Wage and technology dispersion with wage bargaining

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Abstract

We study endogenous wage and productivity distributions with individual ex post (Nash) wage bargaining between workers and firms. We concentrate on a matching technology with directed search and firms make at most one offer per job opening. A "low-wage equilibrium" (LWE) where there is full employment, all workers search one firm, wage and productivity collapse to single points but workers still earn rents, always exists when workers' bargaining power is not too high. A dispersion equilibrium also exists when marginal search costs for workers are low. Here the number of jobs is less than the number of workers' search messages, and each worker may search several firms. The dispersion equilibrium is often but not necessarily more efficient than the LWE.

Key words: Wage distributions; wage bargaining; sequential search; investment choices.

JEL classification: J23, J31, J64.

1. Introduction

The current theoretical analysis of labor market search and matching is dominated by two rather separate literatures. In the first of these one typically assumes firm wage posting. Wage dispersion may then result from either on-the-job worker search (Burdett and Mortensen (1998), Burdett and Coles (2003)), or from nonsequential search whereby workers find it advantageous to search for more than one firm simultaneously. Wilde (1977) and Burdett and Judd (1983) provide product market applications of the latter type of models; more recently, Acemoglu and Shimer (2000) (hereafter AS) and Mortensen (1998) have studied labor market applications.[1] In AS, wage dispersion among identical workers can result from firms' endogenous capital choices, whereby some firms invest more capital than others, and offer a higher wage, in return for greater probability of employing a worker. The other main branch of this literature assumes bilateral matching and bargaining between individual firms and workers, and builds on the work of Mortensen and Pissarides and followers (e.g. Mortensen and Pissarides (1999), Pissarides (2000); see also Acemoglu and Shimer (1999)). This model framework assumes continuous time, unlimited decision horizon and sequential search. Firms' capital choices and wages are here typically identical.

The current paper sets out to integrate these two strands of literature by extending the AS model to situations with (Nash) wage bargaining between individual firms and workers. Such bargaining by construction precludes the possibility that wages may differ for identical firms and workers. Equilibrium wage dispersion may still however arise when workers search nonsequentially and firms make endogenous capital choices prior to hiring and subsequent wage bargaining. Anticipating that the bargained wage splits net ex post output in given proportions between the firm and the worker, we show that firms may select different capital intensities. Nash bargaining over different total surpluses then results in different wages.

We study two versions of this model, differing in the more detailed assumptions about the technology for matching workers and firms. The first version, discussed in section 2, adopts the AS (and Burdett-Judd) matching technology whereby workers search one or two firms at random, and each firm may make several job offers. The second, studied in section 3, is less standard and assumes that initial worker search is directed in spreading workers' search messages evenly out across active firms, and that firms make only one job offer for each job opening. Whenever the number of established firms then is at least as great as the number of search messages sent by workers, there is always full employment.

Under both matching technologies we demonstrate that for some (reasonably low) range of workers' search costs, there exists an equilibrium distribution of productivities for established firms in the market, which corresponds to a distribution over workers' wages, all workers searching at least one firm and some more than one. Under the AS search technology (model 1), such an equilibrium implies (as in AS) that workers search either one or two firms. Under our alternative search technology, a wage dispersion equilibrium may imply that all workers search two firms or more. We also demonstrate that there always exists a low-wage equilibrium (LWE, similar to the "Diamond equilibrium" in AS) where each workers searches only one firm, firms face no effective competition for workers, and the wage distribution collapses to a single mass point. At an LWE, the wage (and firm productivity) always corresponds to the level at the lower support of the distribution, for the case where the distribution is spread out. As opposed to the wage-posting case, in an LWE workers earn positive rents by virtue of their ex post bargaining strength, and choose to enter the market given (small) positive search costs. While the lower support is the same in all equilibria, the distribution is more spread out under more competition (more workers obtain job offers from at least two firms).

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[1] A survey of the recent literature in this field is provided by Mortensen and Pissarides (1999).

