A continuous-time value function approach
to VSL and VSI valuation

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Abstract

I present a simple analytical model in continuous time where individuals can be either healthy or ill, and have constant transition between the states and constant mortality rates in each state. The values from being in each of the two live states are represented by two Bellman dynamic programming equations. This leads to simple expressions for lifetime values of remaining in each of the states, values of statistical life in good health (VSL) and of statistical illness (VSI), the value of improved health, and the value of health investments. Changes in background risks have intuitively reasonable effects on VSL and VSI, while a “dead-anyway effect” only appears in the valuation of health investment but not in expressions of VSL or VSI. I argue that my model is appropriate for VSL and VSI analysis when interventions have immediate effects on health risks, but less so when risks are delayed. More care than hitherto ought to be taken, in selecting models for such analyses.

Key words: Value of statistical life; value of statistical illness; mortality risks; background risks.

JEL classification: I12, I18, D11.
1. Introduction

The purpose of this note is to expose and clarify certain issues related to valuing changes in mortality and morbidity, based on a simple but instructive continuous-time model with constant transition (or hazard) rates between three mutually exclusive states: healthy, sick and dead; and exponential remaining times in each of the two former states. As is customary in the literature, we define the value of statistical life in good health (VSL) as the rate of substitution between consumption and mortality risk when initially in a healthy state. We similarly define the value of statistical illness (VSI; as coined by Cameron and DeShazo (2003)) as the rate of substitution between consumption and mortality risk, when initially in the state of illness. Under the continuous-time approach, current valuation of current risk change always departs from the current state. This acknowledgement enormously simplifies the notions and derivations of VSL and VSI, relative to some alternative approaches in the recent literature. In particular, issues such as comorbidities (Bleichrodt, Crainich and Eeckhoudt (2003), background risk (Eeckhoudt and Hammitt (2001)), risk aversion (Eeckhoudt and Hammitt (2004)), and “dead-anyway effects” (Pratt and Zeckhauser (1996), Breyer and Felder (2002)) then lose much of their particular roles in VSL and VSI valuation. Such factors are still important under either a discrete formulation of the decision problem facing the individual, or when effects occur with delay, as is typically assumed in these papers. My argument here is however that such discretization or delay is often artificial in the context of VSL or VSI valuation, at least when the interventions considered have immediate effects on mortality and morbidity rates. Relevant examples are medication or treatment for high blood pressure or cholesterol levels (which affect heart conditions immediately), stricter road traffic or workplace safety regulations (which immediately affect accident rates),
or efforts to clean the drinking water (with immediate effects on rates of sickness due to water contamination).¹ For such cases the model framework developed here is particularly relevant. The model is also relevant for cases with “competing risks” as analysed e.g. by Dow, Phillipson and Sala-I-Martin (1999) and Chang (2002), given that the hazard rates are constant.² I however also consider cases where an intervention has a permanent effect on the mortality or morbidity rate, and show that a “dead-anyway effect” then appears, not in VSL or VSI directly, but in the value of such interventions.

To make the model as simple, intuitive and instructive as possible I concentrate on a stripped-down case where an individual’s consumption is constant in a given state (healthy or ill), and risks are time- and age-independent. I first consider instantaneous rate changes in mortality and morbidity rates, and then show that VSL and VSI can be expressed simply as ratios between the expected discounted future utility value of being in the current state, V(H) and V(I) respectively, and the marginal utility of income in the respective states. One implication of this is that the “value of full health” (the value of being in the healthy state instead of in the state of illness) is always greater when the initial state is the ill state, than when one initially is in the healthy state, whenever the marginal utility of income is lower in the ill state (as is considered reasonable). The effects of increases in “background” mortality, morbidity and return-to-full-health rates on VSL and VSI are shown to have intuitively reasonable signs and values (the two first negative and the last positive). In the final section I also note (although do not formally prove) that many of the basic properties of the solutions, such as the basic formulae describing VSL and VSI and the effects of

¹ See also the empirical study by Breyer and Grabka (2001) who find no empirical basis for a dead-anyway effect, on a large German data set.
² See e.g. David and Moeschberger (1978) and Elandt-Johnson and Johnson (1980) for the theoretical arguments supporting the current model, under competing risks.
background risks, are not altered when risk levels change throughout an individual’s lifetime.

