Non-discriminating renegotiation in a competitive insurance market

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Abstract

A variant of the Rothschild-Stiglitz model of a competitive insurance market is considered, where each uninformed firm is allowed to renegotiate the contracts that its customers initially sign, subject to the restriction that renegotiated contracts be offered to all the firm's customers. Such non-discriminating renegotiation is shown to weaken the profitability of cream skimming to the extent that there exists a unique equilibrium outcome. This outcome is that of Miyazaki and Spence, i.e., the incentive-compatible pair of zero-profit contracts, if efficient; and the incentive-compatible, zero-profit pair of contracts maximizing low-risk utility, otherwise.

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1. Introduction

Since the seminal work of Rothschild and Stiglitz (1976), researchers on the economics of competitive markets with asymmetric information have struggled with two intertwined problems that seem to reappear in such markets: First, an equilibrium may not exist, at least not in pure strategies; and second, an equilibrium, if it exists, may not be efficient (subject to informational constraints). The
first to suggest a model exhibiting the existence of an efficient equilibrium was Miyazaki (1977). An insurance-market version of this model was presented by Spence (1978). In a model with two types, high-risk and low-risk individuals, the Miyazaki–Spence outcome is the incentive-compatible, zero-profit contract menu that maximizes low-risk utility. The present work is part of a research programme aiming at providing a sound game-theoretic foundation for this as an equilibrium outcome.

Using insurance as the context, we present a model of a market with asymmetric information having the following characteristics: On the one hand, insurers are assumed to be unable to commit not to renegotiate a contract once it has been signed. On the other hand, the flexibility that such renegotiation creates for the insurers is restricted by their inability to discriminate among their customers on the basis of the latter’s initial contract choices: A contract menu offered by an insurer to one of its customers, has to be offered to all its customers. With such a restriction, we say the renegotiation is non-discriminating.

In the terminology of industrial organization, we allow firms to do second-degree price discrimination by offering non-linear prices, as is standard in studies of insurance markets. Renegotiation between a firm and its customers that takes place after initial contracts are signed can, in general, be based on the segmentation of the market induced by the individuals’ initial contract choices. We disallow such third-degree price discrimination by imposing that renegotiation be non-discriminating. Such a requirement may originate in laws prohibiting third-degree price discrimination or in explicit or implicit clauses in the initial contracts, so-called most-favoured-customer clauses. ¹

In the Rothschild–Stiglitz model, with each firm offering a menu of contracts, an equilibrium in pure strategies can never yield an inefficient outcome. The non-existence problem in this model arises when any efficient outcome involves cross-subsidization of the high risks at the expense of the low risks. Cream-skimming offers, attracting low risks only, then undermine the existence of a pure-strategy equilibrium. Introducing renegotiation weakens the profitability of cream skimming, since the prospects of renegotiation make it attractive for high risks as well to choose the cream-skimming offer. In fact, it will be shown that the profitability of cream skimming is weakened to the extent that the Miyazaki–Spence outcome (and this outcome only) survives in equilibrium. Imposing that renegotiation be non-discriminating implies that having only one round of renegotiation is not restrictive, as the market segmentation (high risks and low risks self-selecting separate contracts) that efficiency requires, does not yield any information upon which further renegotiation can take place.

¹ With regard to the legal aspects, the discussion in McAfee and Schwartz (1994, pp. 217–218), on whether price discrimination law, in particular the U.S. Robinson–Patman Act, allows the second-degree kind but not the third-degree one, is relevant.
The rest of the paper is organized as follows: In Section 2, we describe the model. Our model is kept as close as possible to the original Rothschild and Stiglitz (1976) set-up, except for the introduction of non-discriminating renegotiation. In Section 3, the equilibrium analysis of the model is carried out. We apply the concept of a Perfect Bayesian Equilibrium and solve the model by backward induction. First, we characterize the renegotiation stage (Propositions 1 and 2). Second, we show existence of, and partially characterize, equilibrium in the subgame following any set of initial contract offers (Proposition 3). Finally, we are able to demonstrate the existence of a unique renegotiation-proof, symmetric equilibrium of the whole model; this equilibrium yields the Miyazaki-Spence outcome (Proposition 4). Section 4 contains a discussion of why this particular cross-subsidizing outcome, and it alone, survives in equilibrium. Section 5 concludes by relating our analysis to other work. The proofs of all results are relegated to Appendix A.

2. The model

Consider a continuum of individuals uniformly distributed on the unit line [0, 1]. Each individual faces two possible states of nature: In state 1, the good state, no accident is experienced and his endowment is \( w_{01} \). In state 2, the bad state, an accident is experienced, and his endowment is \( w_{02} \), with \( w_{01} > w_{02} > 0 \). The individuals are identical except for the probability of an accident. The high-risk (H) type has probability \( p^H \), the low-risk (L) type has probability \( p^L \), with \( 0 < p^L < p^H < 1 \). For any set of individuals, let \( \varphi \in [0, 1] \) denote the fraction of high risks. The high-risk fraction of the whole population is \( \varphi_0 := \mu^H_0 / (\mu^H_0 + \mu^K_0) \) (= \( \mu^H_0 \)), where \( \mu^K_0 > 0 \), \( K \in \{H, L\} \), denotes the amount of K-type individuals in the whole population.

Insurance is provided by the firms in the set \( J := \{1, \ldots, j, \ldots, n\} \). A firm does not know, \textit{ex ante}, any individual’s type. If an individual buys insurance, then the endowment \( w_0 = (w_{01}, w_{02}) \) is traded for another state-contingent endowment \( w = (w_1, w_2) \gg 0 \); \(^2\) we say the individual buys the insurance contract \( w \). If \( w_1 = w_2 \), then \( w \) provides full insurance; if \( w_1 > w_2 \), then \( w \) provides partial insurance. The set of feasible contracts, \( W \), is given by: \( W := \{ (w_1, w_2) ; w_1 > w_2 > 0 \} \); we assume that moral-hazard considerations prevent \( w_1 < w_2 \).

