UNIVERSITETET I OSLO
Det matematisk-naturvitenskapelige fakultet

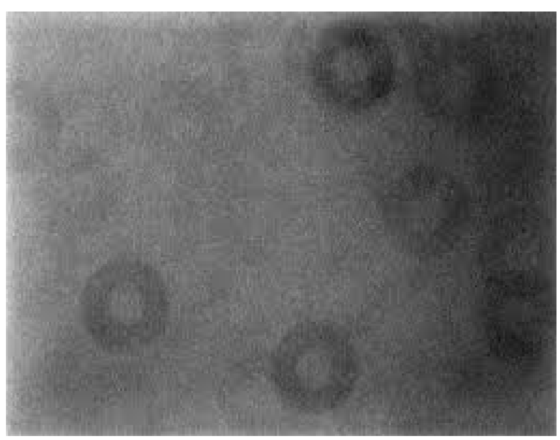
Slutteksamen i AST2210 — Observasjonsastronomi
Eksamensdag: Mandag 6 desember 2017
Tid for eksamen: 14:30 – 18:30
Oppgavesettet er på 6 sider.
Vedlegg: Ingen
Tillatte hjelpemidler: lommekalkulator, matematisk formelsamling,
Øgrim & Lian: Størrelser og enheter i fysikk og teknikk

Kontroller at oppgavesettet er komplett
før du begynner å besvare spørsmålene

Spørmålene kan besvares på enten bokmål, nynorsk eller engelsk. You may answer these questions in either Norwegian or English.

1. Kahoot time! Answer the following questions from lecture Kahoots. *All* correct answers must be marked in cases with more than one correct answer; omitted correct answers will be deducted by 1 point, as will incorrectly marked wrong answers.

   (a) What is the typical angular resolution one achieves from the ground without adaptiv optics?
   A) 0.01 arcsec B) 0.1 arcsec C) 1 arcsec D) 10 arcsec



   (b) Which CCD-related effect/correction is illustrated here?
   A) Blooming B) Bias C) Flat field D) Dark current

(c) What does a *power spectrum* measure?
A) Average wavelength of the signal
B) The signal amplitude as a function of wavelength
C) Average distance between peaks in the signal
D) Gaussianity

(d) Bayes theorem reads
A) $P(\theta|d) = P(d|\theta)P(\theta)/P(d)$
B) $P(\theta|d) = P(d|\theta)P(d)/P(\theta)$
C) $P(\theta|d) = P(\theta)P(d)/P(d|\theta)$
D) $P(d|\theta) = P(\theta|d)P(\theta)/P(d)$

(e) A prism splits light into separate wavelengths because...
A) the edges of a prism are flat
B) photons are both particles and waves
C) part of the light is reflected on the surface of the prism
D) the refractive index depends on wavelength

(f) Inflation predicts...
A) a flat universe
B) non-Gaussian fluctuations
C) a nearly scale-invariant spectrum, but $n_s > 1$
D) a weak background of random gravitational waves

(g) Image reconstruction algorithms (like MOMFBD) use...
A) adaptive optics to recover solar information
B) multiple frames of each object under varying seeing PSFs
C) the same seeing PSF for all frames
D) micro-lenses arrays to determine the wavefronts

(h) A birefringent material...
A) is often used for making glasses
B) increases the intensity of the incoming radiation
C) can be used to convert circular to linear polarization
D) has different speed of light in two orthogonal directions

(i) For diffraction with an infinitely long slit with width a, where does one see the first minimum?
A) $\theta = \lambda/a$
B) $\theta = a/\lambda$
C) $\theta = (\lambda/a)^2$
D) $\theta = \sqrt{a/\lambda}$

(j) When does "Star Wars – the last jedi" premiere?
A) November 13th
B) November 20th
C) December 13th
D) December 20th

2. Give <u>short</u> answers to the following questions:

   (a) What are the two main advantages of observing from space compared to ground?

   (b) Explain *astigmatism* both in words and with a ray-tracing sketch.

   (c) Define "bias", "dark current" and "flat field". In each case, are these multiplicative or additive effects?

   (d) Draw a sketch of a CCD, both seen face-on and in profile. Mark the most important elements.