2. Wage bargaining under the AS search technology

Consider a labor market where all firms and workers operate for one period. Before production starts, firms sink an initial establishment cost C (thus establishing goodwill, market access and legal rights to production such as patent rights), and a capital size k determining output x = f(k), where f'>0, f''<0. In most of the exposition we use a Cobb-Douglas specification, $f(k) = k^{\alpha}$, where $\alpha \in (0,1)$ is a positive constant. After establishment each firm offers one job opening, producing x provided that the job is filled. If no worker is hired, nothing is produced. When production takes place, output is split in fractions β and 1-β to the worker and firm, respectively, through ex post bargaining between the parties; thus w(k) = βf(k), where w(k) is the wage in a firm with capital k. Firms cannot commit beforehand to paying a particular wage, and the resulting (bargained) wages will be higher in firms with larger k. This solution also embeds an assumption that workers' threat points involve no unemployment benefits or leisure foregone.[2]

Assume a continuum of workers of measure 1, and a continuum of firms of measure M established at the start of the period, where M is endogenous and may be greater or smaller than one. Established firms are initially unknown to workers. Assume that a worker selects n search messages at the start of the period, each reaching one established firm by some random process, and each costing c to send for the worker, except that the first message is free.[3]

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[2] Our bargaining solutions here and in the next section will also throughout assume that whenever a worker has an offer from two firms or more, the other offer serves as an outside option, and thus does not affect the disagreement point; see Muthoo (1999) chapter 5.. Since the outside option for the worker will always be worse than that option chosen (since else the worker would apply to another firm), the bargaining solution is not affected by the presence of two or more several offers. The same property is assumed to hold in cases where the firm has more than one applicant.

[3] This assumption is made to make our model correspond as closely as possible to AS. In AS however one free search was necessary to "save" the Diamond equilibrium, something that is not the case here.

Search can only take place at this time. When a search message reaches a firm, the firm makes a conditional job offer to the worker (it is implicitly assumed that the worker will not get the job for certain if he or she applies). The worker thereby obtains information on that firm's existence and its level of k. Given wage bargaining, the wage level obtained in that firm is then also known to the worker receiving the offer. The worker selects one and only one firm at which to apply among the n. Assume that the worker chooses to apply to the firm with the highest level of k, which will be the one with the highest wage among the n.[4] Call this level k_n. A firm offering k_n will in expectation receive $q(k_n)$ applications. Provided that it gets at least one application, the firm selects one of the applicants at random, and employs the worker who had sent this application, while the other applications are turned down.

By the law of large numbers, a firm which receives an expected number q of applications has a probability e^{-q} of receiving no application. This implies that a worker who applies at a firm with $k = k_m$ has a probability $\rho(k_m)$ of becoming employed by this firm, given by

$$\rho(k_m) = \frac{1 - e^{-q(k_m)}}{q(k_m)}, \tag{1}$$

(since each of the q applicants which on average applies to such a firm type has the same probability of becoming employed, and $1-e^{-q}$ is the probability that such a firm type at all employs a worker).

Assume initially that each worker sends either one or two search messages, which will later be proven to hold. Define Q = 1/M as the number of workers per established firm in the

[4] This can be shown to be the worker's optimal strategy (at least weakly), in the same way as in AS for the case of wage posting. Intuitively, if it was instead strictly preferable for the worker to apply to a lower-wage than to a higher-wage firm, all workers would apply to the lower-wage receive more applicants than the higher-wage firm. This would imply that the return to applying to the low-wage firm must be strictly lower than the return to applying to a

economy, and z as the fraction of workers sending only one search message, and a fraction 1-z sends two messages, where $z \in (0,1]$. The average number of sampled firms relative to the number of firms, which is identical to the number of search messages received by firms on average, is then given by s(av) as follows:

$$s(av) \quad = \quad Q[z + 2(1-z)] \quad = \quad Q(2-z). \qquad (2)$$

Note that s(av) is the expected number of messages received by any firm, independent of the firm's capital choice (since messages are sent out without information on this stock). We next want to derive the expected number of worker applications received by the firm, which is generally an increasing function q(k) of k. To derive this relationship, note first that all workers who send only one message apply to the one contacted firm. Those who send two messages apply to a firm with capital stock k_1 with probability $G(k_1)$, where G is the distribution function for k across firms. $q(k_1)$ can then be found as follows:

$$q(k_1) \quad = \quad Q[z + 2(1-z)G(k_1)], \qquad (3)$$

which increases in $G(k_1)$ whenever $z<1$. Intuitively, a higher k_1 (and wage) implies a greater probability that any worker who has contacted this firm and in addition one other firm, actually applies to the particular firm offering k_1.