A contract \( w \in W \) is evaluated by a K-type individual according to the expected utility \( u^K(w) := (1 - p^K) u(w_1) + p^K u(w_2) \), where \( u \) is a strictly increasing, twice continuously differentiable, and strictly concave von Neumann–Morgenstern utility function. Let \( U := \{ (u^K(w), u^K(w)) ; w \in W \} \) denote the set of

\(^2\) We use the following notation for vector inequalities: \( a \gg b \) if and only if \( a_i > b_i \) for all \( i \); \( a \geq b \) if and only if \( a_i \geq b_i \) for all \( i \).
feasible pairs of utility levels, with $u = (u^H, u^L)$ as a generic element. Define, for any finite set of contracts $W' \subset W$, the pair $\bar{u}(W')$ of maximum utility levels associated with it; formally: $\bar{u}(W') = (\bar{u}^H(W'), \bar{u}^L(W'))$, where $\bar{u}^K(W') := \max u^K(w)$, subject to $w \in W'$, $K \in \{H, L\}$. Also define the set $U(W')$ of pairs of utility levels that both types (weakly) prefer to $\bar{u}(W')$; formally, $U(W') := \{u \in U: u \geq \bar{u}(W')\}$.

We may describe the game as follows:

In Stage 1, each firm $j \in J$ offers a finite set of contracts, $W_j \subset W$. The offered sets are observed by all firms and individuals before the beginning of the next stage. We refer to $\bar{u}(W_j)$ as a pair of initial utility levels.

In Stage 2, the individuals choose among available contracts; in order to ensure that individuals prefer some contract to none, and without loss of generality, we make the restriction that self-insurance is included in each firm’s offer: $w_0 \in W_j, \forall j \in J$. Each firm $j \in J$ observes the amount of individuals each of its contracts attracts, i.e., it observes $\mu(w), \forall w \in W_j$; and it forms beliefs about the fraction of high risks in each of its customer segments, i.e. it forms $\phi(w), \forall w \in W_j$. In particular, the firm believes that the fraction of high risks among all its customers equals $\phi_j = \sum_{w \in W_j} \phi(w) \mu(w) / \mu_j$, where $\mu_j = \sum_{w \in W_j} \mu(w)$ is the total amount of customers attracted.

Let $V$ denote the range of $u$. If $V$ is bounded below, then $U$ is a proper subset of $V \times V$. 

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In Stage 3, each firm \( j \in J \) offers a finite set of contracts \( \tilde{W}_j \) to all its customers to replace \( W_j \), subject to the restriction that none of its customers suffers: \( \tilde{u}(\tilde{W}_j) \geq \tilde{u}(W_j) \). We refer to \( \tilde{u}(\tilde{W}_j) \) as a pair of final utility levels.

In Stage 4, for every \( j \in J \), each customer of firm \( j \) chooses a contract in the set \( \tilde{W}_j \).

In Fig. 1, this set-up, in panel (b) of the figure, is compared to that of Rothschild and Stiglitz (1976), in panel (a). The special features of the present model are Stages 3 and 4. By introducing these extra stages, we provide firms with an opportunity to renegotiate. On the other hand, by insisting that any of a firm’s revised offers be presented to all its customers, we ensure that the renegotiation is non-discriminating.

3. Equilibrium

In the subsequent analysis, the concept of a Perfect Bayesian Equilibrium (PBE) is applied. In analysing the game, we proceed by backward induction. Since Stage 4 is straightforward, we go directly to Stage 3. Define \( C(u; \varphi) \) as the minimum expected costs associated with providing a customer, who is believed to be high-risk with probability \( \varphi \), the pair \( u \) of utility levels, where the minimization is done subject to a pair of incentive-compatibility constraints. A formal statement of this definition is in Appendix A. Let \( y \) denote the inverse of the von Neumann–Morgenstern function \( \nu \).

Lemma 1. The expected cost per customer, \( C(u; \varphi) \), can be expressed as

\[
C(u; \varphi) = \varphi y(u^H) + (1 - \varphi) \left\{ (1 - p^L) \left( \frac{p^H u^L - p^L u^H}{p^H - p^L} \right) \right. \\
+ p^L \left( \frac{(1 - p^L)u^H - (1 - p^H)u^L}{(1 - p^L) - (1 - p^H)} \right) \right\}.
\]

The proof of this Lemma, as well as all other proofs, are contained in Appendix A. Straightforward calculus shows that the cost function \( C(u; \varphi) \) has the properties stated in Lemma 2 (in Appendix A). In particular, a typical isocost curve has the shape shown in Fig. 2. Since \( \partial C(u; \varphi) / \partial u^L > 0 \), the shape depends on the sign of \( \partial C(u; \varphi) / \partial u^H \): For \( u^H \) near \( u^L \), \( \partial C(u; \varphi) / \partial u^H < 0 \), and for \( u^H \) small, \( \partial C(u; \varphi) / \partial u^H < 0 \). \(^4\) In between, there is a unique point for which \( \partial C(u; \varphi) / \partial u^H > 0 \).

\(^4\) Note that, for \( \varphi \) 'large', there may not exist \( u \in U \) such that \( \partial C(u; \varphi) / \partial u^H > 0 \).
The locus of points in utility space, for which $\frac{\partial C(u; \varphi)}{\partial u^H} = 0$, is an increasing curve. It coincides with the full-insurance line ($u^H = u^L$) for $\varphi = 0$ and shifts downwards as $\varphi$ increases.

This analysis implies the following Proposition:

**Proposition 1.** Consider a firm $j \in J$ that, in Stage 1, offered $W_j$ and, in Stage 2, attracted $\mu(w)$, $w \in W_j$, and formed beliefs $\varphi(w)$, $w \in W_j$. In particular, the firm believes that $\varphi_j = \frac{\sum_{w \in W_j} \varphi(w) \mu(w)}{\sum_{w \in W_j} \mu(w)}$ is the fraction of high risks among its customers. Then, in Stage 3, firm $j$ minimises the expected cost per customer, $C(u; \varphi_j)$, over all $u = (u^H, u^L) \in U(W_j)$. Furthermore, $\arg\min_u C(u; \varphi_j)$ is a singleton, and $u_j = (u_j^H, u_j^L) = \arg\min_u C(u; \varphi_j)$ is implemented by $W_j = \{w_j^H, w_j^L\}$ given by: $w_j^H = w_j^L$, $u_j^H(w_j^H) = u_j^H(w_j^L) = u_j^H$, and $u_j^L(w_j^L) = u_j^L$.

