Stokes drift in internal equatorial Kelvin waves; continuous stratification versus two-layer models

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The Stokes drift in long internal equatorial Kelvin waves is investigated theoretically for an inviscid fluid of constant depth. While the Stokes drift in irrotational waves is positive everywhere in the fluid, i.e. directed along the phase velocity, this is not always the case for internal Kelvin waves, which possess vorticity. For constant Brunt-Väisälä frequency the Stokes drift in such waves is sinusoidal in the vertical with a negative value in the middle of the layer for the first baroclinic mode. For a pycnocline which is typical of the equatorial Pacific, we find for the first mode that the largest negative Stokes drift velocity occurs near the depth where the Brunt-Väisälä frequency has its maximum. Here, estimated drift values are found to be of the same order of magnitude as those observed in the Pacific equatorial undercurrent (the Cromwell current) at the same level. In contrast, a two-layer model with constant density in each layer yields a positive Stokes drift in both layers. This contradicts the fact that, as shown in this investigation, the vertically integrated Stokes velocity (the Stokes flux) must be zero for arbitrary Brunt-Väisälä frequency.

The mass transport induced by periodic irrotational surface waves in an inviscid fluid was first investigated by Stokes (1847). For irrotational waves the Stokes drift is positive everywhere, i.e. it is in the same direction as the phase velocity (Longuet-Higgins and Stewart 1962; Eames and McIntyre 1999). The presence of vorticity in waves changes this, and may cause the Stokes drift to be negative (oppositely directed to the phase velocity) in some parts of the fluid. The vorticity related to the presence of viscosity is not important in this connection. Although the effect of viscosity induces a mean Eulerian drift velocity in the fluid, it only causes a slow attenuation of the Stokes drift through temporal or spatial amplitude decay (Weber 1983; Jenkins 1986). Thus, inclusion of viscosity does not cause any change of sign in the Stokes drift. Other sources of vorticity, e.g. baroclinicity and the earth’s rotation, have a much more profound influence. The effect of rotation on the Stokes drift has been investigated for two-dimensional horizontal flow by Longuet-Higgins (1969) for double Kelvin waves, and by Weber and Drivdal (2012) for continental shelf waves (as part of a more comprehensive study), yielding positive Stokes drift where the bottom gradients are small, and negative drift over the steeper parts of the bottom topography. Negative Stokes drift has also been reported by Flierl (1981).

In this investigation we consider the effect of baroclinicity. The main purpose of the study is to investigate the apparent contradiction between the results from continuous stratification, which yields negative Stokes drift in parts of the fluid, and a two-layer model with constant density in each layer. In the latter case the Stokes drift is positive in both layers. The theory applies to internal plane waves in general, but we focus here on internal equatorial Kelvin waves since the equatorial regions display some of the strongest baroclinic signals in the ocean.

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