_t(k_t, k_{t+1})$ has to hold for any feasible *utility–bequest pair* $(u_t, k_{t+1})$ of generation $t$. Furthermore, it is assumed that the utility level of each generation cannot fall below a certain lower bound $\underline{u}$. This lower bound serves two purposes. First, $\underline{u}$ can be interpreted as the subsistence level of any generation. Moreover, since there are technological limitations on the accumulation of stocks in the course of one period, $F_t(k_t, k_{t+1}) < \underline{u}$ can be used to capture that the bequest $k_{t+1}$ is infeasible given the inheritance $k_t$. Hence, generation $t$'s utility–bequest pair $(u_t, k_{t+1})$ is said to be *feasible* at $t$ given $k_t$ if $\underline{u} \le u_t \le F_t(k_t, k_{t+1})$. Assuming ordinal level comparability, noth-

[9] Saposnik [36] proves a similar result for the finite number case.

ing is changed if $\underline{u}$, $u_t$ and $F_t$ are transformed by the same strictly increasing function $\phi$ so that the feasibility constraint reads as $\phi(\underline{u}) \le \phi(u_t) \le \phi(F_t(k_t, k_{t+1}))$.

The sequence ${}_1\mathbf{F} = (F_1, F_2, \dots)$ characterizes the *technology* of the economy under consideration. Given the technology ${}_1\mathbf{F} = (F_1, F_2, \dots)$, a utility path ${}_t\mathbf{u} = (u_t, u_{t+1}, \dots)$ is *feasible* at $t$ given $k_t$ if there exists a path ${}_{t+1}\mathbf{k} = (k_{t+1}, k_{t+2}, \dots)$ such that, for all $s \ge t$, generation $s$'s utility–bequest pair $(u_s, k_{s+1})$ is feasible at $s$ given $k_s$. If ${}_t\mathbf{u} = (u_t, u_{t+1}, \dots)$ is feasible at $t$ given $k_t$, then the same holds true for any other path ${}_t\mathbf{v} = (v_t, v_{t+1}, \dots)$ with $\underline{u} \le v_s \le u_s$ for each $s \ge t$ since $\underline{u} \le v_s \le u_s \le F_s(k_s, k_{s+1})$ implies that $(v_s, k_{s+1})$ is feasible at $s$ given $k_s$.

Before providing an ethical justification for sustainability in this technological framework we need to clarify what this concept means. As noted by Krautkraemer [25, p. 2091], "[w]hile there is an abundance of definitions of sustainability, it basically gets at the issue of whether or not future generations will be at least as well off as the present generation." Our definition is in line with this view:

DEFINITION 2. Generation $t$ with inheritance $k_t$ is said *to behave in a sustainable manner* if it chooses a feasible utility–bequest pair $(u_t, k_{t+1})$ so that the constant utility path $(u_t, u_t, \dots)$ is feasible at $t+1$ given $k_{t+1}$. The utility *path* ${}_1\mathbf{u} = (u_1, u_2, \dots)$ is called *sustainable* given $k_1$ if there exists ${}_2\mathbf{k} = (k_2, k_3, \dots)$ such that every generation behaves in a sustainable manner along $({}_1\mathbf{k}, {}_1\mathbf{u}) = (k_1, (u_1, k_2), (u_2, k_3), \dots)$.

This definition corresponds closely to what is usually meant by sustainability, e.g., it can be shown that any path sustainable according to Definition 2 is also sustainable according to a definition of sustainability proposed by Pezzey [29].[10] Furthermore, Definition 2 satisfies a characterization of sustainability suggested by Asheim and Brekke [4].[11]

Definition 2 does *not* entail that it will be desirable to follow *any* sustainable path. In particular, a generation may leave behind a wrong *mix* of capital stocks, leading to the realization of an inefficient path. Such inefficiency may be the result of a sequence of generations performing piece-wise planning rather than an omnipotent and benevolent social planner implementing an overall plan. However, even though it will not be desirable to follow *any* sustainable path, it might be the case that any "good" path is sustainable. Such a justification for sustainability is offered in the following section. For this purpose, we make use of a condition which is sufficient for sustainability of utility paths.[12]

PROPOSITION 3. *If* ${}_1\mathbf{u} = (u_1, u_2, \dots)$ *is a non-decreasing utility path that is feasible given* $k_1$, *then* ${}_1\mathbf{u}$ *is sustainable.*