Firms establish as long as net expected profits from entering the market are positive, such that net profits are dissipated in equilibrium. Each firm selects a profit-maximizing k at the stage of establishment. Denote net expected profits of a firm selecting k by $\pi(k)$. In a free-entry equilibrium, $\pi(k) = 0$ for all levels of k selected by firms, implying that

high-wage firm, a contradiction.

$$\pi(k) = (1-e^{-q(k)})(1-\beta) f(k) - k - C \ = 0. \qquad (4)$$

Here $1-e^{-q(k)}$ is the probability that a firm is active as a function of k. Assuming that the distribution function G is perfectly continuous wherever it is nonzero, we find the following family of first-order conditions with respect to k:

$$\frac{d\pi(k)}{dk} = (1-e^{-q(k)})(1-\beta) f'(k) + e^{-q(k)} q'(k)(1-\beta) f(k) - 1 = 0, \qquad (5)$$

where $q'(k) = 2Q(1-z)g(k)$, and $g(k) \equiv dG(k)/dk$ is the density function for k.[5] In the Cobb-Douglas case, $f(k) = k^{\alpha}$, $f'(k) = \alpha k^{\alpha-1}$, and $kf'(k)/f(k) = \alpha$.

When G has no mass point, (4) and (5) hold for all $k \in [k_0, k_m]$, where k_0 and k_m are the supports of G(k), represent an infinity of optimality and zero-profit conditions.[6] To pin down k_0 and k_m, note that $q(k_0) = Qz$, while $q(k_m) = Q(2-z)$, implying from (4):

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[5] We are here ignoring possible cases where the equilibrium productivity distribution has an atom. AS show that an atom (a mass point) may occur at the upper support of the distribution for k in the wage-posting case. The same result applies here, and the analysis of such a case is quite parallel to that of AS (pages 591-592 and 595-596). A mass point occurs when total worker return to applying at firms with higher w, $w\rho(w)$ (where we here index k by w), has an interior maximum. Workers will then not find it advantageous to apply to firms offering higher wages, because the wage increase is more than offset by a reduction in the probability of obtaining employment at this firm. The main focus of the present paper is on the alternative matching technology of section 3 below, where such a possible mass point turns out not to be an issue. We here skip these issues, without much loss of generality, for comparison of our wage-bargaining case with the wage-posting case of AS.

[6] A condition can here be derived ensuring that G has no mass point, in fashion similar to AS. Since our discussion of the AS model here is only illustrative (and most emphasis will be put on our alternative search technology in section 3) we will not go deeply into this issue. We however note that it can here be shown that no mass point exists whenever the solution value for z is sufficiently large, and the search cost c is not too small. On the other hand, when c is very small, there will also here exist such a mass point, and more of the probability mass will

$$(1-e^{-Qz})(1-\beta)f(k_0) = k_0 + C \qquad (6)$$

$$(1-e^{-Q(2-z)})(1-\beta)f(k_m) = k_m + C. \qquad (7)$$

At the lower support we must have q'(k) = 0.[7] This implies

$$g(k_0) = 0. \qquad (8)$$

For the moment z can be assumed given. (4)-(8) now determine k_0, k_m, $g(k_0)$ and Q, and all g(k) for $k \in (k_0, k_m]$, where M = 1/Q represents the measure of entering firms relative to the number of workers ("labor market tightness"). We will below see that z is also determined endogenously, by workers' search decisions. k_0 is given by

$$k_0 = \frac{\alpha}{1-\alpha}C. \qquad (9)$$

The solution (9) for k_0 is the same as found by AS, and is independent of bargaining or bargaining power. This is a condition for (unconditional) technically optimal (but as will be seen later, generally not socially efficient) production at the bottom of the wage and

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be shifted to this mass point as c falls.

[7] The intuition behind this result is as follows. Note first that slightly below the lower support, the productivity distribution must be zero, as firms do not reduce the probability of attracting a worker by lowering k further. The firm must be exactly indifferent between retaining k and lowering it slightly, when k is at the lower support, and this decision is made without concern for the number of workers attracted (since this number is not affected when the firm is already at the lower support). The firm must however also be indifferent between retaining k at the lower support, and increasing it slightly. This can then hold only when such an increase does not increase the number of workers attracted, which in turn requires the density of q(k) to be zero at k_0.

productivity distribution, and will be valid for all solutions also in this paper.