The use of value functions of the Bellman type in continuous time is now widespread in economics, in particular within theoretical and empirical labor market analysis, see e.g. Mortensen and Pissardies (1999), Pissarides (2000) and van den Berg (2003). This tool has turned out to be quite powerful and at the same time intuitively easy both to understand and to handle mathematically, at least in simple contexts similar to those considered here. Basic ideas are that decisions and evaluations are always made sequentially and based on the current state, and that no current commitments are made to actions at future dates. This framework has however so far had few applications in health economics settings. I claim that it is extremely well adapted to such settings. A main purpose of this note is then also to stimulate others to undertake research following related approaches.

2. The model

Consider an individual who at can be in only one out of three mutually excluding states: alive and well, with current utility \( u(c_1, H) \); alive and ill, with utility \( u(c_2, I) \); and dead with utility normalized to zero. \( c_1 \) and \( c_2 \) denote the consumption levels in the healthy and ill states, which are assumed constant over time for a given state, but may differ between states, typically with \( c_1 \geq c_2 \). The current utility level \( u \) is then

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3 One early exception is Hey and Patel (1983), who use a related two-state transition model to study effects of preventive measures to avoid illness, and care to improve the medical state the patient has become ill. No implications of their model for VSL valuation are however drawn.

4 We here do not discuss the sources of consumption possibilities. These will typically be labor income, initial financial wealth, and possible insurance schemes. Labor income may be lower in the sick than in the healthy state, while initial insurance contracts to insure the individual against sickness may (partly or fully) counteract the reduction in labor income. A related issue not elaborated further here is that when sickness or life insurance contracts can be continuously renegotiated, changes in death and sickness risk will induce a renegotiation of the initial contract so as to ensure an actuarially fair return.
constant over time in a given state. I assume that \( u(c, H) > u(c, I) \), and \( u_c(c, H) > u_c(c, I) \), implying that both the absolute utility level, and the marginal utility of consumption, are higher for given consumption, in the healthy state relative to the ill state. We model time as continuous. An individual who starts from an initial healthy state faces constant and independent Poisson rates \( \lambda_D \) and \( \lambda_I \) at which death and illness occur respectively. When ill the individual faces a constant death rate \( \mu_D \), and a rate \( \mu_H \) at which health is restored to the initial level. This formulation is similar to Cameron and DeShazo (2003), except for two main differences. First, time is here continuous instead of discrete. Secondly, hazard rates are constant here instead of the more general variable rates in Cameron and DeShazo. These changes make for a far simpler formulation, permitting easier derivations and interpretations of results. While our assumption of constant transition rates makes our model less general, it is useful for pedagogical purposes, and for displaying certain principal analytical effects that are far more difficult to derive and interpret under more general formulations. It will also easily be seen that a number of the more important results do not depend on transition rates being constant over time.

Define the Bellman (or value) functions \( V(H) \) and \( V(I) \), as expected discounted lifetime values of current health and current illness, as follows:

\[
\begin{align*}
(1) \quad rV(H) &= u(c_1, H) - \lambda_D V(H) + \lambda_I [V(I) - V(H)] \\
(2) \quad rV(I) &= u(c_2, I) - \mu_D V(I) + \mu_H [V(H) - V(I)].
\end{align*}
\]

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5 The formulation of Bellman functions in continuous time follows from taking limits of the associated discrete-period functional equations, presented e.g. in Bellman and Kalaba (1965). Derivations are straightforward; for a standard application reference which has had great influence on the labor market literature, and where also a simple derivation is offered, see Shapiro and Stiglitz (1984).
$r$ is the (constant) subjective rate of interest. The left-hand sides of (1)-(2) can be interpreted as current returns to two different assets, namely the present discounted value of currently being healthy, and the present discounted value of currently being sick. These returns have three components. The first is the current utility of consumption (or “current well-being”) in each state. The second is a component representing the change in asset value when the state changes from the current to the dead state (which is here simply the loss of the whole initial asset value). The third is a component representing the change in asset value when the state changes, from the current to the alternative live state (from healthy to ill in (1), and from ill to healthy in (2)).

