Topographic effects on current variability in the Caspian Sea

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[1] Satellite-derived surface height fields reveal that variability in the central and southern basins of the Caspian Sea is correlated with topography. Consistently, empirical orthogonal functions from current meter data from the southern basin are aligned with the isobaths. In addition, the gravest mode, which accounts for over 80% of the variance, has an equivalent barotropic structure in the vertical. To what extent this variability can be modeled using a linear analytical model is examined. The latter assumes equivalent barotropic flow aligned with the geostrophic contours, which in turn are dominated by the topography. With ECMWF winds and ETOPO2 topography, the model yields surface height deviations which are significantly correlated with satellite-derived estimates on seasonal and longer time scales in the central basin. The model is somewhat less successful in the southern basin, where the stratification is stronger. Nevertheless, the results are encouraging, given the extreme simplicity of the model.


1. Introduction

[2] The Caspian Sea (CS) is the largest inland water body of the world in both area (379,000 km²) and volume (78,000 km³). The CS is located between 36°N and 47°N in a region with complex bathymetric features [Ismailova, 2004]. There are three distinguished basins: the northern, central, and southern basins. The southern and the central basins have maximum depths of 1025 and 788 m, respectively, and a sill with a maximum depth of approximately 170 m separates two [Peeters et al., 2000]. The northern basin is a shallow extension with maximum depth of 20 m. The CS is classified as a deep inland sea, due to its thermo- haline structure and water circulation [Lebedev and Kostianoy, 2006].

[3] The CS is enclosed and tides are fairly weak. Density-driven and wave-driven flows do occur [Bondarenko, 1993; Ghaffari and Chegini, 2010; Ibrayev et al., 2010; Terziev et al., 1992], but the currents are primarily forced by the winds. Due to the strongly variable topography, the resulting flows are often spatially and temporally variable, with an active mesoscale eddy field [Terziev et al., 1992; Trukhchev et al., 1995].

[4] Diverse observations with floats and hydrography, and the results of numerical models simple hydrodynamic interpretations [Bondarenko, 1993; Terziev et al., 1992], suggest that the circulation in all three basins is predominantly cyclonic. The circulation is thus associated with a predominantly southward flow along the western coast and northward flow along the eastern coast. Recent observations in the shallow coastal area (~100 m depth or less) likewise indicate cyclonic flow, but also reveal exceptions. In particular, the flow near the eastern coast has been observed to reverse episodically [Ibrayev et al., 2010], as has the flow near the southern boundary [Ghaffari and Chegini, 2010]. Further information about the hydrographic structure and general circulation of the CS is given by Kostianoy [2005].

[5] Although the region has been studied extensively, a comprehensive picture of the circulation is lacking. In particular, we do not have a first-order model explaining the response to wind forcing and, more generally, an explanation for the predominantly cyclonic flow. The goal of the present study is to propose such a model.

[6] Our model is relevant in regions where the ambient potential vorticity contours (PV), or geostrophic contours, are closed. For unstratified regions these are the \( f/H \) contours, where \( f \) is the Coriolis parameter and \( H \) is the water depth. Such models have been used before to investigate wind-driven variability [see Kamenkovich, 1962; Hasselmann, 1982; Isachsen et al., 2003]. The main assumption of such models is that the primary component of the flow is parallel to the \( f/H \) contours. The flow is then forced by convergence or divergence in the surface Ekman layer and balanced by divergence or convergence in the frictional Ekman layer at the bottom (details are given below).

[7] As shown in Figure 1, the \( f/H \) contours in the Caspian Sea are indeed closed in both the central and southern basins. We will, therefore, investigate the use of such a model here. However, as there is significant stratification in the CS, it is not certain that a barotropic model is sufficient.
So we extend the existing model to allow for vertical shear. As seen below, observations from the southern basin show that current fluctuations are approximately equivalent barotropic, and taking that into account is straightforward.

The manuscript is organized as follows: relevant observations are described in section 2. In section 3, the equivalent barotropic model is derived. In section 4, the model solutions are presented and discussed and concluding remarks are given in section 5.

