Tax distortions, household production
and black-market work*

By

Jon Strand
Department of Economics
University of Oslo
Box 1095, Blindern
0317 Oslo, Norway
jon.strand@econ.uio.no

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Abstract

We study an economy with high- and low-productivity households, where household services can be produced either by households themselves or purchased in the regular ("white") or "black" market. Black market work is unfavorable when it otherwise would have been done in the white market, and favorable when it else would have been done within the household. When tax wedges are low, the black market reduces welfare by competing with white-market production. When tax wedges instead are high, the black market increases welfare by competing with own-household production. With intermediate tax wedges, welfare may be either improved or reduced by a black market. Welfare is improved when black services are demanded by low-productivity households, and reduced when black services are demanded by high-productivity households. An increase in the tax wedge may then reduce the benefits from a black service market.

Key words: Black markets, household production, tax wedges.

JEL classification: H21, H26, J22, J23.
1. Introduction

The purpose of this paper is to study effects of taxation on the distribution of household work, as performed in three different spheres of the economy: within the own household; as paid work in the legal ("white") sector subject to taxation; or as paid work in the illegal ("black") sector where taxes are evaded. We focus on the degree to which high taxation inefficiently pushes activity out of the paid legal sector and into either the black or own-household sector, and the relationships between marginal taxes, the distribution of work between the three types of activity, and the degree of allocative inefficiency. The magnitude of the potential allocation problem is substantial. For one thing, a large fraction of economic activity consists of services done by or for the household. Scandinavian time use surveys show that own household work activity (as different from leisure activity) in these countries constitutes more than 2/3 of the volume of regular paid work in terms of hours. It seems clear that much of this work can gainfully be carried out by persons outside of the household, who have a comparative advantage in such activities. As noted by Sandmo (1990), however, taxation of regular market work but not household work serves as a tariff on trade between individuals, and creates inefficiencies by pushing activity out of the paid sector and into the household sector. Such tariffs are particularly high in the Scandinavian countries, where also the amount of own household work is high. High taxation rates in addition create a basis for a "black" sector, giving demanders and suppliers incentives to meet secretly in order to avoid paying taxes. This market, most of which is in the personal services sector, is smaller but not negligible, at least on the order 2-10 % of GDP in most OECD countries (and higher in many developing countries).¹

I here consider a simple model where household services can be produced by household members themselves, or purchased on the "white" or "black" market. I assume
that labor productivity is always highest in the white market and lowest in home production, with black market productivity intermediate. Individuals are of two types with different productivities. High-productivity individuals have a comparative advantage in "industrial" production versus household services production. Efficiency implies that all household work is carried out in the white market, since labor productivity is highest here, for all workers. This is also the resulting allocation when the tax rate is low enough to rule out either home production or a black market. Most of our focus is however on cases with higher tax rates. In the black market, sellers and buyers incur costly search and form pairwise probabilistic matches, and enter the black market until private gains from entry are dissipated. At equilibrium either low- or high-productivity workers demand black-market services, but never both.

The welfare implications of permitting a black market depend on whether black services are demanded by households that otherwise would buy such services in the white market, or by households that otherwise would produce their own home services. Welfare increases when the black market "steals" activity from households’ own home production, and decreases when it "steals" activity from the white market. I derive conditions for the different cases to arise. I find no monotonous relationship between the magnitude of the tax wedge and the welfare properties of a black market. As expected, a black market is favorable when the tax wedge is "very high", and unfavorable when it is "very low". A novel and more surprising result is that this relationship is less clear in intermediate cases. In the economy described by our model, going from an intermediate to a somewhat higher level of general taxation may thus make it less desirable to have a black market.

My results are derived assuming risk neutrality, and perfect substitutability between white, black and own household production. These assumptions are made mainly for

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1 See e.g. Pedersen (1998) for an extensive survey. See also de Soto’s (1989) study of the “hidden” markets in Lima, Peru, where he estimates that the “underground” economy (most precisely identified as the aggregate of
tractability and do not pretend to be realistic. In particular, risk aversion is ubiquitous and central to any discussion of tax evasion. It may also imply more general (e.g. nonseparable) utility functions that is likely to alter some of my conclusions. In the final section I discuss these problems, and their implications for the analysis, more extensively.

Considerable attention has been paid by economists to implications of household production activity (e.g. Corlett and Hague (1953), Becker (1965), Boskin (1975), Gronau (1977), and more recently Sandmo (1990), Juster and Stafford (1991), Kolm (2000), Kleven (2004), Kleven, Richter and Sørensen (2000), Pigott and Whalley (1998) and Anderberg and Balestrino (1999)), and the presence of black labor markets (e.g. Allingham and Sandmo (1972), Srinivasan (1973), Feige (1989), Cowell (1990), Jacobsen and Sørensen (1997), Kolm and Larsen (2001), Pedersen (1998), Schneider (2000), Schneider and Eneste (2000); see also the survey in Slemrod and Yitzhaki (2002)). So far, however, to my knowledge no work integrates the two in a coherent analytical model. My model can also be viewed as formalizing some less formal ideas in Ben-Porath (1980), where a barter economy (with specialists exchanging professional favors free of taxes) replaces my black sector.

The black (or “underground”) economy has also been studied extensively in developing country contexts (de Soto (1989), Rauch (1991), Loayza (1996)). Much of this literature argues that a black market may have favorable allocation properties (in particular de Soto’s exposition), but little deals explicitly with analytical welfare issues. Exceptions are Choi and Thum (2002, 2003), who show that a black market may have favorable welfare properties in an economy with corrupt government officials and when going underground the household and black sectors) constitute up to 60 % of the total economy.

\[ Sørensen (1997) \] presents the sketch of a computable equilibrium model where both household and underground production are present. This model however does not address the main issues here, which are the implications and efficiency of a black services market, and how this market interacts with other markets.
limits these officials’ ability to extract entrepreneurs’ rents. Choi and Thum thus give a justification for the black market which is quite different from that given here.

