# Modulated surface gravity waves on water of finite depth with constant vorticity


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Generally, in coastal and ocean waters, the velocity profiles are typically established by bottom friction and by surface wind stress and so are varying with depth. Currents generate shear at the bed of the sea or of a river. For example ebb and flood currents due to the tide may have an important effect on waves and wave packets. In any region where the wind is blowing there is a surface drift of the water and water waves are particularly sensitive to the velocity in the surface layer.

Surface water waves propagating steadily on a rotational current have been studied by many authors. On the contrary, the modulational instability or the Benjamin-Feir instability of progressive waves in the presence of vorticity has been poorly investigated. One can cite Johnson[1], Oikawa, Chow & Benney[2], Li, Hui & Donelan[3], Baumstein[4], Choi[6], Okamura & Oikawa[7], Nwogu[8] and Thomas, Kharif & Manna[9].

The present study deals with the modulational instability of one dimensional, periodic water waves propagating on a vertically uniform shear current. We assume that the shear current has been produced by external effects and that the fluid is inviscid. A NLS equation (vor-NLS equation) for surface waves propagating on finite depth in the presence of non zero constant vorticity is derived by using the method of multiple scales. It is shown that the heuristic method to derive a NLS equation from a nonlinear dispersion relation is not valid when vorticity is present. This is a consequence of the coupling between the mean flow due to the modulation and the vorticity. A stability analysis of a weakly nonlinear wave train as a function of the parameter $kh$ where $k$ is the carrier wavenumber and $h$ the depth and of vorticity magnitude. In Fig.1, it is shown that plane wave solutions are linearly stable to modulational instability for an opposite shear current independently of the dispersive parameter $kh$. Consequences on the Benjamin-Feir index are also considered.

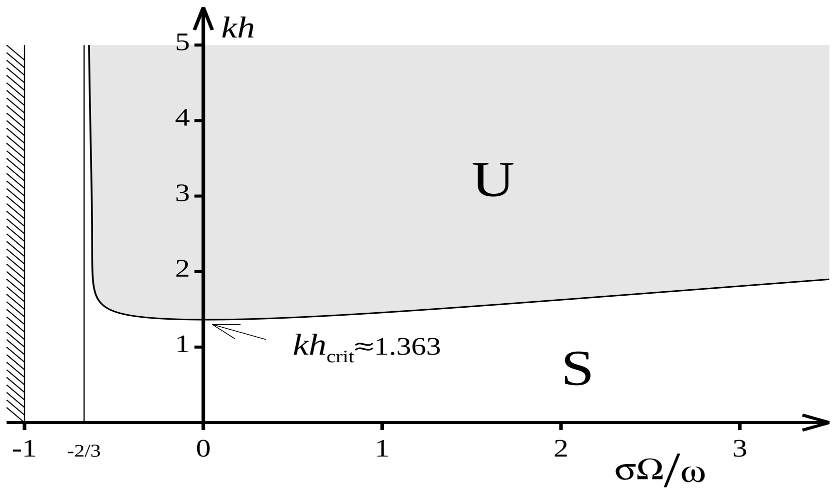


Figure 1. Stability diagram in the $(\overline{\Omega}, kh)$-plane. $\overline{\Omega}$ is the dimensionless current intensity, $\omega$ is carrier frequency and $\sigma = \tanh(kh)$. **S** : stable, **U** : unstable.

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