Runup of interfacial waves on a plane beach

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Abstract

We study theoretically the runup and drawdown of non-breaking long waves in a density-stratified fluid system. Idealizing the density profile by a stairway consisting of N discrete layers, a multi-layer long-wave model based on the Lagrangian description is developed. The model equations are solved numerically to study the splitting of initial disturbances, the reflected waves from variable topography, and the movement of each interfacial tip along a sloping beach. Example results from a three-layer case are reported.

Introduction

In fluids with density stratification, gravity waves may exist at the subsurface density interfaces. Remote-sensing and in-situ observations have reported evidence of the occurrence of internal gravity waves in both natural waters and the atmosphere. Understanding the shoaling process of internal waves is one of the key elements in the study of mixing and transport mechanisms in the ocean. It is also important to the design of many coastal structures. In this study we investigate the evolution of two-dimensional long waves in two-layer and three-layer fluid systems. We simulate the splitting, propagation, runup, and reflection of internal waves over a sloping topography up to the wave breaking point.

Theoretical model

We consider a variable-density fluid system in which the continuous stratification across the water column is idealized by a stairway density profile consisting of N discrete layers. Applying the common long-wave assumption, for the n-th layer the depth-integrated governing equations in the Lagrangian coordinates are obtained:

$$\begin{bmatrix} x_a^{[n]} & y_a^{[n]} \\ x_b^{[n]} & y_b^{[n]} \end{bmatrix} \begin{Bmatrix} x_{tt}^{[n]} \\ y_{tt}^{[n]} \end{Bmatrix} = -g \begin{Bmatrix} \zeta_a^{[n]} \\ \zeta_b^{[n]} \end{Bmatrix} - g \sum_{i=1}^{n-1} \frac{\rho^{[i]}}{\rho^{[n]}} \begin{Bmatrix} \zeta_a^{[i]} - \zeta_a^{[i+1]} \\ \zeta_b^{[i]} - \zeta_b^{[i+1]} \end{Bmatrix},$$

where g is the gravitational acceleration,

$$\zeta^{[n]} = \zeta^{[n+1]} + \frac{h^{[n]}}{\Delta^{[n]}}$$

denotes the position of the upper interface, and $h^{[n]} = c_{\text{up}}^{[n]} - c_{\text{dn}}^{[n]}$ represents the initial layer thickness. In addition, $\zeta^{[N+1]} = -h$ is the water depth. In the absence of density variation, the above equations recover the Lagrangian shallow water equations for surface waves. We solve the model equations by an explicit finite-difference scheme on uniform grids. The time derivative term is approximated by the second-order three-point discretization and the spatial derivatives are evaluated using the five-point formulas. Our numerical scheme has been tested using the existing analytical solutions for runup of surface waves and the propagation of an internal solitary wave in a two-layer system.

Numerical examples

We pursue two-dimensional problems by examining a specially designed initial static deformations in a three-layer stratified system over a uniformly sloping beach. The initial condition is a pair of depression soliton-shaped deformations, which corresponds to the common mode 1 waveform in gravity internal waves. We shall not dwell on the generation mechanism of the initial deformations as our primary goal is to study the detailed propagation, runup, and drawdown of general incident interfacial waves.

Figure 1 plots the temporal variations of interface displacements at the tips. The free surface is initially calm and the density ratio is $\rho^{[1]} : \rho^{[2]} : \rho^{[3]} = 0.99 : 1 : 1.01$. Despite having identical initial amplitudes at both interfaces, it is interesting to see that the runup and drawdown of the second interface, $\eta^{[2]}$, are stronger than those of the bottom one, $\eta^{[3]}$. Figure 2 shows select snapshots of two density interfaces for the same case. In the figure, we identify a pair of mode 2 right-going waves generated by the initial mode 1 disturbances. This complication is due to the evolving of the initial static deformations into two-way propagating waves.

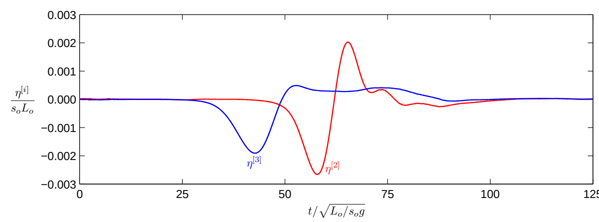

Figure 1: Runup and drawdown of internal tips on a uniformly sloping beach. $\eta^{[i]}$ denotes the vertical displacement at the interfaces.

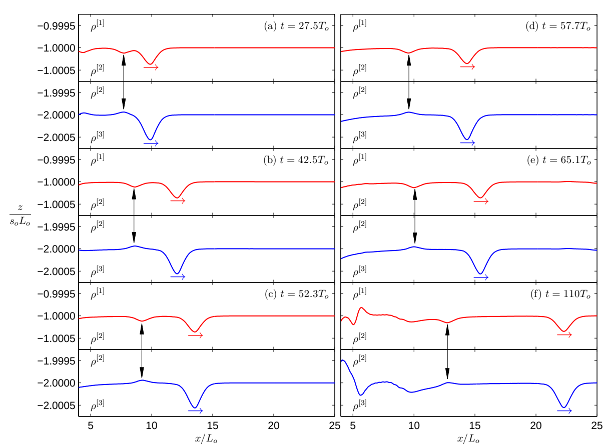

Figure 2: Snapshots of subsurface density interfaces at select times. Horizontal arrows indicate the propagation of mode 1 waves. Double arrows highlight a pair of mode 2 waves.

More numerical results will be presented and discussed at the Symposium.