2. Basic model framework

Consider an economy where individuals may engage in three activities: 1) regular paid work subject to (income, VAT etc.) taxation at constant rate $\tau$, 2) ”black market” work which is not declared as taxable income, and 3) own household work exempt from taxation. The economy produces two goods, household services (in sector 1) and ”industrial” goods (in sector 2), both with constant returns to scale in labor only. Regular paid work is done in both sectors, while black-market and household work are performed only in the household service sector. Individuals are of two types who differ in their relative productivities, in regular paid work in sector 2 versus sector 1. The industrial good is sold on the world market at a given unit price; here productivities for the two worker types are $q_1$ or $q_2$ ($>q_1$). Type 1 (hereafter denoted low-productivity or only “low”) individuals have the same productivity, and wage, $q_1$ in regular paid work in sector 1. Type 2 (high-productivity or only “high”) individuals never choose to do paid work in sector 1 (since their productivity and wage is always greater in sector 2). Assume that both types have productivities in black work in sector 1 equal to $q_{B1}$ (in the same way as their white-market sector 1 productivity is the same), which is intermediate between regular-work productivity in sector 1, $q_i$, and their productivity in home production, $q_{Hi}$, implying $q_1>q_{B1}>q_{Hi}$, $i = 1,2$. Intuitively, activities 1 and 2 imply advantages of specialization over activity 3. Activity 1 may in addition imply economies of scale, which cannot be reaped in activity 2 (since these are likely to be small, one-man, firms).\(^3\) It is then never efficient for high individuals to do any black-market work; thus all this work is done by low

\(^3\) Choi and Thum (2002) correspondingly show that black-market firms are likely to be inefficiently small.
individuals. Assume also that high individuals have a comparative advantage in industrial versus home production, i.e., \( q_2/q_1 > q_{2H}/q_{1H} \). Table 1 gives an overview of the main assumptions regarding productivities.

--- Table 1 in about here ---

Each individual’s (or household’s) total work time, split between white, black and own-household work, is given and normalized to one. The required amount of household work for each household is fixed at \( W_H \). If an individual (or household) of type \( i=1,2 \) itself carries out this work, the time it will take is \( H_i = W_H/q_{iH} \), where \( H_2 \leq H_1 \). Whenever market services are produced and sold in sector 1, the time required to perform the household work for one household is \( H = W_H/q_1 < H_1 \). \( H_B = W_H/q_B \) is the amount of work time required to perform the required black household services for one household.

Under certain assumptions, at an efficient allocation all high individuals produce in sector 2 only, while type 1 individuals produce in both sectors. It is never efficient for any worker to engage in black labor nor do own housework, since labor productivity is everywhere highest in market production.

2.1 Equilibrium given no black market

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4 This will hold also if we assume that type 2 individuals have an absolute advantage in black-market production, but not a comparative advantage relative to the industrial sector.

5 It is here not obvious that type 2 workers have an absolute advantage over type 1 workers in either household or black production, i.e. we may have \( q_{1H} > q_{2H} \), and \( q_{1B} > q_{2B} \). This may follow from specialization activity which makes the “less productive” type 1 workers more productive in home or black market production in which they specialize.

6 The relevant assumptions are with respect to the sizes of the two groups of workers, in particular, there must be sufficiently many low workers relative to high workers for low workers to be able to perform all the required household work for high workers.
We now introduce taxation but still disregard the black market. Given that both leisure time and necessary household work are fixed, households maximize their net after-tax income available for purchasing market (industrial) goods. Denote this income by \( R(i,W) \) and \( R(i,H) \), when a household of type \( i \) purchases its home services in the market, and itself produces these services, respectively, where \( R(i,W) = (1-\tau)q_i - Hq_i \), and \( R(i,H) = (1-\tau)(1-H_i)q_i \). Note that relative price effects here can be ignored, as prices of industrial and home services both are determined by the (given and equal) labor costs. Three cases are relevant: a) both types of households demand home services in the market; b) low individuals produce their own home services, while high individuals demand market home services; and c) both types do their own home services. Case a occurs given that 

\[
1 - \tau > \frac{q_{1H}}{q_1} .
\]  

When (3) holds, \( R(i,W) > R(i,H) \) for \( i = 1,2 \) under this case. Case b holds whenever 

\[
\frac{q_{2H}}{q_2} \leq 1 - \tau < \frac{q_{1H}}{q_1} .
\]  

Under condition (2), (1) fails to hold, and at the same time \( R(2,W) > R(2,H) \), leading type 2 households only to demand home services in the market. Finally, case c, implying that all home services are produced by the household, arises whenever 

\[
1 - \tau \leq \frac{q_{2H}}{q_2}
\]  

holds, which implies that \( R(2,H) \geq R(2,W) \).
To sum up, with no black market and a small tax wedge $\tau$, all household work will be performed as paid work in sector 1. When the tax wedge is higher (consistent with (2)), low individuals find it worthwhile to do their own household work, while high individuals still demand such work in the regular paid market. Finally, when the tax wedge is even higher such that (3) holds, all household work is done within the households.

Net welfare can be identified with the total value of industrial production. Clearly welfare is greatest with maximum specialization and all households demanding home services in the white market. Welfare is lowest in case c, intermediate in case b and highest in case a. See appendix A for a derivation of the respective measures.

2.2. Solutions with black-market service demand

2.2.1 Low individuals demand black services under case b

I now introduce a black market for home services. In sub-sections 2.2.1-2.2.4 I go through a taxonomy of relevant cases starting with case b (only low individuals do their own household work with no black market), and then go on to cases a (none do their own household work) and c (all do their own household work). The model now contains two new elements, relative to the basic framework set out in section 2.1. The first is a mechanism for random matching of black-market demanders and suppliers. Secondly, there is a mechanism for detection and punishment of criminal black-market activity. These elements differ somewhat in detail between cases, as discussed more carefully in appendices B-C. I here find it most natural to present the basics of these additional elements as an integral part of the model.