2. Observations

2.1. Satellite Observations

Satellite observations provide the most comprehensive information about basin-scale variability in the sea. Weekly updated gridded maps of sea level anomalies (SLA) for the region and covering the period January 1992 to December 2011 are available (online at http://www.aviso.oceanobs.com). We used these fields to conduct an EOF analysis of the variability.

The leading EOF, which accounts for 92% of the total variance, is remarkably similar to the f/H field in the central basin (Figure 2). This implies the sea surface is oscillating coherently across the basin. A similar result was found in the Norwegian, Lofoten, and Greenland basins by Isachsen et al. [2003].

The response in the southern basin is less clear. The first mode does have a positive lobe that approximately traces out the f/H contours in the western portion of the basin, but the other lobes do not bear an obvious relation to

![Figure 1. The bathymetric field of the CS (shown as gray shading), and potential vorticity field (contours) in the Caspian Sea. While the f/H contours have semicircular shapes in the central basin, the southern basin encompass relatively complicated feature.]

![Figure 2. Principal component analysis of sea level anomalies (SLA) for the CS. (top) Leading EOF mode and f/H contours, and (bottom) associated PC time series, which accounts for 92% of the total variance in the field.]

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the contours. EOF1 also exhibits distinct variability in the north-west region. Here the depth is 5–6 m and the flow moreover is heavily influenced by the Volga River.

[12] The remaining EOFs account for only 8% of the variance. These structures (not shown) are dominated by higher-wave number features that give little dynamical insight. So we will focus on the first EOF hereafter.

2.2. Current Meter Observations

[13] Three current meter moorings were deployed in a line perpendicular to the coastline off the south-western shelf of the CS, from November 2004 to early May 2005 (Figure 3). The moorings were located over the 20, 50, and 230 m isobaths at (37.505°N; 49.865°E), (37.531°N; 49.866°E), (37.553°N; 49.864°E), respectively.

[14] Recording Current Meters (RCM 9) were deployed near the surface, at middepths (for the 230 m mooring), and near the bottom, to provide measurements throughout the water column. The sampling frequency was 3 cph, but the analysis presented here is based on daily averaged velocities.

[15] Figure 4 shows the variance ellipses for the daily averaged currents for the three moorings. These have high eccentricities and the principal axes of the variability are closely aligned with the \( f/H \) contours, at all three moorings.

[16] The alignment and magnitude of the ellipses suggest the currents are fairly barotropic. To quantify this, we calculated the fraction of energy in the barotropic mode:

\[
R = \frac{H \langle \pi^2 \rangle}{\langle H \pi^2 \rangle + \sum_{i=1}^{N} \langle u'^2 \rangle dz}
\]

where \( N \) is the total number of the depth layers, \( \pi \) and \( u' \) are depth-averaged and depth-varying velocities, respectively. The angle brackets in (1) are time averages and \( H \) is the depth at the mooring. The depth-averaged velocity accounts for 92%, 79%, and 83% of the total variance in the moorings from the south to the north, respectively. This is consistent with previous results which suggested the flow field in the eastern part of the southern basin is nearly barotropic [Ghaffari and Chegini, 2010; Zaker et al., 2011].

[17] The deepest mooring has current meters at 3.5, 65, and 112 m. Thus, the upper two are in the seasonal thermocline (which spans the upper ~30 m in summer) and the lower one is well below it (Figure 3, right). The very high frequency fluctuations are not well correlated between the upper two and the lower one. But the squared coherences between the three time series are 0.6–0.8 at periods longer than 3 days. So the low frequency fluctuations have a reasonably large vertical coherence. As this mooring yields the best information about the flow in the basin interior, we use its currents for the subsequent analysis.

[18] The velocity ellipses suggest that the velocities at the deepest current meter at the third mooring are weaker than at the surface instruments, but also aligned (or counter-aligned) with them (Figure 4). To examine the vertical
structure, we calculated complex EOFs for the three instruments [Kaihatu et al., 1998]. The first mode, which accounts for over 80% of the variance, is shown in Figure 5. This mode has the largest velocities at the surface and decays with depth, but the velocities are approximately parallel throughout the water column. So the velocities are approximately equivalent barotropic (EB). The e-folding scale, determined by nonlinear least squares, is about 350 m. The velocities are also aligned with the topography, consistent with the conclusions from the satellite data that variability is so-aligned.

[19] Of course stratification can cause current fluctuations, as those observed here, to decay with depth. Figure 3 (right) shows the potential density and temperature structures in summer (2008), along a transect extending from the coast toward the offshore region, in the study area. It reveals a strong seasonal thermocline located at ~30 m depth, which is a typical seasonal thermocline depth in summer for almost whole basin [see Kostianoy, 2005]. Ghaffari et al. [2010] using hydrographic data (the same transect) showed that during the unprecedented severe winter (2008), the seasonal thermocline reached almost 100 m, where the water mass was still stratified. In fact, the CS has low salinity and the density stratification largely determined by temperature [Terziev et al., 1992]. Therefore, the density stratification mimics the temperature structure, i.e., the stratification is weaker but does not vanish in winter [Kosarev, 1975]. Figure 6 provides a large-scale picture of the stratification in the CS. Basin-wide potential density profiles reveal that the southern basin is more stratified than the central basin. In the southern basin, the surface densities are significantly less, producing larger near-surface gradients. Correspondingly, estimates of the Burger number, $Bu = \left( \frac{L_d}{L} \right)^2$, where $L_d$ is the internal deformation radius and $L$ is the scale of motion, are 0.057 and 0.027 for the southern and central basins, respectively (assuming a length scale as 50 km).

[20] So both current meter observations and hydrography suggest that the vertical stratification may be important, at

![Figure 5](image1.png)

**Figure 5.** EOF1 for the currents at the deepest mooring in the southern basin. The arrows are from the EOF, at the depths corresponding to the current records. The topographic contours are indicated at the bottom, and an exponential function with an e-folding scale of 350 m is shown for comparison.

![Figure 6](image2.png)

**Figure 6.** The potential density profiles in the central (gray lines) and southern (black lines) basins. The data were collected during the cruise on the CS (organized and conducted by the International Atomic Energy Agency) in September 1995 [IAEA, 1996]. (left) Vertical distributions of the potential density in the southern and central basins. (right) Overlay of the potential density profiles.
least in the southern basin. An equivalent barotropic description of currents seems appropriate, and a simplified model of wind-driven variability should thus assume EB currents. Having such depth-decaying currents is significant because topography will then exert a weaker influence than in a purely barotropic flow. EB models have been studied previously in the context of the Southern Ocean [Ivchenko et al., 1999; Krupitsky et al., 1996; LaCasce and Isachsen, 2010]. Below, we develop such a model in the context of closed-PV contours.

3. Theoretical Model

[21] The dynamics of flows with closed-PV contours differ from those with blocked contours. With blocked contours, forcing is required to support a steady circulation, as with the Sverdrup circulation. With closed contours, steady flows parallel to the PV contours can exist in the absence of forcing and dissipation [Kamenkovich, 1962; Killworth, 1992; Hasselmann, 1982; Greenspan, 1990; Young, 1981]. Such flows can be very strong and can be excited by wind forcing.

[22] Isachsen et al. [2003] derived a linear, barotropic model to study such flows. In the model (described below), an imbalance between the net transport in the surface and bottom Ekman layers results in a change in the geostrophic circulation within the closed contour region. The model was used to study wind-driven fluctuations in the Nordic Seas and was reasonably successful at reproducing variability observed from satellite and in a GCM. The success of such a model at high latitudes is aided by having relatively weak stratification. Whether such a model could work at lower latitudes, as in the Caspian Sea, remains to be seen.

[23] As the current fluctuations in the Caspian have an approximately EB structure, we recast the barotropic solution of Isachsen et al. [2003] for an EB flow. This involves mostly slight modifications, and the barotropic solution is recovered as a limiting case.

[24] As noted, the variables (e.g., pressure and velocity) vary with depth, but the direction of flow does not. So we can write

$$u(x,y,z,t) = u_s(x,y,t)P(z),$$  \hspace{1cm} (2)

and

$$p(x,y,z,t) = p_s(x,y,t)P(z),$$  \hspace{1cm} (3)

where $u$ and $p$ are the horizontal velocity vector and pressure, $u_s$ and $p_s$ are the corresponding surface values, and $P(z)$ is a vertical structure function. Following Isachsen et al. [2003], we assume a linear bottom drag, so that the bottom stress is:

$$\tau_b = Ru(-H) = r_e u_e,$$  \hspace{1cm} (4)

where $R$ is the bottom friction coefficient, $H(x,y)$ is depth, and $r_e = RP(-H)$ is a modified drag coefficient. Similar equations were derived previously by Krupitsky et al. [1996] and LaCasce and Isachsen [2010]. Substituting these into the linear horizontal momentum equation and integrating over the fluid depth yields:

$$\frac{\partial u_s}{\partial t} + f k \times u_s = -g \nabla \eta + \left( \frac{r_e u_e}{\rho_0} - \frac{r_e u_e}{F} \right),$$  \hspace{1cm} (5)

where $\rho_0$ is a constant density, $g$ the acceleration due to gravity, and $k$ is the unit vector in the vertical direction. Furthermore, $\tau_b$ is the surface stress vector, $\eta$ is the sea surface height, and $F(x,y)$ is the vertical integral of the profile function:

$$F \equiv \int_{-H}^{0} P(z) dz.$$  \hspace{1cm} (6)

[25] Substituting (2) into the continuity equation and integrating vertically yields:

$$\frac{\partial}{\partial x} (F u_x) + \frac{\partial}{\partial y} (F v_y) = 0,$$  \hspace{1cm} (7)

where $u_x$ and $v_y$ are eastward and northward velocity components. Thus, we may define a transport stream function as:

$$F u_x = -\psi_x, \quad F v_y = \psi_y.$$  \hspace{1cm} (8)

[26] Taking the curl of (5) and using (8), we obtain the equivalent barotropic potential vorticity equation:

$$\frac{\partial}{\partial t} \nabla \times u_s + f (\psi_s \frac{F}{H}) - \nabla \times \frac{r_e u_e}{\rho_0 F} = \nabla \times \frac{r_e u_e}{F},$$  \hspace{1cm} (9)

where the Jacobian term $J(\psi, f/F) = (\psi_x \frac{F}{H})$ is the advection of PV by the flow. According to (9), a steady flow must be parallel to the $f/F$ contours in the absence of forcing. These are the geostrophic contours in the EB model.

[27] As in Krupitsky et al. [1996], Ivchenko et al. [1999], and LaCasce and Isachsen [2010], we take $P(z)$ to be an exponential function

$$P(z) = \exp \left( \frac{z}{\delta_0} \right),$$  \hspace{1cm} (10)

so that:

$$F = \delta_0 \left[ 1 - \exp \left( -\frac{H}{\delta_0} \right) \right].$$  \hspace{1cm} (11)

[28] We see that $F$, and hence the PV contours, are affected by vertical shear. For strongly sheared flows, i.e., for $\delta_0 \ll H, F$ is approximately constant and the PV gradient is dominated by planetary beta. For deeper currents, i.e., for $\delta_0 \gg H, F \rightarrow H$ and the barotropic model is recovered. So the PV contours are intermediate between latitude lines and $f/H$ contours, depending on the e-folding scale. Note too that the shear affects the effective bottom drag, as the latter is proportional to the bottom velocity. In deeper waters, where the depth is much greater than the e-folding scale, the drag is weak.

[29] The solution then follows that of Isachsen et al. [2003]. Assuming the forcing and friction are weak, and that the variations occur on long times scales, the dominant balance in (9) is:
This implies that the first-order flow follows the $f/F$ contours (which we assume are closed). The model thus filters out, for example, topographic waves, which entail cross-contour motion. So:

$$\psi = G\left(\frac{f}{F}\right),$$

where $G$ is some function. The surface velocity is then given by:

$$u_s = \frac{1}{F}s \times \nabla G = \frac{1}{F}G' s \times \nabla \left(\frac{f}{F}\right),$$

where $G'$ is the derivative of $G$ with respect to its argument, $f/F$.

To determine $G$, we integrate equation (9) over a region bounded by an $f/F$ contour. This eliminates the Jacobian term, after invoking the continuity equation, and yields:

$$\frac{\partial}{\partial t} \int_\Omega u_s \cdot d\Omega = -\frac{\partial}{\partial t} \int_\Omega \frac{r}{\rho_0 F} \cdot d\Omega - \int_\Omega \frac{r u_s}{F} \cdot d\Omega,$$

after applying Stokes’ theorem. The first term on the RHS is the net transport into the surface Ekman layer and the last is the net transport in the bottom layer. An imbalance between these two results in a change in the circulation around the contour.

We solve (15) by Fourier transforming the variables in time:

$$\tau_s(x, y, t) = \sum_\omega \tau_s(x, y, \omega)e^{i\omega t}, \quad u_s(x, y, t) = \sum_\omega u_s(x, y, \omega)e^{i\omega t}.$$

With this, (15) is:

$$\int_\Omega \left(\frac{i\omega + \frac{r_s}{F}}{F}\right) u_s \cdot d\Omega = \frac{\partial}{\partial t} \int_\Omega \frac{r}{\rho_0 F} \cdot d\Omega + \int_\Omega \frac{r u_s}{F} \cdot d\Omega.$$  \hspace{1cm} (16)

The surface circulation depends on the bottom friction and the forcing frequency. At low frequencies ($\omega \ll r_s/F$), the circulation is in phase with the winds and has an amplitude which is inversely proportional to the modified bottom friction coefficient. At high frequencies, the circulation lags the wind by 90° and is independent of friction.

Finally, inserting (14) into (16) gives:

$$G' = \frac{\int \tau_s/(\rho_0 F) \cdot d\Omega}{\int (i\omega/F + r_s/F^2) \nabla (f/F) \cdot \hat{n} d\Omega},$$

which is the EB equivalent to equation (8) in Isachsen et al. [2003]. They also discussed the equivalent barotropic model, but neglected that baroclinicity alters the PV contours. Assuming the flow is zero outside the region of closed contours, (14) can be integrated sequentially inward, yielding $G$ and hence the stream function (applying the inverse Fourier transform). Furthermore, since the stream function is proportional to the surface pressure, i.e., $G = F/(g/f)\eta$, the result can also be used to find sea surface height deviations between the inner and outer contours.

4. Results

First, we compare the model prediction against sea level anomaly (SLA) measurements from the satellite. To do this, we estimated time series of SLA differences across both the central and southern basins, between an “inner” $f/F$ contour in the deep basin and an “outer” contour near land. This SLA difference is proportional to the geostrophic transport between the two contours. The satellite-based estimate was calculated as the difference between SLA averaged over the two contours while the model estimate was calculated by integrating $\eta = f/(gF)G'$ over a set of closely spaced contours between the inner and outer ones.

The 0.125° × 0.125°-resolution European Centre for Medium-Range Weather Forecasts (ECMWF) operational analysis (available online at http://www.ecmwf.int) was used as wind forcing for the model. Topographic data were obtained from the ETOPO2 0°2' × 0°2' data set (available at http://www.ngdc.noaa.gov). The raw topographic data were smoothed to comparable to the internal deformation radius here [see also Isachsen et al., 2003; Lacasse, 2000; Krupitsky et al., 1996].

The model’s free parameters are the bottom friction coefficient $R$ and the e-folding scale of the vertical shear, $h_0$. We set the bottom friction coefficient, $R = 5.0 \times 10^{-4}$ m s$^{-1}$, following Gill [1982] and Isachsen et al. [2003].

4.1. Central Basin

To do this, we calculated the correlation between the observed and modeled sea level differences, between an inner and an outer $f/F$ contour, as a function of $h_0$; the result for the central basin is shown in Figure 7. The correlations are low at small values, confirming that topography is important for the response (recall that with small values of $h_0$, the PV contours are essentially latitude lines). As $h_0$ is increased, the correlations grow and they are constant above $h_0=800$. The correlation coefficients are 0.68, implying reasonably good agreement.

The latter is confirmed by comparing the two time series (Figure 8). Both are dominated by the seasonal signal, with negative values (SLA lower on inner contour than on outer contour) during winter months, indicating anomalous cyclonic circulation. The ratio of the model’s RMS value to the observed value is 1.1, so the model captures both the variability and the amplitude of the signal.

The fact that the correlations do not decrease as $h_0$ gets large suggests that a model based on $f/H$ would do equally well as the equivalent barotropic model here. We confirmed this. This in turn implies topography exerts a controlling effect in the central basin, regardless of the stratification.

4.2. Southern Basin

Figures 9 and 10 show the corresponding results for the southern basin. As in the central basin, the model-data
correlations are lowest with small values of \( h_0 \) and they are larger and approximately constant for large values. But they are also slightly higher at an intermediate decay scale, around \( h_0 = 500 \). So it would seem the EB model is slightly better than a barotropic \( f/H \) model here. The correlations nevertheless are somewhat lower than in the central basin, reaching a maximum value of about 0.58.

[43] The time series from the satellite and the model, with \( h_0 = 500 \), are shown in Figure 10. The seasonal cycle is less pronounced than in the central basin, in both time series. But the two series are less similar than in the central basin. Moreover, the ratio of the RMS amplitudes is 0.7; so the model somewhat under-predicts the amplitude of the response.

[44] The preceding results suggest perhaps that the agreement is superior in the central basin because the seasonal signal is more pronounced there. This is partly true. Shown in Figure 11 are the coherences as functions of frequency. Results are shown both for the EB model and the barotropic (large \( h_0 \)) version of the model. The coherences are insignificant on time scales less than about 5 months, in both basins. In the central basin, the coherence peaks at around 10 months, or roughly one year; this is consistent with the model capturing the seasonal cycle. It decays at longer time scales. And the barotropic model performs equally well as the EB model, as noted before. In the southern basin the response is similar, but the coherences are also lower. They are largest at around 1 year too, but the difference is less marked from the lower frequencies. Furthermore, the barotropic model produces consistently lower coherences than the EB model.

[45] The fact that the coherences are low on the short time scales is to be expected from the model. With \( H \sim 1000 \text{ m} \) and \( R = 5 \times 10^{-4} \text{ m} \), the barotropic decay time scale, \( T_d = H/R \), is on the order of a month. The time scale in the EB model is longer because the velocity shear reduces the bottom velocities; with \( h_0 = 500 \text{ m} \), it is roughly five times longer. The model assumes that time variations in the circulation are equally long (otherwise the first-order flow would not be along \( f/F \) contours). So it is not surprising we capture only time scales exceeding a few months.

[46] That the coherences also decrease for the long time scales is consistent with the results found by Isachsen et al. [2003]. At these scales, baroclinic effects presumably come into play, effects which cannot be captured in an equivalent barotropic model. So the model is most effective on the intermediate time scales, particularly at the seasonal frequency.

### 4.3. Comparison With Current Meter Observations

[47] Lastly we revisit the mooring data from the southern basin and compare the along- \( f/F \) velocities at the deepest mooring with the model predictions. The model velocities were calculated from (14) and (17). As before, we use a drag coefficient of \( R = 5.0 \times 10^{-4} \text{ m s}^{-1} \) and an e-folding scale of \( h_0 = 500 \text{ m} \).

[48] The observed and modeled velocity time series (Figure 12) show some similarities. The correlation coefficient, 0.48, is slightly lower than that obtained with the SLA data in the southern basin. This is to be expected, as the entire current meter record is just over six months long. From the preceding discussion, the model is better at capturing the month-to-month variations than those on shorter time scales. The ratio of the RMS velocities is \( \sim 0.8 \), so the predicted amplitude is also reasonable.

![Figure 7](image-url)

**Figure 7.** The correlation coefficient \( (R_c) \) between the satellite observations and the EB model as a function of e-folding scale \( (H_0) \) for the central basin.

![Figure 8](image-url)

**Figure 8.** Time series of the relative sea level displacements between the rim and inner \( q_e \) contours in the central basin where \( H_0 = 800 \). The thin and thick lines indicate weekly time series of the sea level anomalies based on the satellite observations and the EB model predictions, respectively.
Note that the assumed e-folding scale of 500 m is somewhat larger than the e-folding scale deduced from the first EOF from the current meter, which was 350 m. Using $h_0 = 350$ m yields a slightly lower correlation $\approx 0.4$ and also a lower amplitude ratio $\approx 0.6$. Nevertheless, this is fairly inconclusive, given the short record length.

Interestingly, the observed fluctuations lag the EB model prediction, by roughly 1 week. Accordingly, adding a 5 day lag to the model time series raises the correlation coefficient from 0.48 to 0.64, and most of the increase is due to a better match at shorter time scales. But it is clear that such comparisons should be done with a longer time series before any speculation is made.

5. Summary and Conclusions

We have used a linear analytic model to study current fluctuations in the central and southern portions of the Caspian Sea. The model assumes the flow is wind driven and that dissipation is entirely by bottom drag. Motivated by current meter satellite altimeter observations, the velocities are assumed to be equivalent barotropic (EB) and aligned with EB PV contours, the $f/F$ contours (where $F$ is a modified function of depth which takes the vertical decay of the velocities into account). These contours close on themselves in both the central and southern basins of the Caspian Sea and allow us to estimate the flow variability from a simplified expression. A barotropic version of the model was used previously to study the response in the Nordic Seas and Arctic Ocean [Isachsen et al., 2003]. The modification to EB flow and the application to the midlatitude Caspian Sea is new.

The model was reasonably successful at simulating current variability on time scales exceeding a few months.

![Figure 9](image1.png)  
*Figure 9.* The correlation coefficient ($R_c$) between the satellite observations and the EB model as a function of e-folding scale ($H_0$) for the southern basin.

![Figure 10](image2.png)  
*Figure 10.* Time series of the relative sea level displacements between the rim and inner $q_e$ contours in the southern basin where $H_0 = 500$. The thin and thick lines indicate weekly time series of the sea level anomalies based on the satellite observations and the EB model predictions, respectively.

![Figure 11](image3.png)  
*Figure 11.* Coherence estimates between the models and the satellite measurements as functions of the frequency for (top) the central and (bottom) the southern basins. The dark and light lines represent the coherence between the EB model and observation and the barotropic model and observation, respectively. The dashed line shows 95% confidence level.
In the central basin, where the stratification is weaker, the \( f \)-
model and the barotropic \( f \)-limit yield comparable results. Both agree with the satellite observations of low frequency (seasonal to annual time scales) sea surface height variability. In the southern basin, the stratification is stronger. As such, the EB model is more successful than the barotropic model. We find reasonable correlations with both the satellite data and with the current meter data, again primarily on time scales of seasons to a few years.

[53] The coherences here are also somewhat less than those observed in the Northern Seas. In some cases there, the correlation coefficients exceeded 0.8. This is likely due to having weaker stratification at those latitudes. Nevertheless, the dependence of the coherences on frequency was similar. The (barotropic) model was most successful at intermediate frequencies, capturing primarily seasonal variations.

[54] It should be emphasized what the analytical model leaves out. The assumption that the flow is dominated by a component which is purely along the PV contours removes all dynamics with cross-contour flow. This includes topographic and internal Kelvin waves. The associated time scales are on the order of several days in the Caspian Sea (J. E. H. Weber and P. Ghaffari, Mass transport in internal coastal Kelvin waves, submitted to European Journal of Mechanics B, 2013.) The assumption of barotropic or quasi-barotropic motion neglects baroclinic effects, and this evidently decreases coherences on interannual time scales. To take such effects into account will require additional assumptions about the density field. As suggested by the findings of Nöst and Isachsen [2003] and Aaboe and Nöst [2008], a time-mean solution may then be found in which the connection between the top and bottom Ekman layers is modified by the divergence of the meridional thermal wind transport.

[55] Nevertheless, the model is appealing for its extreme simplicity. A first-order assessment of intermonthly variability can be estimated using a single equation and a simple Matlab routine. The model moreover should be applicable in other regions, and possibly in large inland lakes.

[56] Further assessments in the Caspian Sea would clearly benefit from more observations, particularly from longer-term current measurements in both basins. Studies using primitive equation models would also help unravel the possible role of nonlinearities and eddy momentum transport, processes that have been ignored here.

[57] Acknowledgments. We thank Andrey Trebler and Paula Pérez Brunius for helpful comments.

References
Bondarenko, A. L. (1993), Currents of the Caspian Sea and Formation of Salinity of the Waters of the North Part of the Caspian Sea [in Russian], 122 pp., Nauka, Moscow, Russia.