*Proof.* By feasibility there exists ${}_1\mathbf{k} = (k_1, k_2, \dots)$ so that $\underline{u} \le u_t \le F_t(k_t, k_{t+1})$ for all $t \ge 1$. Hence, for all $t$, ${}_{t+1}\mathbf{u} = (u_{t+1}, u_{t+2}, \dots)$ is feasible at $t+1$ given

[10] His definition is: The path ${}_1\mathbf{u} = (u_1, u_2, \dots)$ is sustainable given $k_1$ if there exists ${}_2\mathbf{k} = (k_2, k_3, \dots)$ such that, for all $t \ge 1$, there exists a constant path $(\bar{u}_t, \bar{u}_t, \dots)$ with $\bar{u}_t \ge u_t$ that is feasible at $t$ given $k_t$.

[11] Their characterization is: Generation $t$ behaves in a sustainable manner given $k_t$ by choosing a feasible pair $(u_t, k_{t+1})$ if and only if it is possible for generation $t+1$ to choose $(u_{t+1}, k_{t+2})$ with $u_{t+1} \ge u_t$ *even if* generation $t+1$ behaves in a sustainable manner given $k_{t+1}$.

[12] Pezzey [29, p. 451] refers to a non-decreasing utility path as sustained development. See [29, pp. 451–452], for a discussion of the distinction between sustainability and sustainedness.

$k_{t+1}$. If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ is non-decreasing, then $u_t \le u_{t+1}, u_t \le u_{t+2}, \dots,$ and it follows that $(u_t, u_t, \dots)$ is feasible at $t + 1$ given $k_{t+1}$, implying that any generation $t$ behaves in a sustainable manner by choosing $(u_t, k_{t+1})$. Thus ${}_1\mathbf{u} = (u_1, u_2, \dots)$ is sustainable. ∎

The converse of Proposition 3 does not hold; i.e., it is not the case that only non-decreasing utility paths are sustainable. In particular, it is not in conflict with sustainability that generation $t$ makes a large sacrifice to the benefit of future generations, leading to its own utility being lower than that of generation $t - 1$.

To give a justification for sustainability we will show that, whenever utility paths are ethically justified according to efficiency and equity, they fulfill the sufficient condition for sustainability provided by Proposition 3; i.e., they are non-decreasing. This is not possible without imposing a restriction on the technology which, however, is not very demanding. In an intertemporal context one usually considers technologies that exhibit some kind of productivity. Such productivity can be based on the assumption that consumption can be costlessly postponed to later periods by transforming consumption sacrifices into stocks of man-made capital or by not depleting natural capital. This means that it will be possible to switch consumption between two periods when originally there is higher consumption in the earlier period. This is the starting point of our assumption of immediate productivity, where productivity for a certain technology is defined, not in terms of consumption, but directly in terms of utility.

*Immediate Productivity (of* ${}_1\mathbf{F}$*) Assumption.* If ${}_t\mathbf{u} = (u_t, u_{t+1}, \dots)$ is feasible at $t$ given $k_t$ with $u_t > u_{t+1}$, then $(u_{t+1}, u_t, u_{t+2}, \dots)$ is feasible and inefficient at $t$ given $k_t$.

By the postulated inefficiency of the permuted utility path, there is even a utility gain when the excess utility enjoyed by generation $t$ in comparison to generation $t + 1$ is deferred one period. But even if immediate productivity holds, efficient sustainable paths need not exist. To ensure existence of such paths we make another assumption, which is fulfilled for technologies usually considered in the context of sustainability.

*Eventual Productivity (of* ${}_1\mathbf{F}$*) Assumption.* For any $t$ and $k_t$, there exists a feasible and efficient path with constant utility.

Hence, if ${}_1\mathbf{F}$ satisfies eventual productivity, there is, for any $t$ and $k_t$, a utility level $m_t(k_t)$ such that the path $(m_t(k_t), m_t(k_t), \dots)$ is feasible and efficient at $t$ given $k_t$. Thus, the utility level $m_t(k_t)$ is the *maximum sustainable utility* that can be attained if capital $k_t$ is inherited in period $t$. Note that the assumptions of immediate productivity and eventual productivity are both invariant w.r.t. the same positive transformation of each $F_t$ for $t = 1, 2, \dots,$; i.e., they are also compatible with $u_t$ being only an ordinal measure that is level comparable to the utility level of any other generation.

The following examples show that the general framework described above includes many important classes of technologies as special cases. As a first example, which also shows the logical independence of immediate productivity and eventual productivity, consider the class of linear technologies, which have been used by, e.g., Epstein [17].