In the subsequent analysis, we want to focus on equilibria that will not give rise to renegotiation in Stage 3. Therefore, define a pair of utility levels as renegotia-
tion-proof if, for a given fraction of high risks, it is not profitable for a firm in Stage 3 to replace the pair with another pair that is at least as good for both types of individual:

Definition (Renegotiation-proofness). For a given high-risk fraction \( \varphi \), a pair of utility levels \( \tilde{u} = (\tilde{u}^H, \tilde{u}^L) \) is renegotiation-proof if

\[
\tilde{u} = \arg\min_u C(u; \varphi), \quad \text{subject to: } u^H \geq \tilde{u}^H, \text{ and } u^L \geq \tilde{u}^L.
\]

The firm’s expected revenue comes from the initial endowment, as discussed below. Therefore, minimizing costs amounts to maximizing profits, and this concept of renegotiation-proofness is equivalent to that of interim efficiency.

Definition (Interim efficiency, Holmström and Myerson (1983)). For a given high-risk fraction \( \varphi \), a pair of utility levels \( \tilde{u} = (\tilde{u}^H, \tilde{u}^L) \) is interim efficient if there does not exist another pair of utility levels that is preferred to \( \tilde{u} \) both by the firm and by the individual, whatever his type may be.

The properties of the cost function make it easy to verify the simple characterization of a renegotiation-proof pair of utility levels provided in part (i) of Proposition 2 below. Part (ii) of the Proposition implies that the outcome of any renegotiation that takes place in Stage 3 is a pair of utility levels that is renegotiation-proof.

Proposition 2. (i) For a given high-risk fraction \( \varphi \), a pair of utility levels \( \tilde{u} \) is renegotiation-proof if and only if: \( \frac{\partial C(\tilde{u}; \varphi)}{\partial u^H} \geq 0 \).

(ii) A pair of utility levels \( \tilde{u} = (\tilde{u}^H, \tilde{u}^L) \) that is not renegotiation-proof for a given high-risk fraction \( \varphi \), i.e., where \( \frac{\partial C(\tilde{u}; \varphi)}{\partial u^H} < 0 \), will be renegotiated to a renegotiation-proof pair \( \tilde{u} = (\tilde{u}^H, \tilde{u}^L) \) with \( \tilde{u}^H > \tilde{u}^H \), \( \tilde{u}^L = \tilde{u}^L \), and \( \frac{\partial C(\tilde{u}; \varphi)}{\partial u^H} = 0 \).

The next Proposition shows existence [part (i)], and provides a partial characterization [part (ii)], of equilibrium in all Stage-2 subgames.

Proposition 3. (i) There exists, for any vector of Stage-1 offers, a Perfect Bayesian Equilibrium in the resulting subgame.

(ii) For any vector of Stage-1 offers and any Perfect Bayesian Equilibrium in the resulting subgame, all low risks sign contracts in Stage 2 with firms that give the low risks a best Stage-1 offer. Renegotiation does not change the utility level of the low risks.

We now turn to Stage 1 and our main result, which concerns the equilibrium outcome of the entire game. First, we need to introduce firms’ revenue side. In exchange for offering individuals insurance, a firm takes over the individuals’
original risks and earns an expected revenue that depends on the fraction of high risks among the customers. Let $R(\varphi)$ denote the expected revenue per customer; it is defined as

$$R(\varphi) = \varphi \left[ (1 - p^H) w_{01} + p^H w_{02} \right] + (1 - \varphi) \left[ (1 - p^L) w_{01} + p^L w_{02} \right].$$

In utility space, the Rothschild and Stiglitz (1976) outcome, $u_{RS}$, is the pair of utility levels having the property that revenue equals cost on each customer type. In other words, $u_{RS}$ is the pair of utility levels yielding zero expected profit such that the high-risk utility level (and therefore the low-risk utility level) in isolation earns zero expected profit; formally, $u_{RS} = (u_{RS}^H, u_{RS}^L)$ solves

$$C(u; 1) = R(1), \quad \text{and} \quad C(u; \varphi_0) = R(\varphi_0).$$

Let the low-risk preferred outcome, $u_{LR}$, be the pair of utility levels that maximizes low-risk utility among those that yield a zero expected profit on the whole population of individuals; formally, $u_{LR} = (u_{LR}^H, u_{LR}^L)$ solves

$$\max u^L, \quad \text{subject to:} \quad C(u^H, u^L; \varphi_0) = R(\varphi_0).$$

Now, we can define the Miyazaki–Spence outcome, $u_{MS}$. This coincides with the Rothschild–Stiglitz outcome when the latter equilibrium exists, and equals the low-risk preferred outcome otherwise. Existence of the Rothschild–Stiglitz equilibrium hinges upon whether its outcome is interim efficient. By definition, $u_{LR}^H \geq u_{RS}^H$. Given the properties of $C$ (see Lemma 2 in Appendix A), the efficiency of the Rothschild–Stiglitz outcome is therefore determined by the relationship between $u_{RS}^H$ and $u_{LR}^H$. Thus:

**Definition.** The Miyazaki–Spence outcome, denoted $u_{MS}$, is defined as follows:

$$u_{MS} = u_{RS}^H, \quad \text{if} \quad u_{RS}^H \geq u_{LR}^H;$$

$$u_{MS} = u_{LR}^L, \quad \text{otherwise.}$$

We concentrate on equilibria that are symmetric and renegotiation-proof and are able to show that there exists only one such equilibrium in this model.

**Definition.** Let $(W_1, \ldots, W_n)$ be stage-1 offers of an equilibrium of this model.

1. An equilibrium is symmetric if $\bar{u}(W_j) = \bar{u}(W_{j'}), \forall j, j' \in J$.
2. A symmetric equilibrium is unique if there does not exist another symmetric equilibrium with stage-1 offers $(W'_1, \ldots, W'_n)$ satisfying $\bar{u}(W'_j) \neq \bar{u}(W_j), j \in J$.
3. A symmetric equilibrium is renegotiation-proof if $\bar{u}(W_j)$ is renegotiation-proof given $\varphi_0, \forall j \in J$.

