The combined effect of rotation and variable depth on internal solitary waves.

Roger Grimshaw[1], *Karl Helfrich*[2], *Chuncheng Guo*[3] *and Vasyl Vlasenko*[3]
[1]Department of Mathematical Sciences, Loughborough University, U.K.
[2] Department of Physical Oceanography, Woods Hole Oceanographic Institution, U.S.A.
[3] School of Marine Science and Engineering, Plymouth University, U.K.

Internal solitary waves commonly observed in the coastal ocean are often modelled by the Korteweg-de Vries (KdV) equation. In a reference frame moving with the linear long wave speed c, the KdV equation is

$$\eta_t + \mu\eta\eta_x + \lambda\eta_{xxx} = 0\,. \tag{1}$$

Here $\eta(x,t)$ is the amplitude of the linear long wave mode $\phi(z)$ corresponding to a linear long wave with phase speed c. The coefficients μ and λ are determined by certain integrals involving the modal function. Since oceanic internal waves are often observed to propagate for long distances over several inertial periods, the effect of the Earth's background rotation is potentially significant. The relevant extension of the KdV equation (1) is then the Ostrovsky equation,

$$\{\eta_t + \mu\eta\eta_x + \lambda\eta_{xxx}\}_x = \gamma\eta\,. \tag{2}$$

The background rotation is represented by the coefficient $\gamma = f^2/2c$ in the absence of a background current, where f is the Coriolis parameter. For oceanic internal waves $\lambda\gamma > 0$, and then it is known that equation (2) does not support a steady solitary wave solutions. The simplest explanation is that then the additional term on the right-hand side of (2) removes the spectral gap in which solitary waves exist for the KdV equation, and hence no solitary waves are expected to occur. Recently Grimshaw and Helfrich (2008), Grimshaw and Helfrich (2012) and Grimshaw et al. (2013) have shown through a combination of asymptotic theory, numerical simulations and a laboratory experiment hat the long-time effect of rotation is the destruction of the initial internal solitary wave by the radiation of small-amplitude inertia-gravity waves, and the eventual emergence of a coherent steadily propagating nonlinear wave packet, see figure 1.

However, in the ocean, internal solitary waves are often propagating over variable topography, and this alone can cause quite dramatic deformation and transformation of an internal solitary wave, see the recent review by Grimshaw et al. (2010). Hence, here we examine the combined effects of background rotation and variable topography. Then the Ostrovsky equation (2) is replaced by a variable coefficient Ostrovsky equation whose coefficients depend explicitly on x. We will present some numerical simulations of this equation, together with some analogous simulations using the MITgcm, for a certain cross-section of the South China Sea. A typical case for the latter is shown in figure 2, which demonstrates that the effect of rotation is to induce a secondary trailing wave packet, induced by enhanced radiation from the leading wave.

References

Grimshaw, R., Helfrich, K., and Johnson, E. (2013). Experimental study of the effect of rotation on large amplitude internal waves. *Phys. Fluids*, 25:056602.

Grimshaw, R. and Helfrich, K. R. (2008). Long-time solutions of the ostrovsky equation. *Stud. Appl. Math.*, 121:71–88.

Grimshaw, R. and Helfrich, K. R. (2012). The effect of rotation on internal solitary waves. *IMA J. Appl. Math.*, 77:326–339.

Grimshaw, R., Pelinovsky, E., Talipova, T., and Kurkina, A. (2010). Internal solitary waves: propagation, deformation and disintegration. *Nonlinear Processes in Geophysics*, 17:633–649.

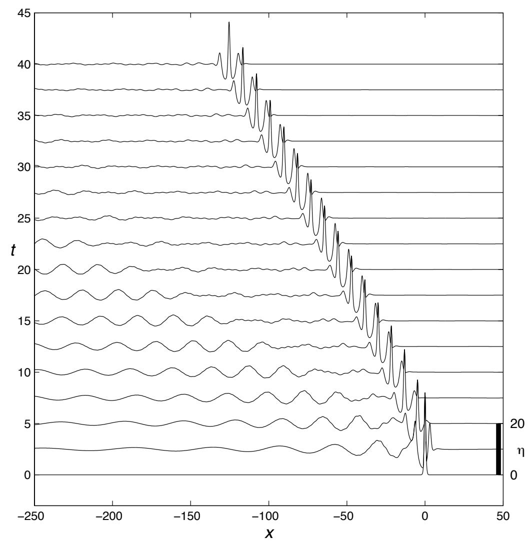

Figure 1: Numerical solution of the Ostrovsky equation with $\lambda = \mu = \gamma = 1$ for an initial condition given by a KdV solitary wave with amplitude 32.

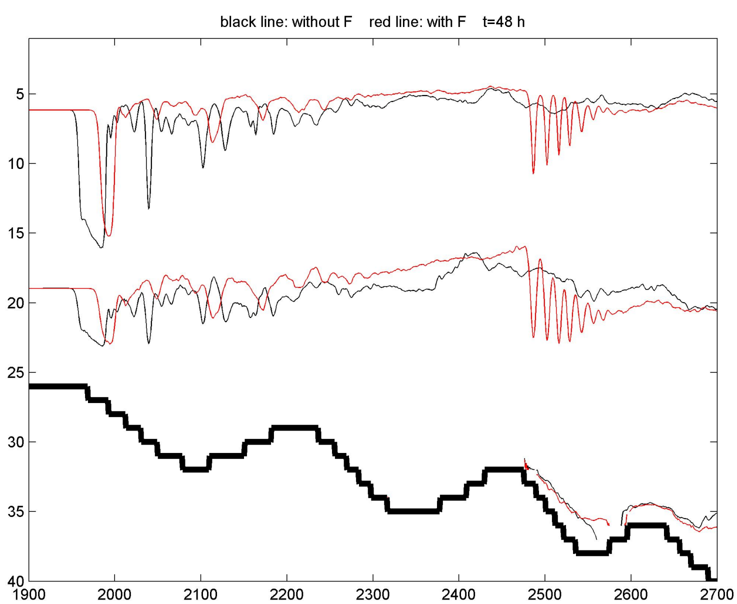

Figure 2: Evolution of a mode 1 internal solitary wave with initial amplitude of $90\,m$ using the MITgcm. The red (black) lines are with (without) rotation.