Consider first case b above where, absent a black market, only low households do their own household production. Assume that some household work for low households is done

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9 This implies assuming that labor allocated to household production is a cost. Since the value of this production is constant, it would change nothing fundamental to include it in the measure of welfare.
in the black market (by low individuals), while high households still purchase their household services in the white market. Both buyers and sellers in the black market are now of the low type.\textsuperscript{10} Defining $w_B$ as the wage paid for black work, supplementary income from a black work assignment is $H_Bw_B$. A buyer of black services has a net gain from this trade equal to

$$G_D = H_1(1-\tau)q_1 - H_Bw_B,$$

where the first term is the gain in net after-tax income (working in regular paid work) when time $H_1$ is freed up for market work. The supplier has net expected gain equal to

$$G_S = H_Bw_B - H_B(1-\tau)q_1 - \gamma F,$$

where the second term is the supplier’s opportunity cost in terms of possible net work income from regular paid work. The last term is the expected fine when caught doing black work.\textsuperscript{11} The two parties bargain over the net match surplus, in an asymmetric Nash bargain, deciding on compensation for black work $w_B$, with relative bargaining strengths $\beta$ and $1-\beta$ to the demander and supplier, respectively. The solution to this bargain yields the following expressions for $G_D$ and $G_S$:

$$G_D = \beta[(1-\tau)(H_1-H_B)q_1 - \gamma F]$$

$$G_S = (1-\beta)[(1-\tau)(H_1-H_B)q_1 - \gamma F].$$

The square brackets here represent total expected gain to the demander and supplier of black services, over doing ones own household work for the demander, and over doing regular paid work in sector 1 for the supplier.

\textsuperscript{10} Although both are of type 1, the properties of demanders and suppliers for a given service generally differ in the sense that there is specialization in black home supply among suppliers, where one supplier serves several demanders, while each demander demands several different services from different suppliers. Individuals thus take on dual roles as demanders and suppliers, and may e.g. have different bargaining powers and search costs in the two different roles. We here simplify the presentation of this market, by assuming that search takes place only once for a given demander, and with respect to only one type of service. Note also that a supplier of black services will most likely supply his or her particular service to ones own household. As long as the particular service supplied constitutes only a small fraction of total service demand, this does not fundamentally alter the set-up of the model.
By its nature a black market involves secrecy and thus informational costs for suppliers and demanders. I represent these in the simplest possible way by assuming that one unit of search in the market for black services takes a fixed amount $H_S$ of time for a demander, and time $\lambda H_S$ for a supplier.\(^{12}\) This unit gives a demander a probability $\rho = h(0)$ of finding a supplier of black services, and gives a supplier a probability $\pi = h(1/0)$ of finding a demander for black services, where $\theta = N_{SB}/N_{DB}$, and $N_{SB}$ and $N_{DB}$ denote the numbers of active suppliers and demanders in the black market, and $h' > 0, h'' < 0$. The $h$ function is similar to the Pissarides (2000) labor market matching function. It is assumed to exhibit constant returns to scale, such that a doubling of the numbers of demanders and suppliers of black services renders the matching probability constant. Assume that each demander and supplier searches only once. Note that since $\rho N_{DB} = \pi N_{SB}$ (= the number of realized black market jobs), $\theta = \rho/\pi$, and $\pi = h(0)/\theta$.

The full analytical solution for this matching market is derived in appendix B. We there find that $G_D$ and $G_S$ in a search equilibrium obey the conditions

\[
\rho G_D = H_S(1-\tau)q_1 \\
\pi G_S = \lambda H_S(1-\tau)q_1.
\]

An equilibrium solution with black-market activity can be found if and only if

\[
\frac{(1-\tau)H_S q_1}{h\left(\frac{1-\beta}{\lambda \beta}\right)} \in \left[G_{D_{-min}}, G_{D_{max}}\right],
\]

where $G_{D_{min}}$ and $G_{D_{max}}$ are given by (6) setting $\gamma$ equal to 0 and 1, respectively. Type 2 individuals will choose not to demand black-market services given that

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\(^{11}\) We assume that only the supplier is subject to this penalty. In most countries it is in fact not illegal to buy black services. One could still of course picture the possibility of a related psychological cost also for the demander.

\(^{12}\) In principle, $\lambda$ could here be either greater or smaller than one. If the market mainly takes the form that suppliers search for potential customers door-to-door, $\lambda$ could be greater than one. On the other hand if
\[ \rho \beta [H q_1 - H_B(1-\tau)q_1 - \gamma F] < H_S(1-\tau)q_2. \]  \hspace{1cm} (11)

In appendix C I derive the behavior of the agency in charge of penalizing offenders who trade in the black market. The agency is taken to maximize net penalty revenues given by

\[ R = \gamma F \pi N_{SB} - C = \gamma_1 (C/N_B) \gamma_2 (N_B) N_B F - C, \]  \hspace{1cm} (12)

where \( C \) is total enforcement cost, \( N_B \) the number of realized trades in the black market, and \( \gamma_1 \) and \( \gamma_2 \) are components in the agency’s enforcement cost function. The first-order condition for the agency is found as

\[ \gamma_1'(C_B) \gamma_2(N_B)F = 1. \]  \hspace{1cm} (13)

These elements are now combined in deriving the first main result, as follows.

**Proposition 1:** Assume that (10) holds, and that the government sets the enforcement effort according to condition (13). Then given that

\[ \rho \beta [H_{11} - (1-\tau)q_1] < H_S(1-\tau)(q_2 - q_1) \]  \hspace{1cm} (14)

holds, a unique equilibrium exists, in which only some fraction of low individuals, who would else do their own household work, purchase black-market services.

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demands look for potential suppliers whenever a particular need arises (as when the house, car or household equipment needs repair or maintenance), \( \lambda \) could be less than one.
Proof: (14) follows from combining (6), (8), (11) and (B5) in appendix B. It is verified that (13) solves uniquely for $\gamma$. (6) and (8) then hold simultaneously, implying a unique solution for $N_B$. Q.E.D.

The left-hand side bracket in (14) represents the gain from a high individual demanding black services, over the gain from a low individual demanding the same services, as considered jointly by the seller and buyer of black services. When (2) holds (only high individuals demand white services), this difference is positive. It is however small when $q_{1H}$ is close to $(1-\tau)q_1$, i.e., there is little to gain by type 1 households producing their own services. The right-hand side of (14) represents the difference in black-market search costs for type 2 versus type 1 households, which are positive. (14) is more likely to hold when $\tau$ is smaller.