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6 In fact, given the first equality, the second equality will hold for any $\varphi$. 

Proposition 4. There exists a unique renegotiation-proof, symmetric equilibrium of the model. It is characterized by each firm, in Stage 1, offering the Miyazaki–Spence outcome, i.e., \( u_j(W_j) = u_{M5}, \forall j \in J \).

4. Discussion

Proposition 4 considers only equilibria that are symmetric and renegotiation-proof. The limitation to symmetric equilibria is not restrictive, since the only asymmetric equilibria involve firms renegotiating or being inactive. The limitation to renegotiation-proof equilibria is done by invoking the so-called Renegotiation-Proofness Principle (RPP): When \( u^H_{RS} \leq u^L_{LR} \), it can be shown that there exist other (symmetric) equilibria than the one in Proposition 4. But they all lead to the Miyazaki–Spence outcome. The difference is that these other equilibria involve renegotiation at Stage 3. According to the RPP, we should in such a case concentrate on equilibria that do not involve renegotiation.

Non-discriminating renegotiation allows for the possibility that the renegotiation-proof equilibrium is implemented by the individuals choosing, in Stage 2, what turns out to be their final contracts. Hence, even though renegotiation-proofness has been defined in terms of utility, it can be implemented at the level of contracts. Clearly, if firms could differentiate their offers depending on contract choices in Stage 2, then such separation of high risks and low risks in this stage would be impossible. Renegotiation-proofness at the contract level conforms with what is observed empirically, that there does not take place any renegotiation of contracts.

As mentioned in the Introduction, non-discriminating renegotiation ensures the existence of a pure-strategy equilibrium in the present model of a competitive insurance market by weakening the profitability of cream-skimming offers, since the prospects of renegotiation make such offers attractive for high risks as well. How is the profitability of cream skimming weakened to the effect that the Miyazaki–Spence outcome, and this outcome only, survives in equilibrium?

Consider a situation with \( u^H_{RS} < u^L_{LR} \), such that any efficient outcome involves cross-subsidization; this is exactly the condition under which the Rothschild–Stiglitz model does not have a pure-strategy equilibrium, and under which the Miyazaki–Spence outcome differs from the Rothschild–Stiglitz outcome. Fig. 3 illustrates such a situation in contract space.

Here, \( H \) and \( L \) are lines consisting of contracts that, if bought by high risks and low risks, respectively, yield zero (expected) profit to firms. In the Rothschild–Stiglitz outcome, \((w^H_{RS}, w^L_{RS})\), firms earn zero profit on each type; the high risks obtain full insurance; and the high-risk incentive constraint binds. There is a continuum of cross-subsidizing pairs of contracts in which the high risks obtain full insurance, the high-risk incentive constraint binds, and firms earn zero profit when such a pair is offered to a set of individuals with a fraction of high risks.
equaling that of the whole population. Each such pair is characterized by its 
low-risk contract; the ‘zero-profit curve’ in Fig. 3 is the locus of such low-risk 
contracts.

In a situation where any efficient, zero-profit outcome involves cross-subsidiza-
tion, the Miyazaki–Spence outcome, \((w_{MS}^{H}, w_{MS}^{L})\), is characterized by the low-risk contract that maximizes low-risk utility subject to the contract being on the zero-profit curve. Efficient, zero-profit outcomes involve contract pairs with the low-risk contract being on the zero-profit curve between \(w_{MS}^{L}\) and the full-insurance pooling contract \(w_{F}^{L}\), the end points included. Efficient outcomes are in general found by varying the profit level and repeating the above exercise. In Fig. 
3, an outcome is efficient if and only if its characterizing low-risk contract is in the 
shaded efficiency region.

Renegotiation favours high risks more when the renegotiating firm believes the 
fraction of high risks is low, since, then, increasing the utility of high risks in 
order to relax the high-risk incentive constraint is less costly. If all firms in Stage 1 
offer \((w_{MS}^{H}, w_{MS}^{L})\), then a cream-skimming deviation will be renegotiated to a 
contract that the high risks prefer to \(w_{MS}^{H}\) even if all high risks are attracted. (This 
is caused by the property that any cream-skimming offer is outside the efficiency 
region of Fig. 3.) Therefore, all high risks will, in an equilibrium of the subgame
following such a deviation, be attracted by the deviating firm and, hence, the 
cream-skimming deviation fails.

If all firms in Stage 1 offer an efficient, zero-profit pair of contracts other than 
\((w^H_{MS}, w^L_{MS})\), then a cream-skimming deviation will be renegotiated to a contract
that the high risks weakly prefer to the offers of the non-deviating firms only if
not all high risks are attracted by the deviating firm. Therefore, not all high risks
will, in equilibrium, choose the deviating firm. This partial success of the
cream-skimming deviation is sufficient for the deviation to be profitable.

5. Related literature

The equilibrium outcome discussed above was first suggested by Miyazaki
(1977). He finds that this outcome is achieved in the following structure: First,
firms offer preliminary contracts; second, each firm may withdraw any contract
that is unprofitable given the prevailing contracts from other firms; and third,
individuals choose among those contracts still available. Miyazaki’s work, written
in a labour-market context, was extended to the insurance market by Spence
(1978); hence, we refer to this outcome as the Miyazaki–Spence outcome.

Cross-subsidization survives in Miyazaki’s set-up because, when firms are
allowed to withdraw contracts, cream-skimming deviations are not profitable since
the non-deviating firms withdraw all their contracts in the second stage. According
to Fernandez and Rasmusen (1993), though, cross-subsidization in this set-up
implies an asymmetric equilibrium with only one active firm. The reason is that, if
several firms offer the cross-subsidizing contract pair in the first stage, each of
them would like to withdraw the loss-bearing high-risk contract in the second
stage if any of the other firms does not withdraw it. On the other hand, if a single
firm offers this contract pair at the first stage, it has no incentive to withdraw at
the second stage. Thus, the analysis of Fernandez and Rasmusen points to a
problem with obtaining symmetric cross-subsidization when withdrawals are
allowed. Our construction, on the other hand, overcomes this problem by allowing
individuals to sign the initial contracts. At the same time, we obtain the necessary
weakening of cream skimming by allowing non-discriminating renegotiation.

