Scale effects and stability of viscous boundary layers in wave tank experiments

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Introduction

The linear stability of boundary layers under surface solitary waves and internal solitary waves was investigated. A change of frame of reference allowed to use the nonlinear steady boundary layer equations to obtain a solution for this otherwise unsteady boundary layer flow. This boundary layer flow was then used to investigate its linear stability properties. For the linear stability the Orr-Sommerfeld equation and the parabolic stability equation were used. Direct numerical simulations using a Legendre-Galerkin spectral element Navier-Stokes solver were done to allow a comparison to the results by the model equations.

Surface solitary waves

Surface solitary waves are characterized by the amplitude to depth ratio ϵ. For the viscous boundary layer under the solitary wave, the viscosity of the fluid is another governing parameter and can be characterized by the viscous length scale δ [6]. A plot of the horizontal velocity profiles in the boundary layer under a surface solitary wave is given in figure 1. In the acceleration part the flow is in the same direction as in the bulk flow, whereas in the deceleration part, where the external pressure gradient changes sign, the flow reverses its direction. This was also obtained by [4] for their linear model of the boundary layer flow. Using this boundary layer flow solution, the linear stability of this flow was analyzed by means of the Orr-Sommerfeld and the parabolic stability equation [2]. This allowed to compute regions of stability for this flow, cf. figure 1. The flow is always unstable and for common wave tank dimensions, perturbations will grow in the deceleration part of the wave. Increasing the amplitude of the solitary wave will lead to larger unstable regions.

Internal solitary waves

The boundary layer under an internal solitary wave is not only determined by the amplitude ϵ and the viscous length scale δ, but in addition, the depth of the upper and lower layer, h_2 and h_1, respectively, and the density of the upper and lower layer ρ_2 and ρ_1. The set up is taken from [1], cf. subsection 4.2. The density is constant in each layer and the number of layers is two. Both, waves of elevation and depression are considered. A plot of the horizontal profiles in the lower boundary layer of a internal solitary wave is given in figure 1. The profiles are qualitatively similar to the ones of surface solitary waves. However, they are less in magnitude. This has also implications for the stability of the boundary layer flow under an internal solitary wave. In figure 2, the unstable regions for internal solitary waves are smaller than for corresponding surface solitary wave.

Conclusion

Remarkably good agreement between the results of the direct numerical simulations and the model equations was found. The results of this stability analysis indicate that under laboratory conditions instabilities should only be observable at the rear of the solitary wave, both for surface and internal solitary waves, a result contradicting the findings by [5] for surface waves. However, for equilibrium heights a multiple of times larger than the water depth of a wave tank, instabilities might appear in front of the crest of the solitary wave. A major result of the present analysis is that contrary to findings in the literature [5, 6] the flow cannot be characterized by a critical Reynolds number. Amplification criteria [3, 2] can explain the discrepancies between different experimental and numerical results. An example of the difficulty of obtaining repeatable results was shown by laboratory experiments performed at the university of Oslo.

References

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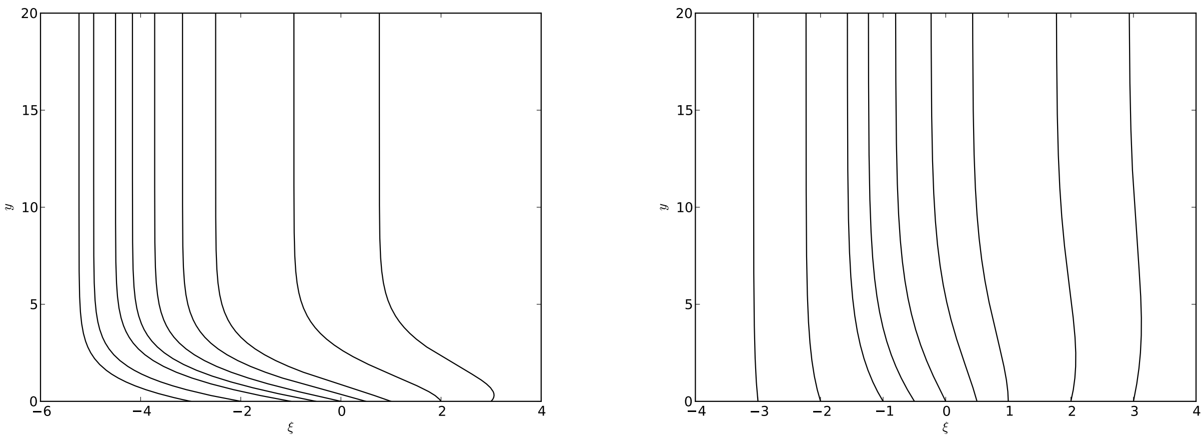

Figure 1: Boundary layer flow under a solitary wave traveling from right to left with $\epsilon = 0.1$ and $\delta = 8 \times 10^{-4}$. The profiles are multiplied by 40. Left: Surface solitary wave. Right: Internal solitary wave of elevation, $\rho_1 = 1.03$, $\rho_2 = 1.$, $h_1 = 1/3$, $h_2 = 2/3$.

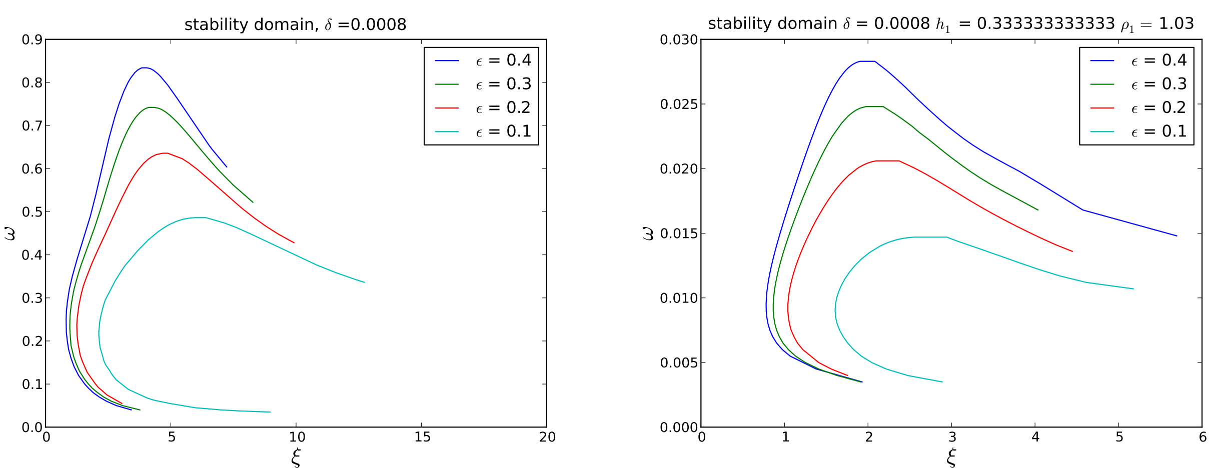

Figure 2: Stability domains for the boundary layer flow under a solitary wave with $\delta = 8 \times 10^{-4}$. Left: Surface solitary wave. Right: Internal solitary wave of elevation, $\rho_1 = 1.03$, $\rho_2 = 1.$, $h_1 = 1/3$, $h_2 = 2/3$.