Nonlinear absorption of surface acoustic waves by composite fermions

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Abstract. – Absorption of surface acoustic waves by a two-dimensional electron gas in a perpendicular magnetic field is considered. The structure of such system at the filling factor \( \nu \) close to 1/2 can be understood as a gas of composite fermions. It is shown that the absorption at \( \nu = 1/2 \) can be strongly nonlinear, while small deviation form 1/2 will restore the linear absorption. The study of nonlinear absorption allows one to determine the force acting upon the composite fermions from the acoustic wave at turning points of their trajectories.

Introduction. – The interaction with surface acoustic waves (SAW) is an important tool in the study of two-dimensional electron gases (2DEG) in various regimes [1], in particular, under conditions of the fractional quantum Hall effect [2, 3]. As is well known, the two-dimensional electron system exhibits a metallic phase in strong magnetic fields, near the half-filled Landau level, \( \nu = 1/2 \). This phase has been understood as a gas of so-called composite fermions (CFs). This concept formulated in the framework of the Chern-Simons theory [4] has appeared successful to explain the acoustic properties of 2DEG [5–7] and to extract quantitative information about CFs’ trajectories.

So far, both experimental and theoretical studies have concentrated on the linear response regime, which is valid for low sound intensities. On the other hand, a very peculiar nonlinear response of 3DEG to acoustic waves (AW) has been predicted [8] and experimentally observed in InSb [9] and in Ga [10]. A striking feature of this nonlinear response is its anomalous sensitivity to an external magnetic field [11]. In fact, the nonlinear response appears suppressed by so weak a magnetic field that does not affect linear absorption at all [10]. The theory of nonlinear response in external magnetic field has been elaborated in [12,13], a comprehensive review is given in [14]. What is important is that an external magnetic field provides an intrinsic scale to measure the force acting upon electrons from the AW.

It is therefore natural to investigate whether the nonlinear effects observed for electrons are also present in the two-dimensional CF liquid, and what new information they provide. This is the aim of the present paper.

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Background and qualitative discussion. — To begin with, let us recall the qualitative picture of nonlinear acoustic response of 3DEG. If wave vector $\mathbf{q}$ is much greater than the electron mean free path $\ell$, then an electron traverses many acoustic periods before being scattered. Consequently, it contributes to the absorption as a free particle. Since for typical electron the $\mathbf{q}$-projection of the electron velocity $\mathbf{v}$, $v_q \equiv (\mathbf{q} \cdot \mathbf{v})/v_s$, is much greater than the sound velocity, $v_s$, it “feels” a rapidly oscillating field of the acoustic wave, the contribution to the absorption being small. As result, only a small electron group with $v_q \approx v_s$ appears important. These resonant electrons determine linear absorption. The situation is very similar to the well-known Landau damping of plasma waves \cite{15}. Turning to the nonlinear effects, one has to discuss the dynamics of the resonant electrons in the finite-amplitude field produced by the AW. As a result, a part of resonant electrons with small $|v_q - v_s|$ become trapped by the acoustic field. Moving fully synchronously with the AW, the trapped electrons do not contribute to the absorption, the total absorption being decreased. This is the reason for the nonlinear behavior of the absorption in the absence of the external magnetic field. The situation is illustrated in fig. 1(a).

Now let us turn to the linear absorption in the presence of the magnetic field. The most interesting behavior takes place at

$$ q^{-1} \ll r_c \ll \ell, \quad (1) $$

where $r_c = v/\omega_c$ is the classical cyclotron radius while $\omega_c$ is the cyclotron frequency. The electron orbit embeds many acoustic wavelengths, as is shown in the right panel of fig. 1. Again, only vicinities of the turning points 1 and 2 are important because only in these regions $v_q$ can be of the order of $v_s$. Correlation of acoustic phases at the turning points leads to well-known geometric oscillations \cite{16,17}. For composite fermions geometric oscillations have also been clearly observed \cite{2}. The effect of trapping in an external potential has been discussed in \cite{18}.