Most of the recent research on game-theoretic foundations for the Miyazaki–
Spence outcome considers how individuals of different types will group when the
type is private information of each individual. Thus, in a sense, firms are
determined endogenously as equilibrium groups of individuals. The seminal work
is by Boyd et al. (1988). After any deviation in their model, the remaining group
of individuals will have to rearrange such that non-negative profit is maintained.
Hence, if high risks are subsidized, then a cream-skimming offer is not compared
by these individuals with the status quo, because the status quo will not be
available if low risks choose to deviate. Thus, cream skimming is eliminated by
construction in their framework.
In related work, and predicting the same outcome, Lacker and Weinberg (1994) investigate which deviations will not be subject to further deviation. Independently, and in a cooperative game-theoretic framework, they present an analysis that is very parallel to ours. In fact, the two papers complement each other. The present paper presents an extensive form and thus a non-cooperative foundation for the cooperative model that Lacker and Weinberg analyze using a modified core concept. Interestingly, Lacker and Weinberg do not permit membership in a deviating group to depend on contract choice; this is similar to our imposition of non-discrimination.

Our work is also related to that of Grossman (1979), although there are differences in the equilibrium outcome. Like us, he suggests that firms are allowed to make a second move after having observed some action by the individuals. In his model, individuals submit applications for a contract at stage 2, while firms at stage 3 decide whether they should accept or reject these applications. The outcome of Grossman's construction coincides with that of Rothschild and Stiglitz when the latter's equilibrium exists and is a pooling outcome otherwise (the zero-profit pooling outcome maximizing low-risk utility). Since there are gains to separation, this outcome is not interim efficient.

Among previous studies of renegotiation in competitive markets with asymmetric information, Nalebuff et al. (1993, pp. 29-35) study a labour market where, first, firms offer wages; secondly, workers choose where to apply for work; thirdly, each firm may offer its applicants a revised wage level; and fourthly, hiring takes place. Their results are similar to ours, as they find an outcome which is always interim efficient. Dionne and Doherty (1994) study a multi-period insurance market where firms, but not individuals, are able to commit to long-term contracts but unable to commit not to renegotiate contracts after a period. Finally, Hillas (1987, Ch. 4) discusses renegotiation in a competitive labour market, introduces a restriction on renegotiation similar to ours (in his sctn. 4.6), and argues that the Miyazaki-Spence outcome is the solution in this market. Hillas does not succeed in carrying his analysis over to the insurance market; see his Ch. 5.

Non-discriminating renegotiation is studied by McAfee and Schwartz (1994) in a monopoly context: A manufacturer faces multiple, privately informed retailers who compete with each other in the retail market. In evaluating the terms of an offered contract, a retailer need information on the terms offered to rival retailers. As McAfee and Schwartz show, this relationship among informed agents creates a scope for non-discrimination clauses when the manufacturer is unable to commit not to renegotiate the initial contracts. Our analysis complements theirs by
showing the effect of non-discriminating renegotiation in a competitive context. With competition, such renegotiation has an effect even if the privately informed individuals do not interact strategically.

In Asheim and Nilssen (1994), we study the effect of introducing renegotiation, without a non-discrimination clause, in an insurance monopoly. While the restriction to one round of renegotiation is, as argued in the Introduction, innocent if the renegotiation is non-discriminating, such a restriction must be lifted if renegotiation is allowed to be discriminating. An interesting topic for further research would be to study a competitive insurance market with renegotiation that is unrestricted with respect both to discrimination and to the number of renegotiation rounds. In the present analysis, we have found that non-discriminating renegotiation leads to an efficient outcome. It would seem natural to conjecture that allowing for unrestricted renegotiation would lead to incomplete separation of individuals by type, i.e., to an inefficient outcome. We leave this topic for future research.

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Appendix A

Formal definition of expected costs. By a now standard argument (Dasgupta and Maskin, 1986, note 10), a firm need no more than one contract for each customer type. Thus, we express the minimum expected costs associated with providing an individual, who is believed to be high-risk with probability \( \varphi \), the pair \( u \) of utility levels, as

\[
C(u^H, u^L; \varphi) = \min_{x} \varphi \left[ (1 - p^H) y(x^H_1) + p^H y(x^H_2) \right] + (1 - \varphi) \left[ (1 - p^L) y(x^L_1) + p^L y(x^L_2) \right];
\]
subject to:

\[(1 - p^H) x_1^H + p^H x_2^H = u^H,\]
\[(1 - p^L) x_1^L + p^L x_2^L = u^L,\]
\[(1 - p^H) x_1^H + p^H x_2^H \geq (1 - p^H) x_1^H + p^H x_2^H,\]
\[(1 - p^L) x_1^L + p^L x_2^L \geq (1 - p^L) x_1^L + p^L x_2^L,\]

where \( x = (x_1^H, x_2^H, x_1^L, x_2^L) \) and \( x_s^K = v(w_s^K) \) (i.e., \( w_s^K = y(x_s^K) \)), \( s \in \{1, 2\}, K \in \{H, L\}. \) The last two constraints are the incentive-compatibility (IC) constraints for high risks and low risks, respectively.

**Proof of Lemma 1.** In the solution to the above minimization problem, the low-risk IC constraint will not bind, since we exclude over-insurance by assumption. The high-risk IC constraint, however, will clearly bind. Moreover, the high risks will be fully insured, implying: \( x_1^H = x_2^H = u^H. \) Thus, we may insert the first constraint into the objective function; turn the third one into an equality, noting that its left-hand side equals \( u^H; \) and disregard the fourth one. This leaves us with the following rewriting of the cost function:

\[
C(u^H, u^L; \varphi) = \varphi y(u^H) + (1 - \varphi) \left[ (1 - p^L) y(x_1^L) + p^L y(x_2^L) \right],
\]

where:

\[
(1 - p^H) x_1^H + p^H x_2^H = u^H, \quad \text{and}
\]
\[
(1 - p^L) x_1^L + p^L x_2^L = u^L.
\]

We can now solve this equation system to find expressions for \( x_1^H \) and \( x_2^L, \) which can be inserted in the cost function to get the expression in the Lemma. Q.E.D.