(1)-(2) can be solved with respect to $V(H)$ and $V(I)$ as follows:

$$V(H) = \frac{1}{D}[(r + \mu_H + \mu_D)u(c_1, H) + \lambda_I u(c_2, I)]$$

$$V(I) = \frac{1}{D}[(\mu_H u(c_1, H) + (r + \lambda_D + \lambda_I)u(c_2, I)]$$

where

$$D = (r + \lambda_D + \lambda_I)(r + \mu_H + \mu_D) - \hat{\lambda}_I \mu_H.$$

$V(H)$ in (3) has a natural interpretation as the “present value of health”, and $V(I)$ in (4) as the “present value of illness”, at current and constant transition rates between states. $V(H)$ and $V(I)$ are both simple functions of $u(H)$ and $u(I)$, only with different

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Note the overly simplified structure where mortality and morbidity rates are constant through time. By construction, VSL and VSI will then also be invariant through an individual’s life. This is counterfactual; see e.g. Johansson (2002), Aldy and Viscusi (2003), and Viscusi and Aldy (2003), for recent discussions of models where individuals’ VSL values vary over their lifetimes, and the empirical evidence in this regard.
weights to the two utility levels, where \( V(H) \) has a relatively greater weight on the current level \( u(H) \), and this relative difference is greater when the discount rate \( r \) is higher. We see also e.g. that when \( \mu_H \) is very large compared to the other parameters, \( V(H) \) and \( V(I) \) are close. (A spell of illness then lasts for a short time since ill individuals get well fast, and the value of current illness to a large extent reflects the value of current health.) Only the dead state is absorbing in general. In the special case of \( \mu_H = 0 \), the ill state is also absorbing (as when “illness” represents permanent disability).

Expected remaining lifetimes, \( EL(H) \) and \( EL(I) \), are also constant over time for a given state, \( H \) or \( I \), and are found as (using techniques similar to those used for deriving (3)-(4))

\[
(6) \quad EL(H) = \frac{\mu_H + \mu_D + \lambda_I}{\lambda_D \mu_H + (\lambda_D + \lambda_I) \mu_D}
\]

\[
(7) \quad EL(I) = \frac{\mu_H + \lambda_D + \lambda_I}{\lambda_D \mu_H + (\lambda_D + \lambda_I) \mu_D}.
\]

\( EL(H) > EL(I) \) if and only if \( \lambda_D < \mu_D \) (as is plausible). The steady-state fractions of healthy and sick individuals in society, \( F(H) \) and \( F(I) \), given that all have identical hazard rates as given in (6) and (7), can now also be found, assuming that a fraction \( \alpha \) start out as healthy and a fraction \( 1-\alpha \) as sick. We find:

\[
(8) \quad F(H) = \alpha \frac{\mu_H + \mu_D}{\mu_H + \mu_D + \lambda_I} + (1-\alpha) \frac{\mu_H}{\mu_H + \lambda_D + \lambda_I}
\]

\[
(9) \quad F(I) = \alpha \frac{\lambda_I}{\mu_H + \mu_D + \lambda_I} + (1-\alpha) \frac{\lambda_D + \lambda_I}{\mu_H + \lambda_D + \lambda_I}.
\]
These expressions simplify further in the (often plausible) case where all start out as healthy ($\alpha=1$). With constant population, the (constant) ratio of sick to healthy individuals in society is then $\lambda/(\mu_H+\mu_D)$. This in turn e.g. implies that the ratio of healthy to sick is independent of the death rate of healthy individuals. Note also that the average time an individual remains in the healthy and sick states (the average “spells of health and sickness”), are $1/(\lambda_D+\lambda_I)$ and $1/(\mu_D+\mu_H)$, respectively.

3. Willingness to pay for mortality and morbidity changes

We now derive measures of the individual’s willingness to pay (WTP) for changes in value of statistical life in good health (VSL), and in illness (VSI), respectively. This can be identified as the individual’s current willingness to pay to reduce the current rate at which death occurs, in each of these two states.\(^7\) Holding $V(H)$ respectively $V(I)$ constant and differentiating with respect to the respective death rates and consumption levels, we find the following simple expressions:

\[
(10) \quad V_{SL} = \frac{dc_1}{d\lambda_D} = \frac{V(H)}{u_i(H)}
\]

\[
(11) \quad V_{SI} = \frac{dc_2}{d\mu_D} = \frac{V(I)}{u_i(I)}.
\]

$u_i(i)$ denotes the marginal utility of income in state $i = H, I$. We find VSL and VSI as simply the present discounted value of currently being in each of the states, evaluated