The value of k_0 from (9) can now be inserted into (6) to yield an implicit solution for Q:

$$(1-e^{-Qz}) = \frac{1}{(1-\beta)}\left(\frac{C}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}. \tag{10}$$

Since all terms on the right-hand side of (10) are constants, Qz must be a constant, which implies that Q is proportional to 1/z. Thus equilibrium firm entry is proportional to the share of workers who search one instead of two firms. To study different cases, consider first $z \to 1$ (all workers sample one firm each). The distribution of k then collapses to a single point given by (9), which in turn collapses to zero as C tends to zero. Firms are then in a monopsony situation versus any applying worker. There is no competition among firms with respect to k, and k is set at the unconstrained optimal level. Under the Cobb-Douglas technology, average ex post profits per unit of k are decreasing in k. Free entry would then increase the number of firms without bounds, thus driving k to zero, whenever C is zero.

$z\to1$ represents a "low-wage equilibrium "(LWE), which has properties similar to the "Diamond equilibrium" (from Diamond (1971)) found in AS. There are however differences between the two types of equilibria. Ex post (when the wage is determined) the firm is here not in a monopoly situation in its relationship with the employed worker, but rather in one of bilateral bargaining, and the resulting wage is not monopolistic. Ex ante, when determining capacity levels, firms however behave monopolistically in the sense that they face no ex ante competition in terms of attracting prospective workers in determining k.

Consider now the possible other extreme case, where z tends to zero. Then Q would tend to infinity and thus M to zero, and no firms would enter in the limit. It is then never advantageous for workers to search (more than once), since the probability of a search message resulting in a job would tend to zero (in particular q(av) would tend to infinity). This demonstrates that all workers searching twice cannot be an equilibrium.

The implication is that a wage dispersion equilibrium must have the property that at least some workers sample more than one firm; thus some workers sample one and some sample two firms. This was inherent in Stigler (1961), and equivalent to results demonstrated elsewhere by Burdett and Judd (1983), and by AS. While the lower support of the distribution over k here is fixed, the upper support is raised when z is reduced. Thus when more workers search twice, the distribution of productivities and wages become more spread out.

To derive a possible interior solution for z, i.e. $z \in (0,1)$, at a wage dispersion equilibrium, note that workers must be indifferent between searching once and twice. Denote the distribution over w by $\Phi(w)$, assumed perfectly continuous with density $\varphi(w)$, and corresponding to the distribution over k (each k corresponding to one w). The expected return to searching once is given by

$$ES(1) = \int_{w_0}^{w_m} w\rho(w)\varphi(w)dw, \tag{11}$$

where $\rho(w)$ is given in (1) and now written a function of w.[8] This corresponds to AS (q is parameterized by w instead of by k as above, where $w(k) = \beta f(k)$). We here assume that $w\rho(w)$ does not have an interior maximum on its domain. Consider next the return to searching twice. The probability that the higher of two wage offers is less than w_1 equals $(\Phi(w_1))^2$; the density of the higher of two offers is then $2\varphi(w)\Phi(w)$. The expected wage obtained from searching twice is

[8] Again we are glossing over the issue of a possible mass point at the upper support of the distribution; cf. footnote 3.

$$ES(2) = \int_{w_0}^{w_m} w\rho(w)2\varphi(w)\Phi(w)dw \,. \qquad (12)$$

Equilibrium implies that the condition

$$\Delta\mathrm{ES} \equiv \mathrm{ES}(2) - \mathrm{ES}(1) = \mathrm{c} \qquad (13)$$

holds, where c is the cost of one additional worker search. (13) states that workers are indifferent between one and two searches, at such an equilibrium.

We sum up the main results in this section, as follows.