**Lemma 2 (properties of the cost function).** Define the sets \( U^F := \{ u \in U : u^H = u^L \} \) and \( U^P := U \setminus U^F = \{ u \in U : u^H < u^L \}. \)

(i) \( \forall u \in U, \forall \varphi \in (0, 1), \partial C(u; \varphi)/\partial u^L > 0; \forall u \in U, \partial C(u; 1)/\partial u^L = 0. \)

(ii) \( \forall u \in U^F, \exists \varphi^* \in (0, 1) \) such that: \( \partial C(u; \varphi^*)/\partial u^H = 0, \partial C(u; \varphi)/\partial u^H < 0 \)

if \( 0 \leq \varphi < \varphi^*, \text{ and } \partial C(u; \varphi)/\partial u^H > 0 \) if \( \varphi^* < \varphi \leq 1. \)

(iii) \( \forall u \in U^F, \forall \varphi \in (0, 1), \partial C(u; \varphi)/\partial u^H > 0; \forall u \in U^F, \partial C(u; 0)/\partial u^H = 0. \)

(iv) \( \forall u \in U, \forall \varphi \in [0, 1], \partial^2 C(u; \varphi)/\partial (u^H)^2 > 0. \)

(v) \( \forall u \in U, \forall \varphi \in [0, 1], \partial^2 C(u; \varphi)/\partial u^H \partial u^L < 0; \forall u \in U, \partial^2 C(u; 1)/\partial u^H \partial u^L = 0. \)

(vi) \( \forall u \in U, \forall \varphi \in [0, 1], \partial^2 C(u; \varphi)/\partial u^H \partial \varphi > 0. \)
Proof. From differentiation in the expression for \( C(u; \varphi) \) in Lemma 1, we get

\[
\frac{\partial C}{\partial u^L} = (1 - \varphi) \frac{p^H(1 - p^L)}{p^H - p^L} \left[ y'(x^L_1) - \frac{p^L(1 - p^H)}{p^H(1 - p^L)} y'(x^L_2) \right],
\]

\[
\frac{\partial C}{\partial u^H} = \varphi y'(u^H) - (1 - \varphi) \frac{p^L(1 - p^L)}{p^H - p^L} \left[ y'(x^L_1) - y'(x^L_2) \right],
\]

\[
\frac{\partial^2 C}{\partial (u^H)^2} = \varphi y''(u^H) + (1 - \varphi) \frac{p^L(1 - p^L)}{(p^H - p^L)^2} \left[ p^H y''(x^L_1) + (1 - p^H) y''(x^L_2) \right],
\]

\[
\frac{\partial^2 C}{\partial u^H \partial u^L} = - \frac{p^L(1 - p^L)}{(p^H - p^L)^2} \left[ p^H y''(x^L_1) + (1 - p^H) y''(x^L_2) \right],
\]

\[
\frac{\partial^3 C}{\partial u^H \partial u^L \partial \varphi} = y'(u^H) + \frac{p^L(1 - p^L)}{p^H - p^L} \left[ y'(x^L_1) - y'(x^L_2) \right].
\]

Note that \( x^L_1 \geq x^L_2 \), since \( w^L_1 \geq w^L_2 \) and \( \nu' > 0 \), and that \( x^L_1 - x^L_2 \) if and only if \( u^H = u^L \). Moreover, \( u'' < 0 \) implies that \( y'' > 0 \). Now,

\[
y'(x^L_1) \geq y'(x^L_2) > \frac{p^L(1 - p^H)}{p^H(1 - p^L)} y'(x^L_2),
\]

where the first inequality follows from \( y'' > 0 \) and \( x^L_1 \geq x^L_2 \) and the second from \( p^H > p^L \). This implies that the square-bracketed term in the expression for \( \frac{\partial C}{\partial u^L} \) above is positive, and part (i) of the Lemma follows. Parts (iv) and (v) are straightforward from the above expressions for \( \frac{\partial^2 C}{\partial (u^H)^2} \) and \( \frac{\partial^2 C}{\partial u^H \partial u^L} \), respectively, since \( y'' > 0 \). Part (vi) follows from the above expression for \( \frac{\partial^2 C}{\partial u^H \partial \varphi} \), since \( y' > 0 \) and \( y'(x^L_1) \geq y'(x^L_2) \). Consider the above expression for \( \frac{\partial C}{\partial u^H} \). If \( u^H = u^L \), then \( x^L_1 = x^L_2 \) and the second term is zero. Since \( y' > 0 \), the expression is positive in this case, unless \( \varphi = 0 \) in which case it is zero; this proves part (iii). Suppose instead that \( u^H < u^L \), so that \( x^L_1 > x^L_2 \). Now, if \( \varphi = 0 \), then the expression for \( \frac{\partial C}{\partial u^H} \) is negative; if \( \varphi = 1 \), then it is positive. By part (vi) and continuity, there exists a unique \( \varphi^* \in (0, 1) \) such that it is zero at \( \varphi^* \), negative below \( \varphi^* \), and positive above \( \varphi^* \); this proves part (ii). Q.E.D.

Proof of Proposition 1. First, minimising expected costs amounts to maximising expected profits. To see this, note that the firm’s expected revenue per customer comes from the initial endowment \( w_0 \) and is independent of utility levels. Thus, for a given \( \varphi \), maximising profits means minimising costs. Second, argmin \( C(u; \varphi), u \in U(W_i) \), is a singleton because \( C \) is strictly increasing in \( u^L \) and strictly convex in \( u^H \) [Lemma 2(i) and (iv)]. Third, the implementation of the cost-minimising pair of utility levels follows from applying the results of Stiglitz (1977) to the present problem. Q.E.D.
Proof of Proposition 2. Given the properties of $C$ [Lemma 2(iv)], we need only perform a local analysis to establish our results.

(i) If $\frac{\partial C(\tilde{u}; \varphi)}{\partial u^H} \geq 0$, then an increase in the utility level for any customer type increases costs [Lemma 2(i) and (iv)], implying that $\tilde{u}$ will not be renegotiated. If $\frac{\partial C(\tilde{u}; \varphi)}{\partial u^H} < 0$, then an increase in the high-risk utility level lowers costs, implying that $\tilde{u}$ will be renegotiated.