\(^7\) Note that this procedure provides us with an appropriate scaling of effects: the scale is such that when $\lambda_D$ and $\mu_D$ respectively are scaled to unity, the expected remaining lifetime of the individual is one period given that no other transition between states occur; and within a very short time span the probability of more than one transition events tends to zero. This implies that the derivatives in (10) and (11) represent VSL (respectively VSI) as defined e.g. by Rosen (1989).
at the respective marginal utilities of consumption in the two states. We have above assumed $V(H) > V(I)$, but $u_1(H)$ may be either greater or smaller than $u_1(I)$ (and always greater when $c_1 = c_2$). Whenever $u_1(H) > u_1(I)$, VSL can be either greater or smaller than VSI. This is a well-known principal result, but is still perhaps not intuitively obvious as the overall value of a longer lifetime is unambiguously higher for a healthy than for a sick individual. It follows because a higher marginal utility of consumption in the healthy state implies that the individual is less willing to give up current consumption in order to gain a longer expected lifetime.\(^8\) Note also that when $\mu_H$ becomes large relative to the other parameters, the ratio of $V(I)$ to $V(H)$ approaches unity. If then $u_1(H) > u_1(I)$, VSI > VSL unambiguously. The individual then experiences relatively short spells of illness on the average. The state of illness will then be escaped relatively rapidly, and future lifetime utility is evaluated to a large extent on the basis of healthy-state current utility. The individual is then willing to pay more at the margin to reduce mortality in the ill state, since his or her marginal utility of consumption is lower in that state.

Following e.g. Rosen (1989), the appropriate interpretations of VSL and VSI are in terms of current willingness to pay to enjoy a small change in the parameters $\lambda_D$ and $\mu_D$ for a possibly short period of time, so as to keep $V(H)$ respectively $V(I)$ constant. Note that the expressions (10)-(11) do not depend on the mortality rates $\lambda_D$ and $\mu_D$ being constant over time. In (1)-(2), $\lambda_D$ and $\mu_D$ can be interpreted as current rates of

\(^8\) Note that this model does not incorporate a competitive market for health insurance, nor the possibility of savings by individuals. Typically, in the absence of saving and borrowing, an optimal and actuarially fair health insurance scheme will here imply that $u_1(L) = u_1(H)$, i.e., it is optimal for individuals to spread their consumption in such a way that marginal utility of income is independent of the current live state. When the individuals must rely on imperfect insurance to provide their own income in the ill state, insurance will typically be underprovided and as a result $u_1(L) > u_1(H)$. When instead health insurance is provided publicly, income maintenance in the ill state may be sufficient to make $u_1(L) < u_1(H)$, in particular when consumption possibilities and needs are significantly reduced when sick. This implies that our model in principle opens up for the possibilities that $u_1(L)$ may be either smaller, greater or equal to
death which do not have to correspond to rates expected farther into the future; such 
rates only affect the values of $V(H)$ and $V(I)$.

Consider next an individual’s “value of full health” (VFH), as the current 
willingness to pay to reduce the rate of transition from the healthy to the sick state, $\lambda_S$ 
(i.e. to postpone sickness), or alternatively, as the willingness to pay to increase the 
current rate of transition from the sick to the healthy state, $\mu_H$ (to “get well faster”).

Differentiating (1), we find this value in the healthy state as:

\begin{equation}
V FH(H) = \frac{dc_1}{d\lambda_s} = \frac{1}{u_1(H)}[V(H) - V(I)].
\end{equation}

The corresponding value in the sick state, from (2), is

\begin{equation}
V FH(I) = -\frac{dc_2}{d\mu_H} = \frac{1}{u_1(I)}[V(H) - V(I)].
\end{equation}

Since $V(H) – V(I) = [(r+\mu_D)u(H) - (r+\lambda_D)u(I)]/D$, (12)-(13) can alternatively be 
written as follows:

\begin{equation}
(12a) \quad V FH(H) = \frac{dc_1}{d\lambda_s} = \frac{1}{u_1(H)} \frac{1}{D}[(r + \mu_D)u(H) - (r + \lambda_D)u(I)]
\end{equation}

\begin{equation}
(13a) \quad V FH(I) = -\frac{dc_2}{d\mu_H} = \frac{1}{u_1(I)} \frac{1}{D}[(r + \mu_D)u(H) - (r + \lambda_D)u(I)]
\end{equation}
(12) and (13) (respectively (12a) and (13a)) differ only in terms of the marginal utility of consumption in the two initial states. This implies that the willingness to pay to avoid sickness is unambiguously higher in the sick state than in the healthy state, provided (as is argued here) that the marginal utility of consumption is higher in the healthy than in the ill state. This is a different and stronger result than that regarding relative VSL values in the healthy and ill states, from (10) and (11), where these differences were ambiguous. Note from (12a)-(13a) that when \( \mu_D > \lambda_D \) (life expectancy is shorter when sick than when in health), the last square brackets of (12a) and (13a) are greater, and there is thus a greater value of avoiding sickness.

