Competitive effort determination

with heterogeneous workers and team production

By

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Abstract

We consider a competitive labor market with purely firm-specific differences in workers’ disutilities of productive effort, firms’ outputs depend on the joint efforts of many workers, and individual worker characteristics cannot be directly observed by firms. Firms compete in ex ante contracts relating workers’ rewards to overall output, offered prior to the production period. Under assumptions similar to those made by Holmström (1982), we demonstrate that a first-best effort allocation, requiring more productive workers to put up greater efforts than less productive ones, can then be implemented by firms paying the same wage to all workers. Differential worker productivities then imply no adverse selection problem, and no distortions.
1. Introduction

In this note we consider the problem of efficient allocation of labor when only group output can be observed by the employer, and workers differ in ability. Departing from the seminal paper by Holmström (1982), we assume that only total output, and not individual workers’ contributions to this, can be measured by each firm. We also assume, as Holmström does in his basic model version, that firms have no ways of screening workers according to productivity, nor distinguishing more from less productive individuals. We however extend his model by assuming that workers differ in their productivities, in a purely firm-specific way, i.e., workers are risk neutral, and ex ante identical before arriving at a particular firm. After arriving at the firm, their realized productivities differ, according to a commonly known distribution that is identical for all workers. We also assume that there is ex ante competition in attracting workers to each firm, requiring hiring firms to offer a given worker a given market-determined expected utility, and leaving each firm with zero expected profits (given constant returns production functions).

A similar problem has been studied by MacAfee and McMillan (1991), who consider a principal who can observe only group output, hires heterogeneous workers and maximizes profits subject to incentive compatibility and participation constraints for each type of worker.1 They show that the optimal mechanism, which permits truthful revelation of individual worker types, depends in a relatively complex way on group

1 An extension to the case of risk averse workers is made by Veen (1995).
output for each of the worker types, who self select to individual (fixed- and variable-part) wage contracts. Generally, a first best, in which the marginal disutility of effort equals the marginal productivity contribution to total output for each worker, is not attainable, due to the adverse selection problem.

We demonstrate that a first best is attainable under somewhat different assumptions from those used by MacAfee and McMillan. Main differences are that we assume ex ante competition in wage contracts offered to workers; and that workers are ex ante identical prior to joining firms, and have identical reservation wages. We then show that a first best (under certain assumptions which coincide with those made by Holmström) can be implemented through a bonus scheme where all workers receive the same positive bonus whenever observed aggregate output exceeds a given minimum level. Workers’ contributions to output then differ according to ability, in an optimal way, but all workers’ wages are the same. At equilibrium, both marginal and average disutility of effort are greater for those workers who contribute more to total output, than for those who contribute less. With equal wages to all this implies that expected utilities are lower for workers with higher productivities.

2. The effort allocation problem

Consider a single firm facing a competitive contract market with ex ante identical workers, who differ ex post in their ability in the firm in question.
We will first consider the efficient allocation problem for this firm, for a
given stock of n workers (where n may be large). Worker i puts up
subjective effort \( e_i \geq 0 \) on the job, and \( v(e_i) \) is subjective disutility
associated with this effort. The \( v \) function is assumed to be the same for all
workers, and \( v', v'' > 0 \) for all \( e_i \geq 0 \). Assume that workers differ in their
productivities, in the sense that for a given subjective effort, some workers
contribute more to the firm’s output than others. Assume also that workers’
productivity differences are purely firm-specific, i.e., all workers have the
same ex ante productivity distribution in all firms, equal to the market
distribution of productivities, denoted \( H(\alpha) \), which is assumed to be
perfectly continuous on \([0, \alpha_m]\), with density \( h(\alpha) \). At equilibrium, workers
employed by any given firm are then a random sample of all workers in the
economy. With a large number of workers in all firms, average worker
productivity will then be (approximately) equal for all firms. Denote the
productive efficiency of worker i, with productivity coefficient \( \alpha_i \) and
putting up effort \( e_i \) by \( \alpha_i e_i \). Denoting the average measure of productivity
for all workers in the firm by \( a \), we have \( a = \sum \alpha_i e_i / n \), with the sum taken
over all n workers in the firm. In line with Holmström (1982) we assume
that the firm’s output can be expressed as

(1) \[ x = f(\sum \alpha_i e_i) + \eta \]
where \( \eta \) is some random variable representing an imperfect relationship between labor input and the output that can be measured by the firm, which is the relevant concept here. Assume also that expected output as viewed by the firm is given by

\[
(2) \quad E(x) = f(n \int_{a_x} e(\alpha) h(\alpha) d\alpha).
\]