EXAMPLE 1. A *linear technology* is defined by an exogenously given positive price path ${}_1\mathbf{p} = (p_1, p_2, \dots)$. A consumption path ${}_t\mathbf{c} = (c_t, c_{t+1}, \dots)$ is feasible at $t$ given $k_t$ if and only if $\Sigma_{s=t}^{\infty} p_s c_s \le p_t k_t$ which is the intertemporal analogue of the standard budget constraint of a household. This explains the interpretation of $p_t$ as the "price" of consumption in period $t$. In the corresponding linear growth model the ratio $p_t/p_{t+1}$ is the *gross* rate of return $1 + r_t$ on the part of the one-dimensional and non-negative inheritance $k_t$ that is not consumed at time $t$:

$$0 \le k_{t+1} \le \frac{p_t}{p_{t+1}}(k_t - c_t), \qquad c_t \ge 0.$$

If utility of consumption $c_t$ is described by a strictly increasing utility function $u$, a linear technology falls into the framework above, with $\underline{u} = u(0)$ and $F_t(k_t, k_{t+1}) = u(c_t) = u(k_t - p_{t+1}k_{t+1}/p_t)$.

A linear technology satisfies immediate productivity if and only if the exogenously given positive price path ${}_1\mathbf{p} = (p_1, p_2, \dots)$ is strictly decreasing; i.e., if $p_t > p_{t+1}$ for all $t \ge 1$, entailing that $r_t = p_t/p_{t+1} - 1$, the *net* rate of return on the part of the inheritance $k_t$ that is not consumed at time $t$, is positive. Eventual productivity (with $m_t(k_t) = u(p_t k_t / \Sigma_{s=t}^{\infty} p_s)$) is satisfied for a linear technology if and only if $\Sigma_{s=1}^{\infty} p_s < \infty$. As this assumption is compatible with $p_t < p_{t+1}$ for some time $t$, this example also shows that eventual productivity does not imply immediate productivity. Conversely, as a strictly decreasing price path ${}_1\mathbf{p} = (p_1, p_2, \dots)$ with $\Sigma_{s=1}^{\infty} p_s = \infty$ exists (e.g., when $p_t = 1/t$ for any $t$), immediate productivity does not imply eventual productivity.

EXAMPLE 2. A second example is given by the *one-sector model* where $f$ is a strictly increasing and concave production function, depending solely on the non-negative stock of man-made capital $k_t$ which is physically identical to the consumption good. Here,

$$c_t + k_{t+1} \le f(k_t) + k_t, c_t \ge 0, k_{t+1} \ge 0,$$

implying that $\underline{u} = u(0)$ and $F(k_t, k_{t+1}) = u(f(k_t) + k_t - k_{t+1})$, where $u$ is again a strictly increasing utility function as above. Since the technology is not time-dependent, the functional value of $F$ depends only on inheritance and bequest. The one-sector model satisfies immediate productivity since $f$ is strictly increasing in $k_t$. It satisfies eventual productivity with $m_t(k_t) = u(f(k_t))$ since the constant utility path ${}_t\mathbf{u} = (m_t(k_t), m_t(k_t), \dots)$ is feasible at $t$ given $k_t$, while increasing utility in period $t$ to a level $u_t > m_t(k_t)$ but still having $u_s = m_t(k_t)$ in the subsequent periods $s > t$ will eventually tear the capital stock down to zero.

EXAMPLE 3. The third example is a discrete-time version of the *Dasgupta–Heal–Solow model* (cf. [12, 40]), where production also depends on the extraction of an exhaustible natural resource. Here, $k_t = (k_t^m, k_t^n)$, where $k_t^m$ is the non-negative stock of man-made capital, and where $k_t^n$ is the non-negative stock of the natural resource available in period $t$. By letting, in this example, $f$ denote a production function depending on the input $k_t^m$ of man-made capital and the non-negative extraction rate of the natural resource $k_t^n - k_{t+1}^n$, we get that $c_t + k_{t+1}^m \le f(k_t^m, k_t^n - k_{t+1}^n) + k_t^m$. Hence, $\underline{u} = u(0)$ and $F(k_t, k_{t+1}) = u(f(k_t^m, k_t^n - k_{t+1}^n) + k_t^m - k_{t+1}^m)$ for any strictly increasing utility function $u$ as above.