When (14) and (2) both hold, type 1 households never purchase white services and are the only demanders of black services. $\tau$ must then be sufficiently high for (1) not to hold, and sufficiently small for (14) to hold. Thus, an "intermediate" range for the tax rate leads to black-market demand by low households only, given that low households else do their own housework and high households else buy it in the white market. When the tax rate is lower, low households buy home services in the white market in the absence of a black market. When the tax rate is higher, high households are instead black market demanders.

Higher tax rates lead high workers to demand black services instead of low workers, for two main reasons. First, a higher tax makes the time freed up for a low worker, when his household work is instead purchased in the black market, less valuable. Secondly, higher $\tau$ reduces search costs proportionately for all individuals (since labor time becomes less valuable to all), but by more in absolute terms for high individuals who have the higher time cost.
Note that (14) tends to hold when a) the time required to produce one’s own home services, $H_1$, is small relative to the time required to find a match in the black market, $H_S$; and b) the difference in labor productivity, between the two types of paid labor is great.

The mechanism specified in appendix B does not determine the realized number of black-market offenses, $N_B = \pi N_{SB} (= \rho N_{DB})$, nor the equilibrium expected penalty for tax fraud $\gamma F$. $\gamma F$ could be thought of as determined in at least two ways. First, if the government sets $\gamma$ and $F$ optimally, an arbitrarily large number of black-market trades $N_B$ can in the model be encouraged by setting $\gamma F$ ”slightly lower” than the level yielding equality in (8)-(9). At the same time enforcement costs can be made arbitrarily low by setting $F$ ”large” and $\gamma$ ”low”.$^{13}$ Ideally, the government might wish to encourage a high volume of black-market work when such work is seen as gainful, and discourage such work when it is seen as harmful, through appropriate values of $\gamma F$. Alternatively a government agency which does not necessarily maximize overall welfare may be in charge of enforcing tax laws, and be subject to an administratively set fine level $F$. This is the case considered explicitly here, in appendix C.$^{14}$

I now consider welfare effects. Note that all private black-market gains are eroded by the condition of free entry. All net gains then take the form of changes in government revenue. Consider the total gain as a function of black market transaction volume, $N_B = \rho N_{DB}$. From the definition of $G_D$ this can be written as

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$^{13}$ An upper limit to $F$ may appear arbitrary in view of Becker’s (1968) seminal analysis which implies that (risk-neutral) individuals should face an ”infinite” punishment with an ”infinitesimal” probability. In most societies tax cheating is however not viewed as a very serious crime, with legal limitations on a socially acceptable punishment for caught offenders. E.g., in Norway the tax authorities may (for ”moderate” tax evasion offenses) impose only a 60 % addition to the original tax claim. We will here essentially take the limitations on $F$ as given and not go deeply into the issue of whether higher levels are socially desirable.

$^{14}$ The approach taken here, and elaborated in the appendix, is stylized and perhaps unrealistic in some respects. In particular, it gives no role to the enforcement agency in consciously determining the level of tax evasion, since the agency here takes this level as given. Our main purpose is to provide a behavioral model that serves to pin down the equilibrium level of black-market activity. See e.g. Androni, Erard and Feinstein (1998) for more thorough discussion of government objectives in this and similar contexts. A main point in much of the related literature is that a government agency here cannot automatically be viewed as welfare-maximizing; it may be
\[ NR(1) = N_B \left[ (H_1 - H_B) \tau q_1 + \gamma F - \frac{C}{N_B} \right], \] 

(15)

where average surplus created per realized black-market trade is inside the square bracket. Clearly \( NR(1) > 0 \), since both the first and the sum of the two last terms are positive. The first term represents the increase in tax revenue, which has two components. One is the increase in tax revenue as freed-up time for low individuals leads to more white (taxable) work, represented by the term \( H_1 \tau q_1 \) in (15). The other is a drop in regular tax revenue as a result of black instead of white work, represented by the term \( H_B \tau q_1 \). The first of these terms however exceeds the second: the work time freed in the home production sector is always greater than the work time required to perform the black-market work. The two last terms in the square bracket in (15) together represent net government revenue from tax-compliance enforcement, which is assumed positive; see also appendix C.

From (15) welfare increases when more low individuals demand black-market services. Given a uniform tax rate \( \tau \), such reallocation, away from own household work, cannot be accomplished in other ways, and the black market can rightly be said to improve efficiency. Part of the potential allocation gain is however dissipated by two types of "rent-seeking" costs, black-market search costs, and government enforcement costs.

### 2.2.2 High individuals demand black services in case b

We now assume that condition (2) still holds, but high workers, who else would have demanded home services in the white market, are now the black-market demanders. A
A detailed analysis of equilibrium in the matching market in this case is found in appendix B. I there show that the following condition must be fulfilled for such a solution to exist:

$$\frac{(1 - \tau)H_2q_2}{\lambda H_2q_2} \in \left[ G_{D2\text{max}}, G_{D2\text{min}} \right]$$

(16)

where $G_{D2\text{min}}$ and $G_{D2\text{max}}$ are defined in a fashion analogous to that of the previous case. A result similar to Proposition 1 can now readily be derived as follows:

**Proposition 2:** Assume that (16) holds, an inequality opposite to (14) holds, and else that the conditions for Proposition 1 hold. Then there exists an equilibrium where some but not all type 2 households, who else would have produced their own home services, demand black market services.

