Nonlinear generation of harmonics through the interaction of an internal wave beam with a model oceanic pycnocline

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The interaction of an internal wave beam (IWB) with an idealized oceanic pycnocline is examined using two-dimensional fully nonlinear direct numerical simulations based on a spectral multidomain penalty method in the vertical direction [1]. The phenomenon of focus is the nonlinear generation of harmonics. Using a finite amplitude IWB, inclined at $\theta = 45^o$ from the horizontal, a total of 24 simulations have been performed, where the two parameters varied are the normalized pycnocline thickness (h/λ_x ; λ_x is the incident IWB's horizontal wavelength) and the ratio (r) of peak pycnocline Brunt-Väisälä frequency to that of the stratified lower layer. The parameter range of $0.1 \leq h/\lambda_x \leq 1$ and $1.5 \leq r \leq 10$ is chosen as such to bridge the associated gaps across previous experimental and numerical studies of a finite amplitude IWB incident on a pycnocline [2, 3, 5].

Harmonics at the point of IWB entry into the pycnocline increase in amplitude and number with a measure of the maximum gradient of the Brunt-Väisälä frequency, suggesting refraction as an important factor in harmonic generation. Among the simulations performed, two distinct limits of pycnocline thickness are identified. For thin pynoclines, whose thickness is 10% of the incident IWB's horizontal wavelength, harmonics trapped within the pycnocline have maximum amplitude when their frequency and wavenumber match those of the natural pycnocline interfacial wave mode, as suggested by weakly nonlinear theory for harmonic generation by plane wave refraction (figure 1). For thicker pycnoclines, whose thickness is equal the incident IWB's horizontal wavelength, IWB refraction results in harmonic generation at multiple locations in addition to pycnocline entry, giving rise to complex flow structure inside the pycnocline (figure 2).

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References

[1] P. J. Diamessis, J. A. Domaradzki, and J. S. Hesthaven. A spectral multidomain penalty method model for the simulation of high Reynolds number localized incompressible stratified turbulence. *J. Comp. Phys.*, 202:298–322, 2005.

[2] N. Grisouard, C. Staquet, and T. Gerkema. Generation of internal solitary waves in a pycnocline by an internal wave beam: a numerical study. *J. Fluid Mech.*, 676:491–513, 2011.

[3] M. J. Mercier, M. Mathur, L. Gostiaux, T. Gerkema, J. M. Magalhaes, J. C. B. DaSilva, and T. Dauxois. Soliton generation by internal tidal beams impinging on a pycnocline: laboratory experiments. *J. Fluid Mech.*, 704:37–60, 2012.

[4] S. A. Thorpe. Turbulence in the stratified and rotating world ocean. *Theor. Comput. Fluid Dyn.*, 11:171–181, 1998.

[5] S.E. Wunsch and A. Brandt. Laboratory experiments on internal wave interactions with a pycnocline. *Exp. Fluids*, 53(6):1663–1679, 2012.