To preserve the picture of nonlinear absorption discussed above, the force component along $\mathbf{q}$, $e v_q B$, should be much less than the typical force from the acoustic wave, $F_0 = q \Pi_0$. Here $\Pi_0$ is the amplitude of the AW-induced potential profile. Otherwise, some electrons appear de-trapped, and the absorption returns to the linear one. The situation near a turning point in this case is illustrated in fig. 1(b). Comparing the forces we arrive at the estimate for the critical magnetic field, $B_c = F_0/v_B$. Measuring $B_c$ one immediately finds the force $F_0$ near the turning points. To reproduce the above-discussed scheme for CFs is the main idea behind this work.
Now let us turn to the case of interests namely to 2DEG interacting with SAW. In many experimental situations one can neglect the deformational interaction due to strains created by SAW in the plane of 2DEG, and the force experienced by the electrons is purely electromagnetic. It is created by the external magnetic field and by the AC electric field of the surface acoustic wave. The latter arises because of piezo-electric effect in the materials used to create the 2DEG (GaAs-AlGaAs heterostructures). The “bare” piezo-electric field is parallel to the propagation direction, $q \parallel \hat{x}$. Here $\hat{x}$ is the unit vector along the $q$-direction. The effective electric field is then given by $E(x,t) = E_0 \sin \xi = -\nabla \Phi$ with $\Phi = \Phi_0 \cos \xi$. Here $\Omega$ is the SAW frequency, and $\xi = qx - \omega t$ is the wave coordinate; $E_0 \parallel q \parallel \hat{x}$. By $\Phi$ we mean the screened electrostatic potential. The relationship between this and the bare potential is discussed in ref. [3]. The CF-picture arises after the Chern-Simons transformation which attaches an even number of flux quanta of a fictitious magnetic field to each electron. The resulting particles are called the composite fermions. For definiteness we will consider the case of two attached flux quanta, appropriate for the state around $\nu = 1/2$. At the mean-field level, the composite fermions will then feel an effective magnetic field $B^* = B + b$, where $B$ is the external (real) magnetic field and $b = -2\phi_0 n$ is the Chern-Simons field. Here $\phi_0 = 2\pi \hbar/e$ is the magnetic flux quantum and $n$ is the electron density which includes the density modulation by SAW, $n = n_0 + \delta n$. Correspondingly, there will be a modulation of the Chern-Simons field, $b = b_0 + b^{ac}$. We will let $B^* = B + b_0$ represent the average effective field, so that the total magnetic field acting on the composite fermions is $B^* + b^{ac}$.

In addition, the motion of the CFs will create an AC electric Chern-Simons field which is given by $e^{ac} = (2\phi_0/e)[\hat{z} \times j]$. The $y$-component of $e^{ac}$ is given by the $x$-component of the current. We can find this from the density modulation using charge conservation. Assuming that the density modulation is $\delta n = (\delta n)_0 \cos \xi$, we get $j_x = e v_t \delta n$. We will later see that this assumption is justified. We then have $e^{ac}_y = 2\phi_0 v_t \delta n$. This is true under the assumption of a harmonic density perturbation and as long as there is no net current through the sample.

The $x$-component of the CS electric field can, as explained in ref. [19], be considered as a potential field. The corresponding potential will be denoted $\Psi$, and in the following we will calculate the response to this field. However, it can be shown (see below) that in the regime of strong nonlinearity the external electric field is not important at all.

**Theory.** Below we employ the random phase approximation (RPA). To calculate the nonlinear absorption by CFs we employ the Boltzmann equation for the CF distribution function, $f$, considering CFs as particles with charge $-e$ and mass $m$. Consequently, the classical Hamiltonian is

$$\mathcal{H} = (P + eA)^2/2m - e\Psi,$$

(2)

where $P$ is the canonical momentum (the kinematic momentum is $p = P + eA$), $A$ is the vector potential while $\Psi$ is the total scalar potential as explained above. The vector potential consists of two parts. One emerges from the static external effective magnetic field $B^*$, and one from the AC Chern-Simons field that is created by the SAW-induced density modulation. At $\nu = 1/2$, $B^* = 0$, and the magnetic field is then $b^{ac} = 2\phi_0 [n_0 - (2\pi \hbar)^{-2} \int d^2 P f]$. It is convenient to split the distribution function as $f = f_0(\mathcal{H}) + f_1$, where $f_0$ is the Fermi function. Then the Boltzmann equation for $f_1$ is

$$\partial f_1/\partial t + \nabla_P \mathcal{H} \nabla_P f_1 - \nabla_\mathcal{H} \mathcal{H} \nabla f_1 + f_1/\tau = -(\partial \mathcal{H}/\partial t)(\partial f_0/\partial \mathcal{H}).$$

(3)

Here we use the relaxation time approximation $-f_1/\tau$ for the collision operator which significantly simplifies the calculations. As is emphasized in [6], this leads to charge non-conservation. However, this is not expected to give any qualitative change at $q\ell \gg 1$. 