The Dasgupta–Heal–Solow model satisfies immediate productivity if the production function is strictly increasing in its first variable. In this model, Cass and Mitra [9] give a necessary and sufficient condition for the existence of a path with constant and positive consumption given an initial vector of positive stocks. (In the Cobb–Douglas case where $f(k_t^m, k_t^n - k_{t+1}^n) = (k_t^m)^\alpha (k_t^n - k_{t+1}^n)^\beta$ holds, this condition is $\alpha > \beta$; i.e., the elasticity of production of man-made capital has to exceed the elasticity of production of the natural resource input.) Dasgupta and Mitra [14] show that this implies the existence of an *efficient* path with constant consumption so that eventual productivity holds.

There may also be examples where utility at time $t$ is directly dependent on stocks of natural resources available at $t$. Immediate productivity can well be obtained in such an economy if it is based on investing man-made capital and if there is no autonomous depreciation of the stocks of natural capital. An efficient constant utility path which is required for eventual productivity could be given either by substituting man-made goods for the utility value provided by the stocks of natural capital or, if such a substitution is not possible, by leaving the amounts of the natural resource stocks invariant. The technological framework used in this paper also captures this situation, which is the perspective of proponents for strong sustainability.

## 4. A JUSTIFICATION FOR SUSTAINABILITY

Given any technology ${}_1\mathbf{F}$ and any binary relation $R$, say that

- The feasible utility path ${}_1\mathbf{u}$ is *R-maximal* given $k_1$, if there exists no feasible path ${}_1\mathbf{v}$ given $k_1$ such that ${}_1\mathbf{v} P {}_1\mathbf{u}$.
- $R$ is *effective* in ${}_1\mathbf{F}$ if, for any $k_1$, there exists an $R$-maximal path given $k_1$.
- An $R$-maximal path ${}_1\mathbf{u}$ is *time-consistent* if, for any corresponding path of capital stock vectors ${}_1\mathbf{k}$ and for all $t > 1$, ${}_1\tilde{\mathbf{u}}$ is $R$-maximal given $\tilde{k}_1$ in ${}_1\tilde{\mathbf{F}}$ where $\tilde{k}_1 = k_t$ and for all $s \geq 1, \tilde{u}_s = u_{s+t-1}$, and $\tilde{F}_s = F_{s+t-1}$.

We will establish that when social preferences $R$ satisfying efficiency and equity are applied to technologies fulfilling the assumption of immediate productivity, then any $R$-maximal is non-decreasing and thus sustainable. To provide such a justification for sustainability we will first determine the set of utility paths that are maximal w.r.t. the Suppes–Sen grading principle $R_S$. The question of effectiveness will be treated in Proposition 6.

PROPOSITION 4. *If the technology satisfies immediate productivity, then the set of $R_S$-maximal utility paths is equal to the set of efficient and non-decreasing paths, and every $R_S$-maximal utility path is time-consistent.*

*Proof.* (i) Every $R_S$-maximal path is efficient and non-decreasing: Efficiency is obvious by the definition of $R_S$. Suppose that there is an $R_S$-maximal path ${}_1\mathbf{u}$ given $k_1$ in period 1 which is *not* non-decreasing. Then there is a period $t$ where $u_t > u_{t+1}$. If $k_t$ is the capital vector in period $t$, then, by immediate productivity, $(u_{t+1}, u_t, u_{t+2}, \dots)$ is feasible at $t$ given $k_t$, and it is Pareto dominated by another path ${}_t\mathbf{v} = (v_t, v_{t+1}, \dots)$ that is feasible at $t$ given $k_t$. This means that the utility path $(u_1, u_2, \dots, v_t, v_{t+1}, \dots)$ is feasible given $k_1$ and Suppes–Sen dominates ${}_1\mathbf{u}$, contradicting that ${}_1\mathbf{u}$ is $R_S$-maximal. Hence, ${}_1\mathbf{u}$ is non-decreasing.