(ii) By Lemma 2(i), it is never optimum to change the low-risk utility level, so $\tilde{u}^L = u^L$. Hence, the renegotiation affects only high-risk utility, and it is sufficient that $\tilde{u}^H(> \tilde{u}^H)$ satisfies $\frac{\partial C(u^H, \tilde{u}^L; \varphi)}{\partial u^H} < 0$ for $u^H < \tilde{u}^H$ and $\frac{\partial C(u^H, \tilde{u}^L; \varphi)}{\partial u^H} > 0$ for $u^H > \tilde{u}^H$. It follows from Lemma 2 (iii) and (iv) that such a $\tilde{u}^H$ exists, is unique, and satisfies $\frac{\partial C(\tilde{u}^H, \tilde{u}^L; \varphi)}{\partial u^H} = 0$. Q.E.D.

Proof of Proposition 3. Let $J^K \subseteq J$ be a subset of $k$ firms, $K \in \{H, L\}$ and $k \in \{h, l\}$, determined by the property that the firms in $J^K$ give a best offer to $K$-type individuals in Stage 1, i.e.: $u^K(W_j) = u^K(W_{ji}) > u^K(W_j), \forall i, i' \in J^K, j \in J \setminus J^K$.

(i) Existence is shown by construction. Let each firm form the following beliefs:

$$
\varphi_j = \left[ \mu_j - \left( \mu^L_0/\varphi' \right) \right] / \mu_j, \quad \text{if } j \in J^L \text{ and } \mu^L_0/\varphi' \leq \mu_j \leq \mu^H_0 + \left( \mu^L_0/\varphi' \right);
$$

$$
\varphi_j = 1, \quad \text{if } j \in J \setminus J^L \text{ and } 0 \leq \mu_j \leq \mu^H_0;
$$

$$
\varphi_j = 0, \quad \text{if } j \in J^L \text{ and } 0 \leq \mu_j < \mu^L_0/\varphi';
$$

$$
\varphi_j = \mu^H_0 / \mu_j, \quad \text{if } j \in J^L \text{ and } \mu_j > \mu^H_0 + \left( \mu^L_0/\varphi' \right),
$$

or if $j \in J \setminus J^L$ and $\mu_j > \mu^H_0$.

That is, if feasible, firm $j$ believes that it has been chosen by $1/\varphi'$ of the low risks if $j \in J^L$ and by no low risk if $j \in J \setminus J^L$. Proposition 2(ii) provides the rationale: The final utility level of low risks will equal their initial utility level. The problem facing a high risk is much more complicated as his final utility level depends not only on the initial utility level, but also on the high-risk fraction among those signing contracts with the same firm as he does.

Let $u^K_0 = u^K(W_j)$ for $j \in J^K, K \in \{H, L\}$, and $u_0 = (u^H, u^L)$. By Proposition 2(ii), low risks choosing a contract from some firm $j \in J^L$ will obtain the final utility level $u^L$, while low risks choosing a contract from some firm $j' \in J \setminus J^L$ will obtain less than $u^L$. Hence, sequential rationality of the low risks and Bayesian updating of the firms are satisfied if the low risks choose contracts such that they divide themselves evenly among firms in $J^L$.

Let $\varphi^*$ be the unique [by Lemma 2(ii) and (iii)] fraction of high risks such that, if some firm $j \in J^L \setminus J^H$ offers $u_j = (u^H_j, u^L_j)$, with $u^H_j < u^H$, and attracts a set of customers with a fraction $\varphi^*$ of high risks, then $u_j$ will be renegotiated to $u$.

If $\varphi_0 \leq \varphi^*$, then sequential rationality of the high risks is satisfied if all high risks choose contracts such that they divide themselves evenly among firms in $J^L$.
The reason is that, with high risks distributed this way, it follows from Bayesian updating that all firms $j \in J^L$ satisfy sequential rationality by renegotiating to $\tilde{u}$, with $\tilde{u}^H \geq u^H$. The final utility level of high risks would be at most $\tilde{u}^H$ with an alternative choice in $J^L$, at most $u^H$ with a choice outside $J^L$.

If $\varphi_0 > \varphi^* \geq 0$ and $J^H \cap J^L = \emptyset$, then sequential rationality of the high risks is satisfied if their contract choice is such that $(1 - \varphi_0)\varphi^*/(1 - \varphi^*)$ high risks divide themselves evenly among firms in $J^L$ and the remaining $(\varphi_0 - \varphi^*)/(1 - \varphi^*)$ divide themselves among firms in $J^H$. The reason is that, then, it follows, from Bayesian updating by the firms, that $\varphi_j = \varphi^*$ for all $j \in J^L$ and, from sequential rationality of the firms, that all high risks obtain the final utility level $u^H$, while any alternative choice would yield at most $u^H$.

If $\varphi_0 > \varphi^* \geq 0$ and $J^M := J^H \cap J^L \neq \emptyset$, with $m$ firms in $J^M$, then sequential rationality of the high risks is satisfied if their contract choice is such that $[(\varphi^* - \varphi_0)/(1 - \varphi_0)\varphi^*/(1 - \varphi^*)]$ high risks divide themselves evenly among firms in $J^L \setminus J^M$ and the remaining divide themselves among firms in $J^H$, with each $j \in J^M (\subseteq J^H)$ attracting at least $(1/\varphi)[(1 - \varphi_0)\varphi^*/(1 - \varphi^*)]$ high risks. The reason is that, then, it follows, from Bayesian updating by the firms, that $\varphi_j = \varphi^*$ for all $j \in J^L \setminus J^M$ and $\varphi_j \geq \varphi^*$ for all $j \in J^M$ and, from sequential rationality of the firms, that all high risks obtain the final utility level $u^H$, while any alternative choice would yield at most $u^H$.

(ii) In view of Proposition 2(ii), low risks choosing $j \in J \setminus J^L$ violates sequential rationality. Q.E.D.

Proof of Proposition 4. (Existence) If $u^H_{RS} \geq u^H_{LR}$, then $u_{MS} = u_{RS}$. Hence, no profitable deviation exists for any one firm. In particular, no cream skimming is feasible, since there is no cross-subsidization.