(see, e.g., [5]). It should be noted that the Hamiltonian (2) is written in terms of the AC Chern-Simons magnetic field $b^c$. The latter must be expressed through the density modulation as an integral over the distribution function. The Boltzmann equation (3) is then in reality a complicated integro-differential equation for the non-equilibrium distribution function. It is easy to show, however, that the main contribution to the density modulation comes from the equilibrium part $f_0(\mathcal{H})$, so that in calculating $f_1$ we can approximate the density modulation with $\delta n^{(0)}$ coming from $f_0(\mathcal{H})$. Indeed, using the fact that in all the region of acoustic amplitudes $e\Psi \ll \epsilon_F$, where $\epsilon_F$ is the Fermi energy, we can then expand $f_0(\mathcal{H})$ around the point $\mathcal{H} = p^2/2m$. The lowest-order term, $\delta n^{(0)}$, is estimated as

$$\delta n^{(0)} = -e\Psi (2\pi\hbar)^{-2} \int d^2 p (\partial f_0/\partial \mathcal{H}) |_{\mathcal{H}=p^2/2m} = g e\Psi .$$ 

(4)

Here $g = m/2\pi\hbar^2$ is the density of states per spin (as usual, we assume the 2DEG to be fully spin-polarized). Then we can solve eq. (3) for $f_1$ with the assumption that $\delta n = \delta n^{(0)}$, and come back to show that the non-equilibrium correction coming from $f_1$ is small compared to $\delta n^{(0)}$.

We will first consider the case $B^* = 0$. Proceeding to the solution, we note that in the resonant region $v_y \approx v_F$ and $v_x \approx s \ll v_F$. The magnetic force then points mainly in the $x$-direction, and the main part of this will be given by $b^c v_y$. This may be combined with the potential $\Psi$ to give the effective potential $\Pi$ such that $-\nabla \Pi = (\pm v_F b^c + E) \hat{z}$, the sign being + for particles with $v_y > 0$ and − for $v_y < 0$. Using the approximation $\delta n \approx \delta n^{(0)}$ and assuming the electric potential to have the form $\Psi = \Psi_0 \cos \xi$, we can find the explicit expression for $\Pi$, $\Pi(\xi) = \Pi_0 \psi(\xi)$, where

$$\psi(\xi) = \cos(\xi + \theta) , \quad \Pi_0 = \Psi_0 \sqrt{1 + \alpha^2} , \quad \alpha = 2mv_F/q\hbar , \quad \theta \equiv \arctan \alpha .$$ 

(5)

The equation for $f_1$ can then be written as

$$s(\partial f_1 / \partial \xi) + \psi'(\partial f_1 / \partial s) + af_1 = aU ,$$

(6)

with

$$s = (v_x - v_y)/\tilde{v} , \quad \tilde{v}^2 = e\Pi_0/m , \quad U = -\tau e\omega (\partial \Pi / \partial \xi)(\partial f_0 / \partial \mathcal{H}) , \quad a = (q\tilde{v}\tau)^{-1} .$$ 

(7)

The dimensionless parameter $a$ has a clear physical meaning. Indeed, $\tilde{v}$ is just a typical velocity of the particles trapped in the potential $\Pi(\xi)$, while $\omega_0 \equiv q\tilde{v}$ is their typical oscillation frequency. Since each scattering event rotates the particle momentum and leads to its escape from the resonant group, nonlinear behavior exists only if $\omega_0 \tau \gg 1$, or $a \ll 1$. Thus $a$ is the main parameter responsible for nonlinear behavior. Equation (6) is easily solved by the method of characteristics, giving the equations

$$d\xi/s = ds/\psi'(\xi) = df_1/a(U - f_1) .$$

(8)