(ii) Every efficient and non-decreasing path is $R_S$-maximal: Suppose that a non-decreasing path ${}_1\mathbf{u} = (u_1, u_2 \dots)$ is efficient given $k_1$, and that a path ${}_1\mathbf{v} = (v_1, v_2, \dots)$ is feasible given $k_1$ and Suppes–Sen dominates ${}_1\mathbf{u}$. Since by Suppes–Sen dominance there is a finite permutation of ${}_1\mathbf{v}$ that Pareto dominates ${}_1\mathbf{u}$, there exists a $T$ such that ${}_{T+1}\mathbf{v}$ Pareto dominates or is identical to ${}_{T+1}\mathbf{u}$. Let ${}_1\tilde{\mathbf{v}}_T$ be a permutation of ${}_1\mathbf{v}_T$ having the property that ${}_1\tilde{\mathbf{v}}_T$ is non-decreasing. By immediate productivity the path $({}_1\tilde{\mathbf{v}}_T, {}_{T+1}\mathbf{v})$ is feasible given $k_1$, as by a sequence of pairwise permutations it is possible to start a feasible utility path in period 1 with the minimum utility level of ${}_1\mathbf{v}_T$, and so on. By Proposition 2 and the premises that ${}_1\mathbf{u}_T$ is non-decreasing and ${}_1\mathbf{v}$ Suppes–Sen dominates ${}_1\mathbf{u}$, it follows that $({}_1\tilde{\mathbf{v}}_T, {}_{T+1}\mathbf{v})$ Pareto dominates ${}_1\mathbf{u}$, which contradicts the efficiency of ${}_1\mathbf{u}$. Hence, ${}_1\mathbf{u}$ is $R_S$-maximal.

(iii) Every $R_S$-maximal utility path ${}_1\mathbf{u} = (u_1, u_2, \dots)$ with a corresponding path ${}_1\mathbf{k} = (k_1, k_2, \dots)$ of capital stock vectors is non-decreasing and efficient by (i). Then for any time period $t$ ${}_t\mathbf{u} = (u_t, u_{t+1}, \dots)$ is non-decreasing and efficient at $t$ given $k_t$ so that it is $R_S$-maximal by (ii) if $R_S$ is applied to the set of all utility paths feasible at $t$ given $k_t$. This shows time-consistency. ∎

Combining this result with Propositions 1 and 3 gives an ethical justification for sustainability in the following sense: In any technology satisfying immediate productivity, only sustainable paths are maximal whenever efficiency and equity are endorsed as ethical axioms. This is the central result of the paper.

PROPOSITION 5. *If the reflexive and transitive social preferences R satisfy efficiency and equity and the technology satisfies immediate productivity, then only sustainable utility paths are R-maximal.*

*Proof.* If $R$ is a reflexive and transitive binary relation satisfying efficiency and equity, then it follows from Proposition 1 that every $R$-maximal element is $R_S$-maximal and thus by Proposition 4 that it is non-decreasing. By Proposition 3, however, any such path is sustainable. ∎

Proposition 5, which can be illustrated by Fig. 1, means that every unsustainable utility path is unacceptable given any theory of justice within a broad class, as long as a weak productivity assumption is satisfied. The class of theories of justice for which our argument applies is broad, as we have accepted

- incompleteness of the social preferences,
- any informational framework as long as utility is (at least) ordinally measurable and level comparable,
- any consequentialistic theory of distributive justice that satisfies efficiency and equity, which are requirements that many endorse.

Our results resemble those obtained in an earlier work of Asheim [2] where, however, the equality of intergenerational distributions of utility measured in the Lorenz sense provides the basis for social preferences, thereby requiring a cardinal measure of utility that is both unit and level comparable. It is an important feature of Proposition 5 that it makes the assumption of cardinal unit comparability

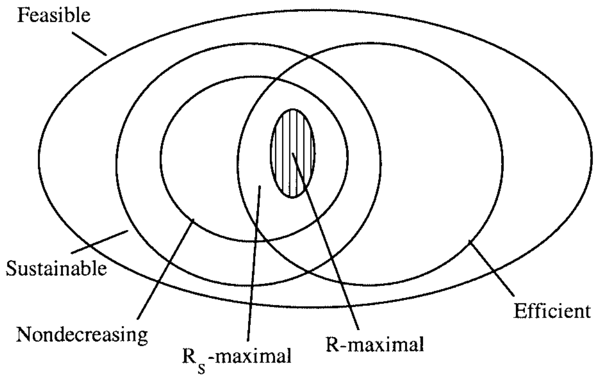


**FIG. 1.** Illustration of Proposition 5.

dispensable. However, if cardinal unit comparability of the utility of different generations is assumed and, in addition, utilitarianism with zero intergenerational discounting—generalized to an infinite number of generations by means of the overtaking criterion[13]—is adopted, then we obtain a special case of social preferences satisfying the efficiency and equity axioms. This in turn means that Propositions 1 and 4 give a generalization of the observation made by Dasgupta and Heal [13, pp. 303–308] and Hamilton [21, p. 407], namely that in the Dasgupta–Heal–Solow model the undiscounted utilitarian maximum will nowhere show decreasing utility.