If $u^H_{RS} < u^H_{LR}$, then $u_{MS} = u_{LR}$, so that $\partial C(u_{MS}; \varphi_0)/\partial u^H = 0$. Suppose that some firm deviates with a cream-skimming offer $u_D = (u^H_D, u^L_D)$ such that $u^H_D < u^H_{MS}$ and $u^L_D > u^L_{MS};$ this is potentially profitable, since there is cross-subsidization at $u_{MS}: C(u_{MS}; 0) < R(0).$ Since $u^L_D > u^L_{MS}$, it follows from Proposition 3(ii) that $u_D$ attracts all the low risks in the population. Suppose that, in equilibrium of the resulting subgame, all high risks are also attracted such that, by Bayesian updating, the deviating firm believes that the fraction of high risks among its customers equals $\varphi_0$. Since $u^L_D > u^L_{MS}, u^H_D < u^H_{MS},$ and $\partial C(u_{MS}; \varphi_0)/\partial u^H = 0$, it follows from Lemma 2(iv) and (v) that $u_D$ is not renegotiation-proof at $\varphi_0$: $\partial C(u_D; \varphi_0)/\partial u^H < 0$. By Proposition 2(ii), it will be renegotiated to an offer $\tilde{u}_D$ such that $\tilde{u}_D^H = u^H_D$ and $\partial C(\tilde{u}_D; \varphi_0)/\partial u^H = 0$. Since $u^L_D > u^L_{MS}$, the latter property, together with Lemma 2(iv) and (v), imply that $\tilde{u}_D^H > u^H_{MS}$. Thus, with the beliefs specified in the proof of Proposition 3(i), all high risks will indeed choose a contract of the deviating firm, thereby satisfying sequential rationality. But it follows from $\tilde{u}_D^H > u^H_{MS}, \tilde{u}_D^L = u^L_D > u^L_{MS},$ and $C(u_{MS}; \varphi_0) = R(\varphi_0)$, that $C(\tilde{u}_D, \varphi_0) > R(\varphi_0);$ the deviation is not profitable.

(Uniqueness) We need to show that there do not exist other renegotiation-
proof, symmetric equilibria. Let \((W_{c1}, \ldots, W_{cn})\) be the symmetric candidate and let \(\tilde{u}_C\) be the corresponding candidate pair of initial utility levels. Consider Fig. 4, where the space \(U\) is depicted and divided into five regions. If \(\partial C(\tilde{u}_C; \varphi_0)/\partial u^H < 0\) [region (i)], then \(\tilde{u}_C\) is not renegotiation-proof, by definition. If \(\partial C(\tilde{u}_C; \varphi_0)/\partial u^H \geq 0\) and \(C(\tilde{u}_C; \varphi_0) > R(\varphi_0)\) [region (ii)], then aggregate profits are negative and it is better for at least one firm not to participate, so this cannot be a symmetric equilibrium. If \(\partial C(\tilde{u}_C; \varphi_0)/\partial u^H \geq 0\) and \(C(\tilde{u}_C; \varphi_0) < R(\varphi_0)\) [region (iii)], then aggregate profits are positive and it is possible for at least one firm, by offering slightly higher utility levels to both high risks and low risks, to increase its profit by attracting all individuals.

Suppose \(C(\tilde{u}_C; \varphi_0) = R(\varphi_0)\) and \(\tilde{u}_C^H > \max(u_{RS}^H, u_{LR}^H)\) [region (iv)]. Since aggregate profits are zero, there is at least one firm with non-positive profit. Now it is possible for this firm to deviate by a profitable cream-skimming attack. To see this, suppose the firm deviates with a cream-skimming offer \(u_D = (u_D^H, u_D^L)\) such that \(u < u_D^H\) and \(u_D^L > \tilde{u}_D^L\), where \((u_D^L - \tilde{u}_D^L)\) is arbitrarily small; this is potentially profitable, since there is cross-subsidization at \(\tilde{u}_C\): \(C(\tilde{u}_C; 0) < R(0)\). Since \(u_D^L > \tilde{u}_C^L\), it follows from Proposition 3(ii) that \(u_D\) attracts all the low risks in the population.

Suppose that, in equilibrium of the following subgame, all high risks are also

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\(^8\) Fig. 4 illustrates the case where \(u_{RS}^H > u_{LR}^H\). In the case where \(u_{RS}^H \leq u_{RS}^H\), region (v) is empty.
attracted, such that, by Bayesian updating, the deviating firm believes that the fraction of high risks among its customers equals \( \varphi_0 \). Since \( \partial C(\tilde{u}_C; \varphi_0) / \partial u^H > 0 \), \( u_D^H < \tilde{u}_C^H \), and \( u_D^H \) is arbitrarily close to \( \tilde{u}_C^H \), it follows from Lemma 2(iv) and (v) that \( u_D^H \) is either renegotiation-proof at \( \varphi_0 \) or is renegotiated to some pair \( \tilde{u} \) with \( \tilde{u}^H < \tilde{u}_C^H \). This implies that high-risk individuals prefer one of the non-deviating firms, and sequential rationality is not satisfied. Instead, only a subset of the high risks is attracted by the deviating firm and its offer is renegotiated to a pair \( \tilde{u}_C \) with \( \tilde{u}_C^H \geq \tilde{u}_C^H \). Since \( \tilde{u}_C \) can be made arbitrarily close to the attacked offer \( u_C^C \), the cost of providing low risks with a better offer can be made negligible, while the reduction in the fraction of high risks among the deviating firm's customers, compared to its customers in the candidate equilibrium, remains non-negligible. Hence, profit is positive; i.e., the deviation is profitable.

If \( C(\tilde{u}_C; \varphi_0) = R(\varphi_0) \) and \( u_{RS}^H > u_C^H \geq u_{LR}^H \) [region (v)], then there is cross-subsidization in the other direction: \( C(\tilde{u}_C; 1) < R(1) \). Now, it becomes profitable for the deviating firm to do cream-skimming towards the high risks: A deviating offer with a slight increase in high-risk utility and a decrease in low-risk utility will attract high risks only and earn a positive profit. Q.E.D.

References


9 The possibility \( \tilde{u}_C^H > \tilde{u}_C^H \) arises when the deviating firm's beliefs off the equilibrium path are such that, if more than the equilibrium amount of customers is attracted, then the firm believes that not all low risks have been attracted by its offer.