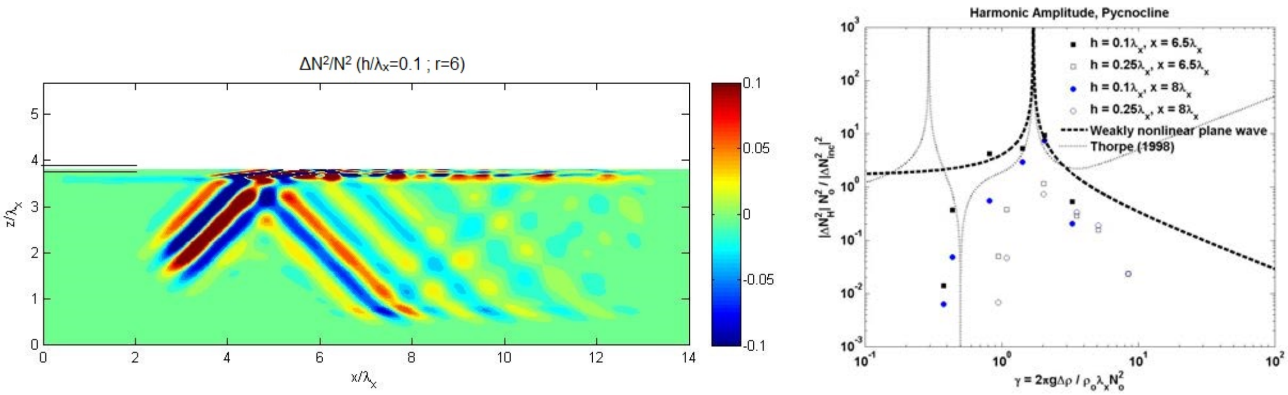

Figure 1: *Left*: Contour plots of the normalized perturbation Brunt-Väisälä frequency $\Delta N^2/N^2(z)$ at steady state for $h/\lambda_x = 0.1$ and $r = 6$. The thin horizontal solid lines delineate the interval $z_0 \leq z \leq z_0 + h/2$, where z_0 is the pyconcline center. *Right*: The dimensionless harmonic amplitude as a function of the parameter γ (defined on the horizontal axis where N_0 is the buoyancy frequency of the lower layer) for the numerical simulations with different pycnocline characteristics, along with the analytic result for a thin pycnocline (shown for $\theta = 45^o$ as a dashed black line). Data are given at two different $h/\lambda_x = 0.1$ locations with harmonic amplitude peaking at value of γ corresponding to $r \in [4, 8]$. Also shown is the analytic result of Thorpe [4].

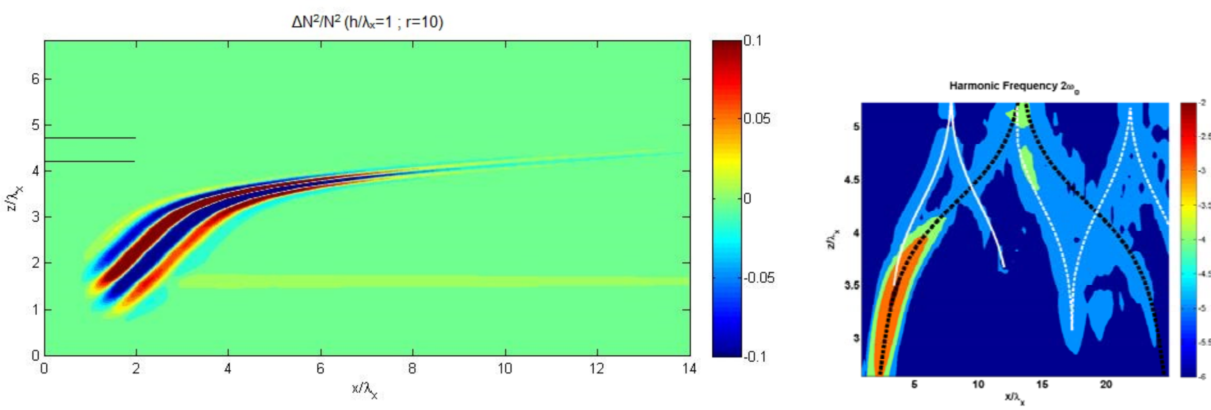

Figure 2: *Left*: Contours of $\Delta N^2/N^2(z)$ at steady state for $h/\lambda_x = 1$ and $r = 10$. The thin horizontal solid lines are equivalent to those in figure 1. Note the strong viscous diffusion of the refracting beam inside the pycnocline. *Right*: The amplitude of $\Delta N^2/N_0^2$ for the first harmonic (right) modes for simulations with $h/\lambda_x = 1$ and $r = 6$. The color scale is logarithmic. The dotted black line is the ray-trace path of the incident wave; the solid and dotted white lines show ray-trace paths for the harmonic frequency 2ω.