Another relevant case is where the individual is faced with a given combined risk of death and illness. A relevant case is when some serious condition, such as a stroke, heart attack, traffic accident or cancer case, which is certain to affect the individual’s health condition adversely but from which the individual may or may not die, occurs at a given rate. When the individual does not die, he or she then enters the sick state instead. We are here interested in valuing an increase in the survival rate of such a condition. We view \( \lambda = \lambda_D + \lambda_I \) as a constant, but each of \( \lambda_D \) and \( \lambda_S \) as variable such that \( d\lambda_D = -d\lambda_I \).

\[
\frac{dc}{d\lambda_D}(d\lambda_I = -d\lambda_D) = \frac{V(I)}{u(H)}.
\]

The value of statistical life when the alternative to dying is surviving in sickness is here simply the value of sickness evaluated at the marginal consumption value under the healthy state. This is lower than the current value of sickness, VSI, from (11), when the marginal utility of consumption is higher in the H than in the I state.
All values so far derived can be interpreted as “current benefits resulting from current payments”. In a number of cases there may however be delays in effects e.g. of treatments for which individuals need to pay currently. I here have in mind factors where there is an average or expected time of delay, but where there is still a possibility that the effect is immediate. Such effects can here be represented by reformulating the model slightly, to allow for a transition to a new state, where the only difference from the initial state is that the death rate is higher. We now simplify further and assume that the individual in question enjoys constant health and utility while alive, permitting us to define utility as simply \( u = u(c) \). We may still define a “state of poor health” (P), simply as a state in which the death risk is higher than initially. Assume that individuals have an initial rate \( \lambda_p \) at which transition to the new state occurs, and that the model is otherwise identical to that used above (e.g., the transition rates \( \lambda_D \) and \( \mu_H \) are as before). An intervention that increases or reduces \( \lambda_p \) then represents an increase of decrease in life expectancy, from some future date. We here want to derive the individual’s willingness to pay to reduce \( \lambda_p \) marginally. This is found, from a calculation essentially similar to (13), as

\[
\frac{dc}{d\lambda_p} = \frac{1}{u} [V(H) - V(P)],
\]

Alternatively we here have

\[
\frac{dc}{d\lambda_p} = \frac{1}{u} \frac{1}{D_i} (\mu_D - \lambda_D) u(c),
\]

(15a)
where $D_1$ equals $D$ from (5) except that $\lambda_I$ is replaced by $\lambda_P$. $V(P)$ now denotes lifetime utility in the high-risk state (corresponding to $V(I)$ above). We see that (15) is quite similar to the VSL expression in (10), as the two expressions differ only in terms of the value changes involved. This implies that the “delay” of mortality effects in the current case has no fundamental implication except in dropping the value increment (from $V(H)$ in the former case, to $V(H)-V(P)$ in the latter). This may appear surprising, but occurs because there is in fact no discrete delay in this case: the drop in $\lambda_P$ occurs instantly, which has an instantaneous effect on (potential) mortality.

4. The value of health investments

So far we have only discussed interventions that affect mortality and morbidity transition rates at the point of time of the actual intervention. Other interventions may instead reduce the mortality rates $\lambda_D$ and $\mu_D$ permanently. In this section we want to derive measures of (once-and-for-all) WTP of an individual, for such interventions, which we may call health investments. Note then that we may write, with sufficiently good approximation,

\begin{align*}
(16) & \quad V(H) = u(c_1, H)\Delta t + \frac{1}{D}[(r + \mu_H + \mu_P)u(c_1, H) + \lambda_Iu(c_2, I)] \\
(17) & \quad V(I) = u(c_2, I)\Delta t + \frac{1}{D}[(\mu_H u(c_1, H) + (r + \lambda_H + \lambda_I)u(c_2, I)],
\end{align*}