Solving first the equation for $s$ and $\xi$, we obtain

$$s^2 = 2(\psi + \eta) ,$$

(9)

where $\eta$ is a constant of integration. It has the meaning of a dimensionless energy for the motion in the $z$-direction. The remaining equation for $f_1$ and $\xi$ is then

$$s(df_1/d\xi) \pm af_1 = \pm aU .$$

(10)
The sign is + for particles with \( s > 0 \) and − for particles with \( s < 0 \). Equation (10) requires boundary conditions. For the untrapped particles, we use periodic boundary conditions, while for the trapped ones we require

\[
f^+_{1, a}(\xi_1) = f^+_{1, a}(\xi_2), \quad f^+_{1, a}(\xi_2) = f^+_{1, a}(\xi_1),
\]

where \( \xi_1 \) and \( \xi_2 \) are the turning points to the left and right of \( \xi \), respectively. The result is

\[
f^+_{1, a}(\xi) = \left[ e^{a \int_0^{2\pi} \frac{d\xi'}{s(\xi')}} - 1 \right]^{-1} \int_\xi^{\xi+2\pi} d\xi' \frac{\psi''(\xi')}{s(\xi')} e^{a \int_\xi^{\xi+2\pi} \frac{d\xi'}{s(\xi')}}
\]

(12)

\[
f^+_{1, a}(\xi) = \left[ \sinh \left\{ a \int_{\xi_1}^{\xi_2} d\xi' \frac{d\xi''}{s(\xi'')} \right\} \right]^{-1} \int_{\xi_1}^{\xi_2} d\xi' \frac{\psi''(\xi')}{s(\xi')} \coth \left\{ a \int_{\xi_1}^{\xi_2} d\xi' \frac{d\xi''}{s(\xi'')} \right\} e^{a \int_{\xi_1}^{\xi_2} \frac{d\xi'}{s(\xi')}}
\]

(13)

where we have defined

\[
f_1(\xi) = -e\Pi_0 \frac{\partial f_0}{\partial H} \frac{\omega}{\omega_0} \tilde{f}(\xi), \quad \omega_0 = q\bar{v}.
\]

The expression for \( \tilde{f} \) can be obtained by changing the sign of \( s \). It is easy to check explicitly, that as \( s \gg a, f_1 \propto s^{-1} \). Consequently, only resonant particles are important.

In the following we will consider the limiting case of strong nonlinearity \( a \ll 1 \). In this limit we can expand the distribution functions (12) and (13) in powers of \( a \) and then compute the non-equilibrium density modulation \( \delta n(1) \). As a result, \( \delta n(1)/\delta n(0) \approx a\omega_0 v_F / v_F \) which is small (\( v_F / v_F \) is of order 1/100 in typical experiments. The approximation could break down if \( a \) were very large). Similarly, we may calculate the current in the \( y \)-direction due to \( f_1 \) in order to determine the \( x \)-component of the CS electric field. Again, this contribution appears proportional to \( a \) and small. This means that any higher harmonics in the CS electric field will be suppressed by a factor \( a \).

**Nonlinear absorption.** We are now able to calculate the absorbed power per length of cross-section, \( P \), from the acoustic wave. It is given by

\[
P = \int \frac{d^2 p}{(2\pi\hbar)^2} \langle \hat{H}_p \rangle = (e\Pi_0 / 2\pi)^2 (v_F / v_F) g\omega P,
\]

(14)

where \( \langle \cdot \rangle \) denotes average over the period of the acoustic wave, while

\[
P = -\sum_{\pm} \int_0^{2\pi} d\eta \int d\eta' \frac{\psi''(\xi)}{|s(\xi)|} \tilde{f}_{1, \pm}(\xi, \eta).
\]

(15)

When evaluating \( P \) one must include contributions from both trapped and untrapped particles in all directions (i.e. for both signs \( \pm \)), and adjust the range of \( \eta \) accordingly. The contributions from particles with different sign of \( s \) will cancel in the order \( \omega^0 \) terms, and the leading contribution will be to order \( a \). After rather tedious calculations we arrive at the result

\[
P = C(e\Pi_0 / 2\pi)^2 (v_F / v_F) g\omega a,
\]