The welfare implications of a black market are now always negative. The change in welfare can, as in section 3, be represented by the change in net total government tax revenue (i.e., gross tax revenue minus enforcement costs). Denote the number of realized black-market trades by $N_{B2}$. This change can then be written as

$$NR(2) = N_{B2} \left[ -H_2\tau q_1 + \gamma F - \frac{C}{N_{B2}} \right].$$

(17)

Since $G_{D2} > 0$, $H_2\tau q_1 - \gamma F > 0$, which implies $NR(2) < 0$. The loss of tax revenue, when household work is done in the black market instead of the regular taxable market, must now be greater than the (gross) revenue raised from fines on black-market tax violations. Intuitively, such a greater loss of tax revenue is required for high individuals to be willing to demand black services in the first place. The loss due to the presence of a black market
has three main components: an allocation loss due to household services production being less efficient in the black than the white market; search costs of black-market agents; and costs of the enforcing government agency.\textsuperscript{15}

2.2.3 Both types demand white services

I now consider case a in section 2.2 above, where both types buy white services in the absence of a black market. Here $\tau$ is sufficiently low for (1) to hold, but sufficiently high to make black-market trade advantageous. Only low individuals are black-market demanders, since these have the lower search costs. The equilibrium condition, making low individuals indifferent about black-market trading and not, is now

$$\rho \beta (H - H_B (1 - \tau)) q_1 - \gamma F = H_S (1 - \tau) q_1.$$  \hspace{1cm} (18)

This condition may hold when $\tau$ is ”not too small”, and $H - H_B$ not too great negative. This leads to the following result in the present case:

**Proposition 3:** Assume that (1) holds together with the condition

$$\rho \beta [H - H_B (1 - \tau)] \geq H_S (1 - \tau),$$  \hspace{1cm} (19)

and that a solution to (18) exists for $\gamma \leq 1$, and all households demand white services. Then only low households demand black services, and the black market has negative welfare effects.

\textsuperscript{15} We assume no enforcement costs for tax compliance in the white sector. This is too simple, but one may at
With no tax enforcement ($\gamma F=0$), the solution described by Proposition 3 is viable whenever (19) holds. Since $H < H_B$, (19) cannot hold when $\tau$ is sufficiently low; there is no scope for a black market when white work is not taxed. For higher values of $\tau$, only provided that (1) holds, a solution to (13) can be found for $\gamma F \geq 0$ whenever the black-market search time cost for demanders, $H_S$, is not too high.

As in subsection 2.2.2, the black sector imposes a social loss, but the welfare loss per black-market trade is smaller. The production loss is the same, but black-market search costs are smaller, since low-productivity demanders have lower time cost of search.

### 2.2.4 Both types do their own household work

In our final case (c in section 2.1), both types produce their own household services in the absence of a black market. The tax rate $\tau$ must then be sufficiently high for (3) to hold. We first need to determine whether high or low individuals are black service demanders. If low individuals are such demanders, (8) still holds for these, with $G_D$ given from (6). If instead high individuals demand black services, (B11) in appendix B (which represents (8) for high individuals) must hold for these except that $G_{D2}$ is determined by

$$G_{D2} = \beta[(1-\tau)(H_2q_2 - H_Bq_1) - \gamma F].$$  (20)

Low individuals now purchase black services, while high individuals purchase white services, given opposite inequality in (B11) with $G_{D2}$ given from (20). We find:

**Proposition 4:** Assume that (3) holds. Then no households demand white household services. Low households demand black services given that least argue that the latter costs per household are far lower than those in the black sector.
\[ \rho \beta (H_2 q_2 - H q_1) < H_S (q_2 - q_1). \]  

(21)

High households demand black services given that an inequality opposite to (21) holds.

(21) follows from (20) and depends only on fundamental parameters of the model (and not e.g. on the tax rate or tax enforcement): the productivity difference between the two types in industrial production, \( q_2 - q_1 \); and the difference between \( H_2 \) and \( H \), which expresses an absolute advantage of high individuals in home production, relative to low individuals producing white household services. In the special (and possible) case of \( H_2 = H \), (16) simplifies to \( \rho \beta H < H_S \). Low individuals are then black-market demanders when black-market search costs are high relative to expected market returns \( (H_S/H \) great relative to \( \beta \rho \)). Whenever \( H_2 > H \) (high individuals are less productive in own household work, than low individuals are in paid white work in the home sector), (21) is less likely to hold the greater is this difference, and the greater is \( q_2 - q_1 \).

When (21) holds, welfare implications are the same as in subsection 2.2.1: the black market is socially gainful by increasing the economy’s total output for a given tax rate \( \tau \). A main difference is that high individuals now produce their own home services instead of buying these in the white market.

When we have an inequality opposite to (21), the presence of a black market is socially gainful, and more so than in subsection 2.2.1, since high individuals now engage in black-market trades only when the social surplus created is greater than that created by low individuals. The distributional implications may however be less favorable, since high-productivity (and –income) workers now are the ones taking advantage of black market access.
3. Conclusions

I have studied a model where household service production can be carried out either within households, as regular paid "white" market work, or as "black" market work where tax is evaded. Households are of two types, where “high” types have a comparative advantage in "industrial" versus household service production. In the absence of a black market, all household work will be carried out in the (more efficient) white market provided that the overall tax wedge in the economy is "low", and done by households themselves when the tax wedge is "high”. When the tax wedge is in an intermediate range, only low households do their own household work. We then introduce a black market with productivity intermediate between own-household and white production, and where suppliers and demanders must incur costly search. Either low or high individuals, but never both, demand black services, which are always supplied by low individuals.

Overall, the black market is socially favorable whenever the work carried out by black-market suppliers otherwise would have been done within the households, but unfavorable when it instead "steals" activity from the "white" market. This is not surprising given the model setup. The more important and surprising conclusions concern the relationship between the overall tax wedge and black-market demand. Table 1 gives a summary of different parametric cases, and their relation to the tax rate. In the table I distinguish between 5 main cases, as follows.

1. When the tax wedge is low, all households demand household services in the white market and there is no basis for a black market. The entry and secrecy costs combined with the allocation losses from services being produced in the black instead of white market, then serve to rule a black market out.
2. When the tax wedge is sufficiently high to make a black market viable, but still sufficiently low for condition (1) to hold, all households demand household services in the white market when a black market is absent. This is represented by cases 5-6 in table 1. In this case only low households demand black services, their lower search cost being the decisive factor.

3. When the tax wedge is higher, corresponding to condition (2), low households, who would else produce their own home services, buy black services. This is represented by case 3 in table 1.