Propositions 4 and 5 do not address the question of *effectiveness* of $R_S$, i.e., the existence of $R_S$-maximal paths. Even if the technology satisfies immediate productivity, utility paths which are both non-decreasing and efficient need not exist so that the set of $R_S$-maximal paths may well be empty. This is indeed the case in the linear technology of Section 3 (Example 1) when the price path is strictly decreasing, but where the sum of the prices diverges. The following proposition, however, shows that effectiveness of $R_S$ can be established by assuming eventual productivity.

PROPOSITION 6. *If the technology satisfies eventual productivity, then the Suppes–Sen grading principle $R_S$ is effective. Furthermore, for any $k_1$ there is an $R_S$-maximal path that is time-consistent and sustainable.*[14]

*Proof.* If the technology satisfies eventual productivity, then, for any given $k_1$, there exists a feasible and efficient path ${}_1\mathbf{u} = (m_1(k_1), m_1(k_1), \dots)$ with constant utility. Since ${}_1\mathbf{u}$ has constant utility, the existence of an alternative feasible path ${}_1\mathbf{v}$ Suppes–Sen dominating ${}_1\mathbf{u}$ would contradict the efficiency of ${}_1\mathbf{u}$. Hence, ${}_1\mathbf{u}$ is $R_S$-maximal given $k_1$. This path is time-consistent, and by Proposition 3, it is also sustainable. ∎

Thus it is seen that in a relevant class of technologies, the equity axiom is useful for intergenerational social evaluation, even in the case of an infinite number of

[13] See von Weizsäcker [49] and, for a more recent discussion in the philosophical literature, [45, 47].

[14] Note that within a technology satisfying eventual productivity only, the Suppes–Sen grading principle is not sufficient to rule out unsustainable utility paths.

generations. This conclusion is somewhat different from the message conveyed by the literature.

## 5. ON THE POSSIBILITY OF TREATING AN INFINITE NUMBER OF GENERATIONS EQUALLY

In most of the literature since Koopmans [24] the view prevails that equity might be difficult to apply in the intergenerational context if there is an *infinite* number of generations. So, e.g., Diamond [16, p. 170] purports to show "the impossibility of treating all time periods the same," and for Dasgupta and Heal [13, p. 280], when summarizing their discussion of the ethical foundation for resource economics, the "key point is that generations cannot be treated identically."[15] A main conclusion of this literature is that the ordinary procedure for establishing effectiveness is blocked when efficiency and equity are postulated in the context of an infinite number of generations. More precisely, the *Weierstrass theorem* cannot be applied in this case since, for relevant classes of technologies, there is no topology that makes the continuity of complete social preferences satisfying the axioms of efficiency and equity compatible with the compactness of the set of feasible paths (cf., e.g., [18]).[16] Based on this finding, a common message of the discussion in the literature is that some kind of impatience or discounting has to be imposed. In the extreme this amounts to saying that a rational evaluation of infinite utility streams will unavoidably lead to discriminating against future generations.

In contrast, the present paper's justification for sustainability indicates that the impression suggested by this literature—that generations cannot be treated equally—is exaggerated. Efficiency and equity can well be applied to filter out the non-empty set of efficient and non-decreasing paths as maximal solutions as long as some fairly weak productivity assumptions hold. To establish this positive result we followed Epstein [17, 18] in changing the focus from the impossibility of having a continuous ordering on a compact set of feasible utility paths to the possibility of having social preferences that are effective (in the sense of having a non-empty set of maximal elements) in a relevant class of technologies.[17]

However, even if one accepts this change in the perception of the problem, an objection might be that the filter provided by the derived incomplete ethical preferences is rather coarse and leads to a set of maximal paths within which no comparison can be made. Apart from the technical difficulties in ensuring effectiveness when completeness is imposed, there is also an ethical problem involved in comparing efficient and non-decreasing paths. Going beyond efficiency and equity is not compatible with our aim at establishing an overlapping consensus as any additional axiom for resolving distributional conflicts between different generations

[15] Similar statements can also be found in the more recent social choice literature (cf. [17, 26, 38]).

[16] The topology is not unambiguously given in the infinite number case. Hence, the question is whether there is a topology large enough to allow for continuity and small enough to make interesting sets of feasible utility paths compact. For a discussion of the relevance of the underlying topology for the continuity of social preferences, cf. [6, 8, 16, 26, 38, 42].