(16)

where \( C \sim 1 \) is some numerical factor that is found from numerical integration. The particles just above and below the threshold of trapping gives similar contributions to the absorption. This contrasts with the case of DC conductivity where there is an abrupt change in the contribution when trapping occurs, leading to a simplified formula relating the conductivity to the number of trapped particles [20].
Effect of a weak effective magnetic field. — Let us now turn to the case where the effective external magnetic field, $B^*$, is not exactly zero, that is, where the filling fraction $\nu$ is close to but not equal to $1/2$. In the resonant region, the Hamiltonian (2) is then changed to

$$\mathcal{H}' = \mathcal{H} + e\nu F B^* x.$$  \hspace{1cm} (17)

Here and in the following we write the expressions for particles with $\nu > 0$, for $\nu < 0$ the sign of the last term must be changed. We still write $f = f_0(\mathcal{H}) + f_1$, but the force acquires a new contribution $F = e\mathcal{V} - e\nu B^*$. The Boltzmann equation (6) is then transformed into the form

$$s(\partial f_1 / \partial \xi) + [b - \psi'(\xi)](\partial f_1 / \partial s) + a f_1 = aU, \quad b = e\nu B^*/q\Pi_0.$$  \hspace{1cm} (18)

The physical interpretation of the parameter $b$ is the ratio of the magnetic force from the external magnetic field to the force from the modified electrostatic potential $\Pi(\xi)$. Solving the equations for the characteristics we get instead of eq. (9),

$$s^2/2 = \psi(\xi) - b\xi + \eta.$$  \hspace{1cm} (19)

Here we write $\xi$ for $\xi - \theta$, i.e. we translate the origin of the coordinates to adjust to the modified potential. In the case $b \gg 1$, in which we are most interested, there will be only one turning point, which we will denote $\xi_0$, and all particles will be untrapped. This will then be the only point where $s = 0$, and we get $\eta = b\xi_0 - \psi(\xi_0)$. The particles will then come from $\xi = -\infty$ at $t = -\infty$, turn at $\xi_0$ and return to $\xi = -\infty$ at $t = \infty$. Of course, this is not the true trajectory of the particles, but it approximates the true trajectory close to the turning point. The nonequilibrium distribution function is

$$f_1(\xi, \eta) = a \int_{-\infty}^{\xi} d\xi' \frac{U}{s(\xi, \eta)} e^{-a \int_0^{\xi} \frac{\eta''}{s}}.$$  \hspace{1cm} (20)

Here the integral over $\xi$ is to be taken along the trajectory defined by the constant $\eta$. That is, for particles that have passed their turning point (and thus have $s < 0$) we must integrate up to $\xi_0$ and then back to $\xi$ remembering the change of sign of $s$. When $a \ll 1$ and $b \gg 1$ the argument of the exponential is very close to 0 in the region of effective interaction near the turning point. Following ref. [11] we will therefore set the exponential to 1 in this region. Changing variable from the integration constant $\eta$ to the position of the turning point, $\xi_0$, and expanding $1/s$ in powers of $1/b$ to lowest order we obtain for the total absorption

$$P = (\pi/2)(e\Pi_0/2\pi)^2(v_b/\nu F)g\omega.$$  \hspace{1cm} (21)

This is equal to the linear absorption, $P_0$, in the absence of an external magnetic field, which can be directly calculated from the linearized Boltzmann equation. Indeed, magnetic field restores linear absorption.

Discussion. — The possibility to restore linear absorption at

$$B^* = B - B_{1/2} \geq B_c \equiv q\Pi_0/\nu F, \quad B_{1/2} \equiv B \big|_{\nu = 1/2},$$  \hspace{1cm} (22)

allows one to determine directly the $q$-component of the force, $q\Pi_0$, acting upon CF at the turning points. Since the effective force is a complicated function of SAW intensity and frequency (a detailed theoretical study of this force will be published elsewhere), the relationship (22) seems useful. On the other hand, the product $q\Pi_0$ can be directly determined from eq. (21) knowing the measured absorbed power, $P$. Consequently, a way to check the above concept is first to reach nonlinear behavior at $B = B_{1/2}$, then restore the linear behavior by changing magnetic field by the quantity $\geq B_c$, and finally measure the absorbed power without changing SAW intensity.
Conclusion. — It is shown that absorption of SAW by 2DEG will show a pronounced nonlinear behavior at $\nu = 1/2$. Small deviations from $\nu = 1/2$ will restore linear absorption. Studies of these deviations allow one to determine the effective force acting upon composite fermions.

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