Bed Shear Stress in the Swash of a Solitary Wave

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The swash zone is a flow of some complication involving wave breaking, wave-generated turbulence, bed-generated turbulence, and interaction between the swash events of one wave to the next. In addition laboratory swash flows may be influenced by viscous forces and surface tension effects if conducted at smaller scales [9]. The role of the bed shear stress over the swash cycle is not well understood, though known to be important for run-up heights, incipient sediment motion, damping of waves. Key to understanding its role is to know how the local bed shear stress evolves in space and time over a single swash cycle. Thus, we propose to study the swash zone using the single swash event of a solitary wave incident on a slope. Solitary waves, recently abandoned as realistic tsunami models [8], are nonetheless convenient to study given their simple characterization in the water of constant depth and propagation without change of form. Furthermore, the swash event from a solitary wave may not be that different to the swash event from a single regular oscillatory wave.

Large-scale experiments are conducted in the Hinsdale Wave Research Laboratory at Oregon State University. Measurements focus in on the swash zone and some of the surf zone. Solitary waves of a wide range of amplitudes (ratio of amplitude to water depth spanning from 0.05 to 0.3) are generated using Goring's method [14] in a still water depth of h=1.72m. The swash events they generate upon interaction with a slope of $s = 1/12$ ($s = \tan(\theta)$) are studied. To measure the bed shear stress, a 'shear plate' is developed that can measure the bottom shear stress directly in the range of 10^{-1} to 10^{2} Pa with accuracy ±5% and a bandwidth of 5Hz. Shear plates have been successful in the past measuring bed shear stress under oscillatory flows [10], swash zones [2] and flows with vegetation [16]. This large-scale study provides an opportunity to achieve high Reynolds numbers and neglect small-scale effects of surface tension and the over-representation of viscous forces normally associated with laboratory studies. It is found that interaction of solitary waves with a slope is most easily characterized by Grilli's slope parameter [4], which is derived as the ratio of the horizontal length scale of the wave to the horizontal length scale of the slope in still water. Thus, the larger the slope parameter, the longer the wave is for that particular slope; or the larger the slope parameter, the milder slope for that wave. The demarcations given by Grilli et al. [4] for breaking and different types of breaking are seen to be accurate in our experiments. The shoaling rate near the toe of the beach is measured via a series of wave gauges there and seen to approach Green's law of shoaling ($H \sim h^{-0.25}$) [15] for solitary waves with large slope parameter. The run-up is measured and for non-breaking waves is seen to be slightly over-predicted by the Synolakis's celebrated run-up law [13], whereas the run-up of both non-breaking and breaking waves matches quite well with the empirical law of Lo et al. [7].

The current state of understanding of the bed shear stress in the swash zone lacks the ability to predict its evolution and effects on the swash flow from the offshore wave characteristics. There are have been numerous efforts recently of measuring the effect of the boundary on the swash, with bore-driven swash particularly popular [1, 2, 3, 6, 12]. Our measurements agree qualitatively with those studies. Given that appropriate non-dimensional quantities are not well established, it is difficult to compare different studies quantitatively. Though currently, it is not able to predict the spatial and temporal evolution of bed shear stress in the swash zone, certain features of the evolution can be pointed out.

For breaking waves travelling onto the dry land: Except near the very front of the bore, the pressure is hydrostatic confirming the validity of shallow water equations. However, it is at the front of the bore that the bed shear stress is the highest. Behind the front, the local stress decays rapidly to a small positive value during the run-up motion. The run-down process begins earlier further down the slope so that the swash tongue becomes stretched with the centre of stretching moving up the slope until it meets the bore front at which time the maximum run-up is reached. There must also be some variation in the depth during the transition from run-up to run-down since the fluid nearest to the boundary must have lower momentum and thus make the transition earlier. This is seen in our measurements by the time history of the calculated local coefficient of friction. The local friction coefficient, $C_f = \tau_b / \left(0.5\rho U^2\right)$ is calculated using the local, time varying shear stress and the local, time varying velocity measured using an ADV with its measurement volume 2 cm above the bed. So C_f in the swash zone is a function of distance on the slope and time making it a less useful concept than in steady flows where it is only a function of Reynolds number. There is variation in its value during the run-up phase: it goes to zero (when the local shear stress becomes zero) and then shoots to infinity (when a small time later the local velocity goes to zero). This small phase difference illustrates the depth variation of velocity during transition from run-up to run-down. During run-down, the local shear stress is seen to increase to its maximum negative value as local velocity also increases under the action of gravity. The run-down flow is closer in character to a steady state flow and we notice that the value of C_f for different locations and different wave conditions is almost the same at $C_f \approx 0.005$ during run-down. The shear stress magnitude fades to back to zero as the height of the water approaches zero at the end of the run-down. The maximum positive shear stress, occurring at the bore front, decreases in value as we follow the bore front up the slope. It's decrease looks to be near linear with distance up the slope. Presumably, it must go to zero at the run-up limit though we only have measurements at discrete locations. The same trend is true for the maximum negative shear stress, which occurs near the end of swash cycle. It also decreases further up the slope because the water has had less distance to accelerate under gravity. It is noted that the maximum positive stresses at locations further down the slope from the breaking point are almost an order of magnitude smaller compared to locations further up the slope than the breaking point. This goes to show how the wave-breaking-generated turbulence affects the boundary layer flow. It is also noted that, in general, the maximum negative shear stresses are smaller than the maximum positive shear stress.

For non-breaking waves travelling onto the dry land: It is interesting to observe that here too, there is stark demarcation in the maximum positive shear stresses before and after the still water line. The magnitudes after the still water line are much higher than the magnitudes before the still water line.

Finally, the motion of the shoreline itself is studied by tracking it with overhead cameras. It is noticed, as it was by Jensen et al. [5], that for the smaller, non-breaking waves, the magnitude of the deceleration of the shoreline is actually less than that due to gravity. This suggests that pressure gradients parallel to the bed must play a role [17, 18, 5]. This is not surprising since the act of wave breaking is to eliminate the pressure gradients in a wave and if a wave doesn't break, they may still exist near the bore front. Jensen et al. report that the shoreline motion close to the run-up limit is well-approximated by gravity dominated deceleration and this would be consistent with the evolution of local bed shear stress described above. For plunging breakers, it is observed that the deceleration of the shoreline is actually very close to that due to gravity. We draw an analogy between the shoreline motions due to bores studied by Shen and Meyer [11] and Yeh et al. [17] and breaking solitary waves. Measurement of the initial velocity of the shoreline motion after breaking and bore collapse, show that roughly only 80% of that initial velocity is responsible for the climb of the shoreline to the run-up limit. Yeh et al. [18] make a similar observation for undular bores. It is postulated that the rest must be lost to the bed shear stress near the bore front. Pedersen et al. [9] recently argued that viscous effects were responsible for the discrepancy in the run-up between a numerical Boussinesq (and BIM) model of an inviscid fluid and their small scale experiment. That was for non-breaking waves and the currently study provides a good opportunity to investigate the role of bed shear stress in run-up of non-breaking waves.

Although the current results are encouraging, clearly there is a lot more work to be done to understand the role of bed shear stress in the singe swash event of the solitary wave, not to mention the swash cycles of a wave train.

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