   (e) Sketch typical E and B mode polarization patterns in terms of linear polarization. What is the curl of a E-mode field?

   (f) Given the Fourier expansion of a function $f(t)$,

   $$f(t) = \sum_k a_k e^{ikt}, \tag{1}$$

   derive a formal expression for a_k in terms of $f(t)$.

   (g) Describe and sketch briefly how a Michelson interferometer work. What observational signature distinguishes a positive detection from a negative signal?

3. The multivariate Gaussian distribution and CMB likelihood:

   (a) Write down the general expression for a multivariate Gaussian distribution with mean equal to μ (which is a vector) and covariance matrix S.

   (b) Draw a *rough* sketch of this distribution for the following two two-dimensional cases:

$$A) \quad \mu = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, S = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}, \tag{2}$$

$$A) \quad \mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, S = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 4 \end{bmatrix}, \tag{3}$$

   What does μ quantify? What do the diagonal elements of S quantify? What do the off-diagonal elements in S quantify?

   (c) The pixel-pixel CMB temperature fluctuation covariance matrix reads

$$S_{ij}^{\mathrm{cmb}} = \frac{1}{4\pi} \sum_{\ell=\ell_{\mathrm{min}}}^{\ell_{\mathrm{max}}} (2\ell+1)(b_\ell p_\ell)^2 C_\ell P_\ell(\cos\theta), \tag{4}$$

   where C_ℓ is the CMB power spectrum, $P_\ell(\cos\theta)$ is the Legendre polynomial of order ℓ, evaluated for the angle θ between pixels i and j, b_ℓ is the Legendre transform of the instrumental beam, p_ℓ is the pixel window, and the sum runs over all multipole moments ℓ between some bandwidth defined by ℓ_{min} and ℓ_{max}. Describe in words what C_ℓ, b_ℓ and p_ℓ means physically, and what each one measures.

   (d) Let $\ell_{\mathrm{max}} = \ell_{\mathrm{min}} = \ell_0$, such that the sum only runs over a single multipole moment. Draw a rough sky map of a possible CMB realization matching this covariance matrix for $\ell_0 = 0$, $\ell_0 = 1$ and $\ell_0 = 2$. Use '+', '-' and '0' to indicate the positions of maxima, minima and zero-levels, respectively. Ignore amplitudes, and focus only on the geometrical structure of the fluctuation maps.

   (e) What happens to the maps sketched in d) if you increase C_ℓ by a factor of 4, but leaves everything else the same?

4. Diffraction through a prism

   (a) What does Fermat's principle state?

   (b) Write down Snell's law for diffraction across a boundary between two media with refractive indices n_1 and n_2. How does n_i relate to the speed of light, and how does this relate to Fermat's principle?

   (c) Consider an equilateral prism with opening angle $A = 60°$. Sketch the path of a light ray through this prism for the special case where the angle of the incident light ray, α, is equal to the emergent light ray. Mark the angle α in the figure. Assume $n_1 = 1$ outside the prism, and $n_2 = n$ inside the prism.

   (d) Let α' be the angle between the light ray inside the prism and the normal to the prism edge. Show that $\alpha' = A/2$.

   (e) Let θ be the *total* angle between the emergent and incoming rays through the prism. Find α given by A and θ.

   (f) Show that

   $$\sin\left(\frac{\theta + A}{2}\right) = n \sin\left(\frac{A}{2}\right) \tag{5}$$

   (g) Assume that the refractive index of the prism as a function of wavelength is given by

   $$n(\lambda) = K + \left(\frac{K_2}{\lambda}\right)^2. \tag{6}$$

   Show that the *angular dispersion* of the prism is given by

   $$\frac{d\theta}{d\lambda} = -\frac{4K_2^2}{\lambda^3}\frac{\sin(A/2)}{\cos\alpha}. \tag{7}$$

   (h) Is red or blue light diffracted the most?

(i) Assume $n = 1.6$ for red light ($\lambda = 700\text{nm}$). What is α for our case?

(j) Assume $K_2^2 = 10^6\text{nm}^2$. What is the angular difference between the emergent rays for red and blue ($\lambda = 400\text{nm}$) light?

PS! Question 1j ("Star Wars" premiere date) will not count towards the final score ;-))