[17] Epstein [18, p. 723] argues that, from a given perspective, "it seems more pertinent to investigate the link between effectiveness and impatience directly, without involving continuity which after all, is at best sufficient and definitely not necessary for existence of optimal paths. Thus, for example, a pertinent question is whether impatience (in some precise sense) is necessary for effectiveness in a relevant set of choice environments."

is likely to be controversial. However, as shown in Proposition 5, basing ethical preferences solely upon the two focal axioms of efficiency and equity proved to be fruitful insofar as it was completely sufficient to give a justification for sustainability.

Nevertheless, the incomplete binary relation $R_S$ generated by efficiency and equity could still be deemed unsatisfactory if there were no possibility at all for comparing the $R_S$-maximal elements in a way that is consistent with efficiency and equity. In that case the efficiency and equity axioms could never be reconciled with the desire to find a solution that is weakly preferred to any other feasible path. However, Proposition 7 below shows that under eventual productivity there exists even a complete[18] binary relation that satisfies efficiency and equity and yields a unique (and sustainable) maximal path.

In looking for a complete binary relation that satisfies efficiency and equity, consider the leximin principle. In the case of infinite utility paths the leximin principle yields a complete binary relation on the class of non-decreasing paths: If ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ are non-decreasing, ${}_1\mathbf{v}$ is (strictly) preferred to ${}_1\mathbf{u}$ (i.e., ${}_1\mathbf{v}$ *leximin-dominates* ${}_1\mathbf{u}$) if there is a $s \geq 1$ with $v_t = u_t$ for all $1 \leq t < s$ and $v_s > u_s$. It is possible to extend the domain of the leximin principle in the infinite case beyond the class of non-decreasing paths (cf. [2, p. 355]). For a statement of this binary relation, for any ${}_1\mathbf{u} = (u_1, u_2 \dots)$ and any $T \geq 1$, write ${}_1\tilde{\mathbf{u}}_T$ for a permutation of ${}_1\mathbf{u}_T = (u_1, \dots, u_T)$ having the property that ${}_1\tilde{\mathbf{u}}_T$ is non-decreasing.

Definition 3. For any two utility paths ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$, the relation ${}_1\mathbf{v} R'_L\, {}_1\mathbf{u}$ holds if there is a $\tilde{T} \geq 1$ such that for all $T \geq \tilde{T}$, either ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$ or there is a $s \in \{1, \dots, T\}$ with $\tilde{v}_t = \tilde{u}_t$ for all $1 \leq t < s$ and $\tilde{v}_s > \tilde{u}_s$.

The binary relation $R'_L$ defined in this way is reflexive, transitive, and satisfies efficiency and equity, implying by Proposition 1 that the Suppes–Sen grading principle $R_S$ is a subrelation to $R'_L$. On the class of non-decreasing paths the binary relation $R'_L$ is complete and coincides with the above mentioned leximin principle, while $R'_L$ may not be able to compare two paths if (at least) one is not non-decreasing. However, by invoking Svensson's [42] Theorem 2,[19] there exists a *complete*, reflexive, and transitive binary relation $R_L$ which has $R'_L$ and thus $R_S$ as a subrelation. Since $R_L$ ranks an efficient path with constant utility above any other feasible path, the following proposition can be established.

Proposition 7. *If the technology satisfies eventual productivity, then there exists a complete, reflexive, and transitive binary relation $R_L$, satisfying efficiency and equity, that is effective. Furthermore, for any $k_1$ there is a unique $R_L$-maximal path. This utility path is time-consistent and, due to its constant utility, sustainable.*

*Proof.* $R_L$ has $R_S$ as a subrelation and thus satisfies efficiency and equity. If the technology satisfies eventual productivity, then, for any $k_1$, there exists a feasible and efficient path ${}_1\mathbf{u} = (m_1(k_1), m_1(k_1), \dots)$ with constant utility. Since ${}_1\mathbf{u}$ is efficient and has constant utility, it follows that ${}_1\mathbf{u} P'_L\, {}_1\mathbf{v}$ (and hence, ${}_1\mathbf{u} P_L\, {}_1\mathbf{v}$ since

[18] *Completeness* means that ${}_1\mathbf{v} R\, {}_1\mathbf{u}$ or ${}_1\mathbf{u} R\, {}_1\mathbf{v}$ for any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$. Hence, a complete binary relation is able to compare any pair of paths.

[19] Svensson [42, Theorem 2] states that any reflexive and transitive binary relation that has the Suppes–Sen grading principle as a subrelation is itself a subrelation to a *complete*, reflexive, and transitive binary relation (i.e., an ordering). In proving this result Svensson refers to a general mathematical lemma by Szpilrajn [43].