Generally, \( E(x) \) will differ from \( x \), for two separate reasons: First, due to the imperfect relationship represented by (a); and secondly, since \( E(x) \) does not exactly represent the value of the labor input in a finite group of workers. Setting \( f(\Sigma a_i \varepsilon_i) = E(x) + \rho \), where \( \rho \) is another stochastic variable. We will find that the error \( \rho \) will become very small (in relation to total output) when \( n \) grows. We also have that

\[
(3) \quad x = E(x) + \eta + \rho = E(x) + \varepsilon
\]

where \( \varepsilon \) expresses the total uncertainty in the firm’s evaluation of output, which is assumed to have zero mean and continuous distribution \( G(\varepsilon) \) on the domain \([-q, \infty)\), where \(-q\) is some lower bound for \( \varepsilon \) (in particular, total measured output cannot be negative. \( x \) is the amount of output that can be sold by the firm at this price. Note here that when \( \rho \) is very small, \( \eta \) will
have approximately the same distribution as $\varepsilon$. Workers are assumed to be risk neutral in money and with utilities given by $u_i = w_i - v(e_i)$.

The efficient allocation of workers’ efforts is given by maximizing the expression

$$W = Ex - \sum v(e_i) = f(\sum \alpha_i e_i) - \sum v(e_i).$$

$W > 0$ at the optimal solutions for the $e_i$ is here a necessary condition for this problem to be meaningful. We find the following first-order conditions

$$\alpha_i f'(na) - v'(e_i) = 0,$$

which must hold for all $i$.

### 3. Implementing the optimal solution

Consider next the possibility of implementing the optimal solution just derived. Implementation requires that both incentive and participation constraints are fulfilled for all worker types. We start with incentive considerations. The firm is assumed not to observe individual workers’ types, efforts, nor their individual contributions to total output. Workers themselves however know their ex ante types in a given firm, and choose their levels of $e_i$ accordingly. Individual workers’ contributions to the total labor input in the firm are assumed to be fully independent. We also assume
that workers do not observe each others’ efforts not seek to cooperate in the
determination of effort. We consider a one-period model, such that
reputation or learning about workers’ abilities are of no concern.

Since the firm can only observe total output, and not the contribution to
output by any given worker, it cannot directly differentiate workers’ wages
except by randomization. It can be shown that randomizing wages cannot
dominate; we accordingly impose the restriction that all workers are paid
the same wage. We will consider a candidate payment scheme whereby
each worker in the firm is paid \( w_1 \) if total output as observed by the firm at
least equals some minimum level, call it \( x^* \), and is otherwise paid \( w_0 \).
Assume \( w_0 \geq 0 \), i.e., workers are never required to make net payments to
the firm. The utility of worker \( i \) can then be written as

\[
\begin{align*}
    u_i &= w_1 - v(e_i), & x \geq x^*, \\
          &= w_0 - v(e_i), & x < x^*.
\end{align*}
\]

From the definition of \( x \) we find, for given \( e_i \), and assuming that the error in
evaluation of the labor force, \( \rho \), is sufficiently small to be ignored, that \( x^* \)
and \( e^* \) are approximately related as follows:

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2 Kandel and Lazear (1992) and Barron and Gjerde (1997) consider cases where workers (or partners)
can observe each others’ efforts or individual outputs (while the firm or principal cannot), and
mechanisms of effort enforcement on this basis. Huck, Kübler and Weibull (2001) study possible
enforcement via social norms.

3 MacAfee and McMillan (1991) study similar schemes where the actual equilibrium wage is the same to
all workers, but where the principal offers a menu consisting of different combinations of marginal and
fixed components, and where individual workers self-select to contracts within those offered in the menu.
The principal structures its contract menu such as to maximize its overall rent extracted from workers.
Due to the informational constraints imposed, this contract is never first best, and different from the wage
scheme considered here.
\[ x^* = f(\Sigma \alpha_i e_i) + \varepsilon^*. \]

Define \( P \) as the probability that \( x \) will exceed \( x^* \), in which case worker \( i \) will receive the wage \( w_1 \) instead of the minimum wage \( w_0 \). We then have

\[ P = \text{prob} (x \geq x^*) = \text{prob} (\varepsilon \geq \varepsilon^*) = 1 - G(\varepsilon^*) = 1 - G(x^* - f(\Sigma \alpha_i e_i)). \]

The effect of an increase in effort for worker \( i \) on this probability is given by

\[ \frac{dP}{de_i} = g(\varepsilon^*) \alpha_i f'(na). \]

The expected utility of worker \( i \) is now given by

\[ Eu_i = P(w_1 - v(e_i)) + (1 - P) (w_0 - v(e_i)) = w_0 + P(w_1 - w_0) - v(e_i). \]

Using (6), worker \( i \)’s first-order condition for optimal effort \( e_i \) is given by

\[ g(\varepsilon^*) \alpha_i f'(na) (w_1 - w_0) = v'(e_i). \]
Comparing the efficient solution, (5), to the solution implementing individual worker effort, (11), we derive the following result:

**Proposition 1**: Assume that the firm observes only aggregate output, and workers have individual productivity parameters $\alpha_i$. Then the efficient allocation can be implemented by the wage scheme

\[(12) \quad w_I = w_0 + 1/g(\varepsilon^*),\]

which implies that all workers are paid the same wage, independent of individual productivities.