4. When the tax wedge is even higher, only high households demand white services when the black market is absent, and only these households also demand black services. This is case 4 in table 1.

5. When the tax wedge is sufficiently high for condition (3) to hold, all produce their own home services in the absence of a black market. Either high or low households may then be black-market demanders, represented by cases 1-2 in the table. Case 2 (high households demand black services) is here more likely when black-market search costs are small.

--- Table 2 in about here ---

The black market is favorable under points 3 and 5 (when the black market “steals” activity from own household production), and unfavorable under points 2 and 4 (when it “steals” activity from the white market). Since households buy their home services in the market when the tax wedge is low, and do their own home production when it is high, then there is, unsurprisingly, a more negative effect of black-market trades when tax rates are lower, with one important exception: For "intermediate” tax rates (such that (2) holds),
results are more surprising and less intuitive. The black market is then favorable only when the tax rate is ”relatively low” within this range.

One should be careful in drawing strong policy conclusions from this simple model. In particular, I ignore differential taxation between sectors, and negative effects on the degree of “morality” from more tax evasion when the black service sector grows.\textsuperscript{16} When black-market trading mainly reduces white-market activity, the existence of a black service market almost certainly reduces welfare and should be discouraged. A related policy implication follows for the tax wedge itself. Higher tax wedges typically reduce efficiency by pushing more activity out of the white market, and into the household and/or black sectors. A novel result here is that this relationship is not uniform for intermediate tax rates. An increase in the tax wedge (moving from point 3 to point 4 in the classification above) may here in principle render black-market activity less favorable to society, as less favorable consumer groups then demand black services. The main impact of this point is probably in terms of how to interpret data on the relationship between black-market activity. It is more questionable whether this ought to have the policy implication that the tax wedge ought to be increased.

Note that the presence of a (possibly large) black market for labor is by itself no clear indication of efficiency or inefficiency in this model. One may here also consult Schneider (2000), who in fact finds no strong relationship between black markets prevalence and indicators of efficiency (such as level of per capital GDP or rates of growth). Obviously, more research is required for settling such issues.

In the model worker preferences over the goods involved, and the technology for producing them, take a particularly rigid and unrealistic form. My assumptions here are made mainly for analytical tractability and to make it possible to derive sharp results even
when integrating search behavior into a three-sector model. I will here discuss two particularly important sets of assumptions, that need modification in future work.

First, my assumption that workers are risk neutral and have separable utilities is very strong.\(^{17}\) This implies that risk per se, associated with punishment of black-market crime, has no impact on behavior, which is of course unrealistic. Black-market activity is inherently risky while white-market and own-production activity are not; indeed, an essential aspect of government-imposed punishment schemes against black-market and other crimes is that such schemes impose substantial risk on those punished (and that enforcement to a large extent relies on the property that individuals are risk averse). The risk neutrality assumption then clearly biases the conclusions in the direction of making black-market outcomes too favorable. Unrealistically also, preferences are considered separable in consumption and other possible arguments (such as leisure and stigma from being caught and punished for black work). More general assumptions here may alter my conclusions, in particular as marginal conditions are likely to be affected by the various cross derivatives.\(^{18}\) One way to approach this problem is to assume an explicit indirect utility function on which particular conditions can be imposed. This was done by Mayshar (1991) in a similar setting, who imposes conditions under which risk aversion will not affect behavior in white-market or own-productive activities (as these are assumed not to be risky here). Whenever a black market arises at all, the presence of a black market will then still be socially favorable (and more efficient except in marginal cases) when the alternative is own household production. When the alternative is white production, black-market production is always less favorable, and the difference greater than in our model.

\(^{17}\) I am grateful to a referee for pointing out to me weaknesses with the risk neutrality assumption here.

\(^{18}\) See Slemrod and Yitzhaki (2002) for further references.
The other main unrealistic aspect of preferences and technology in my model I will note, is that all household goods, whether produced in the white or black market or in the household, are assumed to be perfect substitutes.¹⁹ This is unrealistic as services performed in the white and black markets and within the home are likely to differ in some aspects, and in different ways for different households. With imperfect substitutability there is always likely to be room for a black market in some extent, as some households will always prefer to demand some services in that market. We will then have a smooth relationship between the tax and productivity parameters, and the activity levels in the three service-producing sectors. Factors in disfavor of a black market in my model will lead to a contraction of this sector in the more general model, but not to an elimination of it. In that sense the more general model will yield more general and plausible, but also less sharp, results.

¹⁹ Some of the weaknesses of these assumptions were pointed out to me by Agnar Sandmo.
### Table 1: Overview of productivities

<table>
<thead>
<tr>
<th>Worker type</th>
<th>Productivity in market work, sector 1</th>
<th>Productivity in market work, sector 2</th>
<th>Productivity in home production</th>
<th>Productivity in black-market production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low workers</td>
<td>$q_1$</td>
<td>$q_1$</td>
<td>$q_{1H}$</td>
<td>$q_B \in (q_{1H}, q_1)$</td>
</tr>
<tr>
<td>High workers</td>
<td>$q_1$</td>
<td>$q_2$ (&gt; $q_1$)</td>
<td>$q_{2H} \in [q_{1H}, q_{B2})$, $q_2 /q_1 &gt; q_{2H}/q_{1H}$</td>
<td>$q_B \in (q_{2H}, q_1)$</td>
</tr>
</tbody>
</table>

### Table 2: Overview of main equilibrium cases

<table>
<thead>
<tr>
<th>Basic condition on Taxes</th>
<th>Both types do own home production.</th>
<th>Low workers only do own home production.</th>
<th>None does own home production.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau &gt; 1 - q_{2H}/q_H$</td>
<td>$\tau \in [1-q_{1H}/q_1, 1-q_{2H}/q_2]$</td>
<td>$\tau &lt; 1 - q_{1H}/q_1$</td>
<td></td>
</tr>
<tr>
<td>Low workers in black market</td>
<td>$\rho\beta(H_2q_H - Hq_1) &lt; H_5(q_2^{-} - q_1)$ (case 1)</td>
<td>$\rho\betaH_1\left[\frac{q_{1H}}{- (1 - \tau)q_1}\right] &lt; H_5(1 - \tau)(q_2^{-} - q_1)$ (case 3)</td>
<td>$\rho\beta[H_1q_{1H} - H_B(1-\tau)q_1 - \gamma F] = H_5(1 - \tau)q_1$ (case 5)</td>
</tr>
<tr>
<td>High workers in black market</td>
<td>$\rho\beta(H_2q_2h - Hq_1) &gt; H_5(q_2^{-} - q_1)$ (case 2)</td>
<td>$\rho\betaH_1\left[\frac{q_{1H}}{- (1 - \tau)q_1}\right] &gt; H_5(1 - \tau)(q_2^{-} - q_1)$ (case 4)</td>
<td>Never (case 6)</td>
</tr>
</tbody>
</table>
Appendix A: Welfare measures with no black market

Denote the number of households of type i=1,2 by Ni. Welfare measures are in cases a-c:

\[ V(a) = N_1 q_1 + N_2 q_2 - N W_H \]  
(A1)

\[ V(b) = N_1 q_1 (1 - \frac{W_H}{q_{1H}}) + N_2 (q_2 - W_H) \]  
(A2)

\[ V(c) = N_1 q_1 (1 - \frac{W_H}{q_{1H}}) + N_2 q_2 (1 - \frac{W_H}{q_{2H}}) \]  
(A3)

where \( N = N_1 + N_2 \). Welfare is here greatest in case a and lowest in case c, i.e. \( V(a) > V(b) > V(c) \). \( V(a) \) is here the efficient allocation discussed in subsection 2.1. It requires that \( N_1 q_1 > N W_H \), which since \( W_H = q_1 H \), implies \( H < \alpha \), where \( \alpha \) denotes the share of type 1 workers in the labor force. We find:

\[ V(a) - V(b) = N_1 \frac{q_1 - q_{1H}}{q_{1H}} W_H \]  
(A4)

\[ V(a) - V(c) = (N_1 \frac{q_1 - q_{1H}}{q_{1H}} + N_2 \frac{q_2 - q_{2H}}{q_{2H}}) W_H . \]  
(A5)

Welfare losses in cases b-c are due to the tax wedge pushing productive activity out of the paid market and into the household sector where productivity is lower. In case b this affects low-productivity households only, while in case c it affects both household types. The average allocation loss is generally greater in case c, since the loss due to doing one’s own household work is greater for type 2 than for type 1 households.

Appendix B: Analytical derivation of matching solutions

Case 2.2.1

In this case, suppliers and demanders of black services are in equilibrium when the expected gain from engaging in black services (over the relevant alternatives, which are household work for the buyer and white sector 1 work for the seller) equals the value of the search cost, in terms of net work income foregone when search takes an amount \( H_S \) of time. This requires (referring to equations in the text above, and otherwise using number B1, etc.).

\[ \rho G_D = H_S (1-\tau) q_1 \]  
(8)

\[ \pi G_S = \lambda H_S (1-\tau) q_1 . \]  
(9)

Using (6) and (7) (where we note that \( G_S = [(1-\beta)/\beta]G_D \) and the condition \( \rho = \theta \pi \), we derive the following simple condition:

\[ \theta = \frac{1-\beta}{\lambda \beta} \]  
(B1)
\( \rho \) and \( \pi \) can now be considered as fixed parameters. A possible equilibrium where (8)-(9) are fulfilled with equality now requires a particular level of the expected fine \( \gamma F \). Our model then solves for 0 but not \( N_{DB} \) and \( N_{SB} \). For the moment we just take both the required level of \( \gamma F \), and a particular solution value for \( \rho N_{DB} \), as exogenous. A necessary condition for the existence of such an equilibrium can in this case be derived as

\[
\frac{(1-\tau)H_S q_1}{h}\left[1-\frac{\beta}{\lambda}\right] \in \left[G_{D_{\min}}, G_{D_{\max}}\right],
\]

(10)

where

\[
G_{D_{\min}} = \beta[(1-\tau)(H_1-H_B)q_1 - F]
\]

(B2)

\[
G_{D_{\max}} = \beta(1-\tau)(H_1-H_B)q_1.
\]

(B3)

To see this, note first that \( G_{D_{\min}} \) and \( G_{D_{\max}} \) are minimum and maximum values of \( G_D \) compatible with a solution in the present case, found from (6)-(7) setting \( \gamma \) equal to 1 and 0 respectively. No relevant solution can be found when (10) does not hold. The left-hand side of (10) is less than \( G_{D_{\min}} \) when the gain from evading taxes, \( (1-\tau)H_S \), is small. Black market activity is then always favorable for type 1 individuals. When the left-hand side of (10) exceeds \( G_{D_{\max}} \), black-market trading is unfavorable even with no chance of being caught for tax fraud. This case is uninteresting since it would preclude the very existence of a black sector.

To verify that only type 1 individuals engage in black-market activity, we must consider a possible bargaining solution between a type 2 buyer and type 1 seller of black services. Provided, as assumed, that type 2 individuals would else buy the required services in the white market, the bargaining surplus of a type 2 buyer can then be written as

\[
G_{D2} = Hq_1 - H_B w_B = H_1 q_{1H} - H_B w_B,
\]

(B4)

where as before \( H_{q1} = H_{1q1H} = W_H \). Since the bargaining surplus of the seller has the same form as (5) (only replacing \( w_B \) by \( w_{B2} \)), the bargaining solution with relative bargaining strengths \( \beta \) and 1-\( \beta \) now implies

\[
G_{D2} = \beta[Hq_1 - H_B(1-\tau)q_1 - \gamma F]
\]

(B5)

\[
G_{S1} = (1-\beta)[(H - H_B(1-\tau))q_1 - \gamma F].
\]

(B6)