where $\Delta t$ is a small time interval (and where, formally, $V(H)$ and $V(I)$ on the left-hand sides are defined from a point of time $t$ while the last expressions on the right-hand sides are defined from period $t+\Delta t$). Define now $dR_i = dc_1(i)\Delta t$ as a small change in income affecting current consumption only, when in state $i = H,I$. We may now
consider the individual’s WTP for a small permanent change in $\lambda_D$ starting from the healthy state, and a small permanent change in $\mu_D$ starting from the ill state, i.e., a change $dR_i$ that will make the individual indifferent. Differentiating the two expressions we then find

\begin{equation}
WTP(d\lambda_D, H) = \frac{dR(H)}{d\lambda_D} = \frac{r + \mu_H + \mu_D}{D} \frac{V(H)}{u_1(H)} = \frac{r + \mu_H + \mu_D}{D} VSL
\end{equation}

\begin{equation}
WTP(d\mu_D, I) = \frac{dR(I)}{d\mu_D} = \frac{r + \lambda_I + \lambda_D}{D} \frac{V(I)}{u_1(I)} = \frac{r + \lambda_I + \lambda_D}{D} VSI,
\end{equation}

using (10)-(11). These expressions are of a different dimension than the VSL and VSI expressions, since they can be interpreted as one-time investments to reduce future mortality (and not, as in the cases of VSL and VSI, as current expenses for current mortality reductions). This value may be higher in the H than the I state, but not necessarily so. As argued above VSL is typically higher than VSI. Whenever $\mu_D < \lambda_D$, such that if $\mu_H \geq \lambda_I$, we then have $WTP(H) > WTP(I)$. But we have not put very strong restrictions on $\mu_H$ and $\lambda_I$. Consider e.g. the case where $\mu_H = 0$ (one never gets well after becoming ill, as in the case of a permanent disability), and $\lambda_D + \lambda_I > \mu_D$, which implies that the overall hazard rate from the healthy state (to either illness or death) is greater than the hazard rate from the ill state (to death only). In this case an investment in a lower mortality rate tends to be more valuable in illness than in health, when only the mortality rate in the initial state is affected in both cases.

These last derivations are based on the somewhat artificial assumption that only the mortality rate in health is affected by the investment when the individual is
initially in the healthy state, and vice versa. Most interventions are instead likely to affect the mortality rates in both states. Consider then a different special case where the absolute change in the rate of mortality is the same in both states. This is valued in the following way, departing from respectively the healthy and the ill state:

\[
WTP(d\lambda_D = d\mu_D, H) = \frac{(r + \mu_H + \mu_D) V(H) + \lambda_I V(I)}{D u_i(H)} = \frac{r + \mu_H + \mu_D VSL + \lambda_I u_i(I)}{D u_i(H)} VSI
\]

\[
(20)
\]

\[
WTP(d\mu_D = d\lambda_D, I) = \frac{\mu_H V(H) + (r + \lambda_D + \lambda_I) V(I)}{D u_i(I)} = \frac{r + \lambda_I + \lambda_D VSI + \mu_H u_i(H)}{D u_i(I)} VSL.
\]

\[
(21)
\]

We find the two magnitudes to be quite similar, and typically more similar than the two expressions from (18)-(19). These expressions can also be used to derive some other interesting properties of health investments. Consider e.g. the simplified case where \(\lambda_I = \mu_H = 0\), i.e., there is not transition between the states (individuals are born, and die, either ill or in health). Then (20)-(21) simplify to (using shorthand notation for the WTP terms)

\[
VTP(H) = \frac{1}{r + \lambda_D} VSL
\]

\[
(22)
\]

\[^9\text{Note that, in the model, a person may start our in the healthy state, then enter the ill state, and later return to the healthy state. The change in } \lambda_D \text{ that is valued, occurs over all those time intervals that the individual is in the healthy state, but there is no change in mortality in the intermittent ill states. This is in principal a possible case (e.g. when good health involves certain risky activities, such as parachuting and mountain climbing that are ruled out when ill, and where the rates of fatal accidents involved in such activity may be affected by investment in particular equipment), but it is probably unrealistic in most practical cases.}\]
For given VSL and VSI respectively, the values of such investments are here affected negatively by the mortality rates (as well as by the interest rate). This is a “dead-anyway effect”, as investments are less valuable since there will be fewer future states in which their value is experienced.\textsuperscript{10} It should be stressed that this effect does not appear in the VSL or VSI terms directly, but in the derived investment terms.\textsuperscript{11}