$R'_L$ is a subrelation to $R_L$), where ${}_1\mathbf{v}$ is any alternative path that is feasible given $k_1$. Hence, ${}_1\mathbf{u}$ is the unique $R_L$-maximal path given $k_1$. This path is time-consistent, and by Proposition 3, it is also sustainable. ▮

In Proposition 7 $R_S$ is completed by means of the leximin principle. This is only one possibility for constructing complete social preferences that satisfy efficiency and equity. In a technology that satisfies immediate productivity, a completion of the overtaking criterion may also yield a sustainable path that is preferred to any other path. If we follow this alternative route, however, then we will have to go beyond the framework where utility is only an ordinal measure that is level comparable to the utility level of any other generation. The reason is that use of the overtaking criterion requires that one generation's gain is comparable to another generation's loss. Depending on how we construct a cardinal scale (i.e., how we assign cardinal value to gains and losses at different levels of ordinal utility), a wide diversity of paths can be maximal under the completed overtaking criterion and hence under complete social preferences satisfying efficiency and equity (see, e.g., [20]). In particular, the criterion does not necessarily entail "excessively" high savings rates leading to an unacceptable strain on the present generation (cf. [1, pp. 15–16]). On the other hand, for a given cardinal scale there need not be any maximal path as the assumption of eventual productivity is not sufficient to ensure that the overtaking criterion is effective.

Another approach to making comparisons among $R_S$-maximal paths is to let the choice of an $R_S$-maximal path be a side constraint in a maximization procedure that does not otherwise take into account ethical considerations (cf. [2, 29]). To fix ideas, consider maximizing the sum of discounted utilities subject to the constraint that the chosen path is efficient and non-decreasing. In the one-sector model (cf. Example 2) the *unconstrained* maximum under discounted utilitarianism is non-decreasing for an initial capital stock that does not exceed the modified golden rule size, due to a sufficiently high and sustained productivity of man-made capital. In such circumstances there is no conflict between discounting utilities and the ethical preferences generated by efficiency and equity. Although equity rules out social preferences based on discounted utilitarianism, this axiom does not necessarily rule out *paths* that are maximal under discounted utilitarianism. However, in other technological environments—like the Dasgupta–Heal–Solow model (cf. Example 3), where a sufficiently high productivity of man-made capital cannot be sustained even if eventual productivity is satisfied—any maximal path under discounted utilitarianism (with a constant discount rate) is rejected by efficiency and equity. Thus, a requirement to choose an $R_S$-maximal path necessarily becomes a binding side constraint in this model. Some may claim that the non-decreasing paths resulting from the application of discounted utilitarianism in the former model appeal to ethical intuitions, while the maximal paths of this criterion in the latter model do not, as they impoverish generations in the distant future even though sustainable paths are feasible. If so, our analysis helps to explain these intuitions and provides a way to amend unacceptable paths by justifying sustainability as a side-constraint.[20]

[20] Fleurbaey and Michel [19] provide a criterion for balancing the interests of the different generations which explicitly depends on the underlying technology. One could also use their criterion for making a choice between non-decreasing paths.

## 6. CONCLUSION

The sustainability requirement, which has come to be considered as an important guideline for environmental policy, is a genuinely ethical one as it at least implicitly draws much of its appeal from the desire to be fair toward future generations. It is, however, far less obvious what the precise relation is between intergenerational justice, on the one hand, and sustainability, on the other. There is a long tradition in economics to define justice by referring to the degree of inequality of income distributions, measured, e.g., by Lorenz curves. In trying to give a justification for sustainability such an approach was developed by Asheim [2]. In this paper we have instead directly imposed that every generation be treated equally in intergenerational social preferences, which is tantamount to saying that discrimination against future generations is excluded. The *equity* axiom corresponding to this prevalent ethical norm has a long history in the theory of evaluating intergenerational utility paths. The axiom, however, is considered to cause difficulty, because it might be in conflict with the demand for effectiveness. Here we have shown under weak productivity assumptions how equity combined with the strong Pareto axiom (efficiency) is compatible with effectiveness and can be used to justify sustainability in the following sense: Only sustainable paths are ethically acceptable whenever efficiency and equity are endorsed as ethical axioms. A further question might be how ethics based on only these two axioms can be extended in order to give clearer advice on how to resolve distributional conflicts between generations going beyond the sustainability question.

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