Assume that a potential type 2 buyer, in the same way as a type 1 buyer, spends an expected time \( H_S \) searching for a seller of black services. Since a type 2 individual’s net return to labor is \( (1-\tau)q_2 \), his search cost is \( H_S(1-\tau)q_2 \), which is greater than that of type 1 individuals. The condition under which it is disadvantageous for a type 2 buyer to enter the black market is then the following condition in the text above:

\[
\rho G_{D2} < H_S(1-\tau)q_2.
\]

(11)

**Case 2.2.2**

In this case equilibrium in the black market is characterized by
\[ \rho G_{D2} = H_s(1-\tau)q_2 \]  
\[ \pi G_{S1} = \lambda H_s(1-\tau)q_1 \]  

where \( G_{D2} \) and \( G_{S1} \) are given by (B5)-(B6). We now find

\[ \theta = \frac{1 - \beta q_2}{\lambda \beta q_1}. \]  

(\text{B9})

\( \theta \) is now greater than that in (B1), with type 1 demanders. Search costs of demanders are now higher, and the equilibrium ratio of suppliers to demanders higher. Under an inequality opposite to (14), black work will be demanded by type 2 individuals. This holds when \( \tau \) is "relatively high" while staying within the range defined by (2).

Also here a condition similar to (10) must hold for black market trade to be viable, but now with only type 2 demanders in the black market. This condition is:

\[ \frac{(1-\tau)H_s q_2}{h} \in \left[G_{D2\min}, G_{D2\max}\right], \]  

(\text{B10})

where

\[ G_{D2\min} = \beta [Hq_H - H_B(1-\tau)q_1 - F] \]  
\[ G_{D2\max} = \beta [Hq_H - H_B(1-\tau)q_1]. \]  

(\text{B11}) (\text{B12})

The proof is similar to that for (10). \( G_{D2\min} < H_s q_2 / h \) can, as in (10), be viewed as a minimum condition on \( F \). \( G_{D2\max} > H_s q_2 / h \) is, also as above, a basic "feasibility" condition whereby (for given \( \tau \)) the surplus from engaging in a black-market trade must be sufficiently great given no fines for tax evasion. \( G_{D2} \) can alternatively be written as

\[ G_{D2} = \beta [-H_B - Hq_1 + H_B \tau q_1 - \gamma F], \]  

(\text{B13})

where we have used that \( Hq_{1H} = Hq_1 = W_{1H} \), and \( H_B - H > 0 \). \( \tau \) must here be "sufficiently high" to make the second term in the square bracket in (B13) dominate "sufficiently" over the other terms.

**Appendix C: Analysis of the behavior of a tax enforcement agency**

We will in the following look closer at the behavior of an agency in charge of enforcing tax laws, and possibly catching tax offenders. The average probability \( \gamma \) of catching a black supplier is now viewed as endogenous. We will for simplicity assume that the agency in charge of tax enforcement has the objective of maximizing net revenue from catching black offenders, i.e. it maximizes gross fines minus enforcement costs. Assume also that the agency views \( N_B \) as exogenously given; essentially, the agency has no direct concern for the number of tax offenses as such. Assume that \( \gamma \) is determined by the function \( \gamma = \gamma_1(C/N_B) \gamma_2(N_B) \), where \( C/N_B \) is enforcement cost per committed offense. Assume that \( \gamma_1' > 0, \gamma_1'' < 0, \) and \( \gamma_2' > 0 \), where primes denote derivatives. A greater enforcement cost per offense thus, reasonably, raises the probability of catching a given offense. We also assume that when the overall number of offenses rises, the probability of
catching any given offender rises for a given enforcement cost per offense. This may follow from certain returns to scale in the enforcement technology, e.g. the tasks of enforcement officers can become more specialized when the corps of such officers is larger. The objective function of the enforcement agency can then be written as

\[
R = \gamma F \pi N_{SB} - C = \gamma_1(C/N_B)\gamma_2(N_B) F - C. \tag{12}
\]

R is now maximized with respect to C, where as noted \(N_B\) is viewed as exogenous. This yields the following first-order condition for the agency:

\[
\gamma_1'(C_B) \gamma_2(N_B)F = 1, \tag{13}
\]

where \(C_B = C/N_B\) denotes average enforcement costs per offense. We may now study how increases in \(F\) and \(N_B\) affect \(C_B\) and consequently \(\gamma\). We find

\[
\frac{dC_B}{dN_B} = -\frac{\gamma_1'' \gamma_2'}{\gamma_1''} > 0 \tag{C1}
\]

\[
\frac{dC_B}{dF} = -\frac{\gamma_1'}{\gamma_1''} > 0 \tag{C2}
\]

Increases in the number of offenders and the fine when catching an offender both raise average enforcement costs per offense. More offenders makes enforcement more gainful due to the noted scale economies in enforcement. A higher fine also makes enforcement more lucrative for the enforcement agency. Note that the agency does not take into consideration the effect of an increase in \(F\) on \(N_B\). We find:

\[
\frac{d\gamma}{dN_B} = \left[1 - \frac{(\gamma_1')^2}{\gamma_1''}\right] \gamma_1' \gamma_2' > 0 \tag{C3}
\]

\[
\frac{d\gamma}{dF} = -\frac{(\gamma_1')^2 \gamma_2}{\gamma_1''} + \frac{d\gamma}{dN_B} \frac{dN_B}{dF}. \tag{C4}
\]

\(N_B\) clearly affects \(\gamma\) positively. The effect of increased \(F\) on the level of \(\gamma\) chosen by the agency is however more difficult to sign in general. Still a higher \(F\) has a direct positive effect on enforcement costs, through (C2). The last expression in (C4) is however generally is negative (the number of offenders drops when the fine increases), and the overall effect on \(\gamma\) uncertain.

Assume now that (10) holds. We next need to demonstrate that there exists an equilibrium where the enforcement agency has an incentive to set \(\gamma\) at the level required for equality in (8) and (9). Consider then a level of \(\gamma\) slightly lower than the level required for equality in (8). This will make it strictly advantageous to enter the black market, and \(N_B\) will rise. But by (C3), the enforcement agency will respond to this by raising \(\gamma\). One easily realizes that this leads to a stable equilibrium value of \(\gamma\), whereby (8)-(9) hold with equality.
References


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