5. Effects on VSL and VSI of background risks

For the VSL values derived above one can find effects of changes in “background risks”, represented by changes in $\lambda_D$, $\lambda_I$, $\mu_D$ and $\mu_H$. Since the $u_1(i)$ are assumed constant in each state, these parameters simply alter VSL and VSI in the same ways as they alter V(H) and V(I) respectively. We find

\begin{equation}
\frac{dVSL}{d\lambda_D} = -\frac{r + \mu_H + \mu_D}{D} VSL < 0
\end{equation}

\begin{equation}
\frac{dVSL}{d\lambda_I} = -\frac{1}{u_1(H)} \frac{r + \mu_H + \mu_D}{D} [V(H) - V(I)] < 0
\end{equation}

\begin{equation}
\frac{dVSL}{d\mu_D} = -\frac{1}{u_1(H)} \frac{\lambda_D}{D} V(I) < 0
\end{equation}

\textsuperscript{10}Note that the calculation done on this very simplified case is quite analogous to that performed in the more complicated model set up by Johannesson, Johansson and Löfgren (1997), where they compare the value of “blips” in the mortality rate, versus permanent changes in mortality. They however fail to emphasize the close relationship between these calculations, and the “dead-anyway effect”, as is done here.

\textsuperscript{11}“Comorbidities” can in this framework be represented very simply, as an additional factor affecting the death rate, such that, say in (20), $\lambda_D = \lambda_{D1} + \lambda_{D2}$. The formula for WTP(H) would be the same. This implies that the effect of an exogenous comorbidity on WTP(H) (and WTP(I)) is the same as that of a similar “dead-anyway effect”.
These results are simple, intuitive and easy to interpret. VSL is always reduced when the rate of mortality (in either of the states) or the rate of entering the state of morbidity increases, and is increased when the rate at which the individual returns to the healthy state, after a spell of illness, increases. These results differ from related analyses of background risk (Bleichrodt, Crainich and Eeckhoudt (2003) and Eeckhoudt and Hammit (2001)) and “dead-anyway” effects (Pratt and Zeckhauser (1996) and Breyer and Felder (2002)), where effects typically are ambiguous and often opposite of those found here. The main factor responsible for the difference in results is our continuous-time instead of discrete-time formulation, and our assumption that effects are immediate (an individual current payment is assumed to affect his or her current mortality or morbidity rates) instead of being delayed.

6. Concluding comments

This note has studied a very simple model in continuous time where individuals may transfer between the states of healthy and ill while alive. We use the model to derive values of interventions to change the rates of death and transition between the two states, and the value of statistical life (VSL) and statistical illness (VSI), when effects of the interventions are immediate. This permits extremely simple derivations of effects, and easily interpretable and intuitive results. In particular, the continuous-

\[ \frac{dVSL}{d\mu_H} = \frac{\lambda_i}{u_i(H)D} [V(H) - V(I)] > 0. \]
time formulation permits us to avoid most of the problems created by a number of effects, hereunder the so-called “dead-anyway effect” and problems related to comorbidities, background risk and risk aversion. The main point here is not that such effects are never significant or important. My objective is rather to provide a clearer analysis of the conditions under which they are not important, and what are their implications. In many cases it is clearly appropriate to assume significant time delays in effects (such as with often long minimum latency periods to develop certain cancers, or when current expense has to be made to implement health investment in the future), in which case the current model framework is far less appropriate. The point is however simply to make researchers aware of when a particular type of model is, or is not, the appropriate one.

Note the many strong other assumptions used in the analysis. First, mortality and morbidity rates, as well as consumption levels and the marginal utility of consumption, have all been assumed constant in a given state. We have however also noted above that these assumptions do not appear to lead to very fundamental problems for the principal methods of deriving VSL and VSI in section 3. One implication of relevant alternative assumptions (higher mortality rates, and possibly higher marginal utilities of consumption, for older people) would be for VSL and VSI to drop with age, and perhaps more strongly so than in many competing models. A further analysis of such issues is however left to future research. The assumption of exogenous consumption in each state is also strong and at odds with some approaches in the literature that consider the possibility of efficient insurance or savings.
References