The proof is straightforward, comparing (5) and (11). The implication of Proposition 1 is that the efficient allocation is implemented by paying all workers the same wage, even though their productivities may differ widely.

This result is striking, and perhaps surprising. Note that at an optimal solution just described, the expected utility of worker $i$ is given by

\[(13) \quad Eu(i) = w_0 + P(w_I-w_0) - v(e_i) = w_0 + [1-G(\varepsilon^*)]/g(\varepsilon^*) - v(e_i),\]

where we substitute for $P$ from (8). (13) and (5) together imply that workers’ expected utilities differ, and are lower for higher-productivity
workers, since optimal, and equilibrium, effort levels are greater for these, while the wage is the same for all. Intuitively, a high-productivity worker knows that he or she has a relatively great impact on total output, and consequently a great impact on the probability that group output will exceed the target $x^*$, for any given effort. This makes it optimal for a high-productivity worker to put up a higher marginal (and here thus total) effort, for a given bonus ($w_1 - w_0$). The share of output ascribed to each individual worker will then also differ and be proportional to the individual efficiency coefficients $\alpha_i$.

So far we have shown that an optimal solution, if it exists, must have the properties described by Proposition 1. We will now consider whether such a scheme is feasible. Assume then, following Holmström (1982, page 328) and writing $G(\varepsilon) = G(x-f(na))$:

**Assumption 1**: $G$ is convex in $na$.

**Assumption 2**: $g(x-f(na))/(1-G(x-f(na))) \to \infty$ as $x \to \infty$.

Assumption 1 here assures global optimality of agents’ actions. Assumption 2 has the interpretation that the signal provided to the firm about workers’ efforts, inferred from observed output, is very precise when observed output is very large.4

We next consider whether participation constraints are fulfilled. There

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4 See also Holmström (1979) and Milgrom (1981) for further discussions of these expressions and their interpretations.
are two different types of participation constraints: Ex ante constraints, requiring workers to initially accept a work contract with a particular firm; and ex post constraints, requiring all workers to have incentives to remain with the firm after their firm-specific types have been revealed. Consider a labor market with identical firms, where as before only aggregate output is measured by any firm.\(^5\) We first consider the former of these two constraints. In a competitive contract market with many firms, the ex ante expected utility offered to workers by a particular firm must match that offered by other firms, and be taken as exogenous by each firm. This expected utility is given by

\[ EU = Ew - \int_{a=a_0}^{a_f} v(e(\alpha))h(\alpha)d\alpha. \]

Note that the effort scheme derived under Proposition 1 is Pareto efficient for workers and firm together, and firms act competitively.

Consider next the second requirement. Assume that all workers have ex post opportunities (after being employed by a particular firm) outside of the labor market with common value \(B\) (representing the value of home production, unemployment compensation or leisure).

Consider such a possible solution. Denoting firm \(j\)’s profits by \(R(j)\), the firm’s expected profits are given by

\(^5\) We are thus here implicitly assuming that no firm can profitably reduce its scale of production to a size so small that individual workers’ contributions to output can be identified.
where \( n(j) \) is the number of workers in firm \( j \), \( n(j)a(j) \) is the value of the aggregate (effort-augmented) labor input for firm \( j \), and \( \varepsilon^*(j) \) is the cutoff level for \( \varepsilon \) selected by firm \( j \). Feasibility of the optimal solution now requires \( \text{ER}(j) \geq 0 \) in (15), and \( \text{Eu}(i) \geq B \) for all \( i \). Let \( \alpha_m \) denote the upper support of the productivity distribution over workers. We can then show the following:

**Proposition 2:** Assume a given number of firms \( m \), and a given total number of workers \( N \), no firm can observe individual workers’ productivities, workers have short-run outside opportunities with a common value \( B \), Assumptions 1-2 hold, and firms have unbounded wealth. Then a first-best allocation implies that all workers are employed in firms whenever the following condition holds:

\[
af'(na) \geq v(e_m) + B,
\]

where \( n = N/m, e_m = e(\alpha_m) \), and \( a \) and all \( e_i \) correspond to the optimality conditions (5). Moreover, such an allocation is implemented by firms using the wage scheme (12).
Proof: Consider setting $w_0 = v(e_m) + B$. Then the labor market ex post participation constraint $Eu(i) \geq B$ must hold for all workers, in particular for workers with $\alpha = \alpha_m$. From Assumption 2, for a sufficiently high level of $\varepsilon$, 
$[1-G(\varepsilon)]/g(\varepsilon)$ tends to zero, implying that $Ew = v(e_m) + B$ is sufficient in the limit, to fulfill the participation constraint for all workers. Consider next firms. (16) is the condition that firms’ marginal profits with respect to $n$ are positive given that all firms have $n$ workers and all attract a random sample of workers. This implies that all workers should optimally be employed.

Consider finally possible states where $\varepsilon > \varepsilon^*$. For such states, from (12), $w = w_1 = v(e_m) + 1/g(\varepsilon^*)$. When firms have unbounded wealth, such wages can be paid regardless of $g(\varepsilon^*)$, even when small.

Consider next the effect of competition between firms, in terms of ex ante labor contracts. These must have a given market-determined value for all workers. This value is determined by adjusting the absolute basic wage level $w_0$. Q.E.D.

The first part of Proposition 2 is based on Theorem 4 in Holmström (1982), with the additional requirement that all workers’ participation constraints be fulfilled. Under our assumptions (all workers have the same outside utility $B$ and no intra-firm worker mobility after workers have been hired) these constraints are fulfilled whenever they hold for the highest-productivity workers. Note the role (as in Holmström) of the unbounded wealth requirement for the case where (16) is ”just barely” fulfilled. In this
case implementation requires a large $w_1 - w_0$, and that $\varepsilon$ tends to its upper support. $g(\varepsilon^*)$ could then in principle be small, and the wage premium large (in the rare event that such a premium is paid). Without the unbounded wealth requirement firms would not in general be able to cover their liabilities in such events, and the scheme would break down.6

Proposition 2 goes further than Holmstrom’s analysis by adding considerations about workers’ participation given the implementation schemes offered by firms, and considerations concerning the competitive nature of the labor market. Ex ante, all workers must be provided with a given, and identical, expected contract value sufficient to attract them to the firm. Due to the risk neutrality assumption, such contract value can simply be provided by adjusting the basic level of wages $w_0$. Ex post, high-productivity workers are both hardest to give incentives, and at the same time most attractive to firms. The model however offers no mechanism whereby firms can compete selectively for workers, since workers (and firms) are unaware of any possible firm-specific abilities prior to being hired.

**Final comments**

Under our assumptions, the workers with the highest productivities end up with the worst outcomes, in terms of the combination of wage and effort (the wage is the same to all, but effort is greater for those with high

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6 One might here perhaps argue that the firm could be able to sign a future financial contract insuring firm against the event of a “very high” wage payment, resulting when $\varepsilon > \varepsilon^*$. We will not go into this issue in detail, just mention that moral hazard considerations are likely to make this difficult.
productivities). We thus have the (arguably perverse) situation that high-
productivity workers would have preferred to rather be lower-productivity
ones. The fact (or curse) of being good still leaves them with the incentive
to put up high effort given the wage scheme offered by the firm.

Two further comments are here in order. First note that Proposition 2
gives a sufficient condition for full employment being first-best efficient in
an economy of the type described. The main point is that workers at
productivity levels below maximum enjoy strictly positive utilities in excess
of their outside options, at an equilibrium with full employment. One
problem not addressed here arises when (12) is not fulfilled. In such cases
the implementation of a first-best solution in general requires that workers
with different productivities are to be paid different wages, which is here
not feasible.

A related problem arises when workers differ in their ex post outside
values B. In practice workers with high realized productivities are then
likely to have high alternative values, which typically makes these more
reluctant to take up the types of jobs described here, i.e., the participation
constraint does not hold for these. An adverse selection problem then
generally arises. This is the basic problem addressed by MacAfee and
MacMillan (1991), who demonstrate that the first-best solution then
typically cannot be implemented. Some rents must now go to the high-
productivity workers (as a result of their higher outside values), which
works to modify our solution in the direction of lower implemented effort, through a less powerful incentive scheme.

The final problem to be mentioned arises when workers have different ex ante abilities, prior to joining the firm in question. This creates a different sort of problem whereby firms now have incentives to try to attract those workers who have high productivities. This problem is not addressed by Holmström nor by us, and is analytically challenging.7

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7 See e.g. MacAffee and McMillan (1991) and Veen (1995) for analyses of similar problems.
References:


