Direct measurement of dispersion relation for random surface gravity waves

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The linear dispersion relation for surface gravity waves is often taken for granted for the interpretation of wave measurements. High-resolution spatiotemporal measurements suitable for direct validation of the linear dispersion relation are on the other hand rarely available. While the imaging of the ocean surface with nautical radar does provide the desired spatiotemporal coverage, the interpretation of the radar images currently depends on the linear dispersion relation as a prerequisite, (Nieto Borge et al., 2004, 2008).

Recent numerical results indicate that there are additional branches of the dispersion relation that belong to nonlinear contributions to the wave field (Krogstad and Trulsen, 2010). We distinguish two types of nonlinearity, namely static nonlinearity due to bound harmonics and dynamic nonlinearity due to evolution of free waves. The static contributions leads to energy distributions in various dispersion shells away from the linear dispersion manifold.

Krogstad and Trulsen (2010) simulated the dynamic nonlinear evolution of unidirectional free waves using the cubic nonlinear Schrödinger equation (NLS) and the Dysthe (1979) or modified nonlinear Schrödinger equation (MNLS) and demonstrated that nonlinear evolution of narrow-banded wave fields may render the linear dispersion relation inadequate for proper interpretation of observations. The simulations were carried out over rather long time and distance. Krogstad and Trulsen (2010) found that spectral components above the spectral peak could have larger phase speeds than those anticipated by linear theory and that spectral components above peak tended to have group velocities similar to that of the spectral peak. Krogstad and Trulsen (2010) found that the directly measured dispersion relation deviated from the linear dispersion relation due to dynamic nonlinearity.

Recently we have run experiments and simulations of the Zakharov (1968) equation designed to have sufficient resolution in space and time to measure the dispersion relation for random surface gravity waves directly. The experiments and simulations were carried out for a JONSWAP spectrum and Gaussian spectra of various bandwidths on deep water. From both the experiments and the simulations we have found that the directly measured dispersion relation deviates from the linear dispersion relation above the spectral peak when the bandwidth is sufficiently narrow. These findings are in good agreement with the previous simulations of the (M)NLS equations by Krogstad and Trulsen (2010). The deviation from the linear dispersion relation shows dependence on the Benjamin-Feir index (BFI). From the experiments we also find that the deviation occurs only initially in the wave evolution. On the background of these observations a BFI threshold for the occurence of the deviation is given. The experimental wave fields also exhibit common features such as downshift and dissipation.

References

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