Proposition 1: *Consider the above model with ex post wage bargaining between individual workers and firms. A "Diamond equilibrium", where the productivity distribution collapses to a single point, $k = [\alpha/(1-\alpha)]C$, then exists.*

Proposition 2: *Assume that ΔES is strictly concave and single-peaked, that c is sufficiently small that a solution $\Delta ES = c$ can be found, and that the equilibrium productivity distribution has no mass points. Then there exists a solution to the above model, whereby the relationships (4)-(13) solve for the endogenous variables k_0, k_m, z, M and the distribution of k, where z is given by the lower of the two values solving (13).*

Proposition 1 follows directly from the above discussion, and is a corollary to the equivalent result in AS. The second result is also essentially identical to Proposition 1 in AS. The net gain from additional search has the same basic form as in their model with endogenous k (exemplified by the figures in AS, p. 593). Workers are in equilibrium with respect to z at the lower of the two possible values of z solving (13) (as $\Delta ES-c$ is driven to zero with respect to 1-z, the share of workers making two searches).

3. Solutions under alternative search technology

We now change the assumptions with regard to the search technology. First, we assume that workers' search messages are directed in the sense that a new message always goes to a firm that has received "few" messages at the outset. Search messages will then be evenly spread out across firms. This is in itself not very realistic, but may still be a relevant analytical case as it represents an extreme opposite to the completely undirected search in AS (whereby a firm has a given probability of receiving a given "application", independent of the number of "applications" it has already received).[9] It also has the advantage of providing simple analytical solutions that are relatively easy to interpret. The second difference from AS is that firms are now allowed to offer a job to no more than one worker, after having received a (possibly large) number of search messages. This implies that workers who receive a job offer, know for certain that they will get the job (while in AS a job offer is generally uncertain). We argue that this is a feature which makes our model more realistic than that of AS. This new feature, it turns out, also takes away the unrealistic feature of the AS model, that workers at equilibrium never search more than two firms. Our model however assumes no coordination of firms' return offers: A firm which has received two "applications", gives a job offer at random, i.e., has no information on the number of other job offers received by

[9] One way in which this can be perceived is that there will exist a complete list of established positions (e.g. on the internet), and it is there possible to see how many applications each position has received at any one point in time, but that no other information about each particular position is directly observable prior to applications. Alternatively, one may think of a market administrator who distributes all search messages evenly between firms, but such that each worker does not know a priori whether or not his or her message goes to a firm

those workers who have applied to the firm.[10] A further change from AS is that now a "search message" sent by a worker will be synonymous to an application; and a "return message" from the firm synonymous to a job offer. In AS, by contrast, an "application" could be sent only after the firm had received the initial "search message" and returned this with a message about its wage offer (or capital stock).

Each worker may apply for multiple (n) jobs, but can accept at most one of them. A firm receiving more than one application makes a job offer to one and only one applicant, *selected at random,* and hires this worker if the offer is accepted.[11] Let z_n denote the probability that any given worker sends n = 1, 2, ... search messages, the aggregate number of messages is $N = \sum_{n=1}^{\infty} z_n\ n$ The probability of obtaining a job offer on the basis of one search message equals min[$\frac{M}{N}$, 1], which embeds our assumption of "directed" search, in two distinct ways. First, whenever the total number of search messages, N, is less than the total number of established firms, M, all search messages will be "answered" by firms. Each firm only answers one message, and no firm receives more than one message. Secondly, whenever N>M, all firms receive at least one message, and M job offers are issued (one for each firm). Any given message is here assumed to go to a firm with none (or a minimum number of) previously received messages.

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which has also received other messages.

[10] We consider this asymmetry as realistic, in the sense that it may be easier for a worker to hide how many job offers it has received, than this is for a firm; and it may be easier for firms to credibly signal that they have received few applications.

[11] This assumption implies that there is not "perfect matching" in the search process whereby firms contact workers after messages have initially been received. Here, firms do not know whether a given worker has sent this one particular message, or has also sent another one to another firm. Note in addition that our assumptions here are in one sense opposite to those of AS and our model in section 2 above. There it was assumed that firms may make offers to many workers, but that workers are allowed to apply only to one firm. Here, firms make offers only to one worker each, while each workers may apply at several firms (although no more than one application will be submitted since the worker knows he or she will be

As seen below, the equilibrium worker strategy may here be to send more than two messages. In our analytical discussion we however concentrate on two simple “benchmark” cases, where in both a maximum of two search messages are sent, namely a) N>M, and b) N<M. In case a, the number of search messages sent by workers is greater than the number of firms. Each firm receives at least one message, and the number of messages answered equals the number of firms, M, such that some messages are not answered. Among those workers who send two search messages, some will then receive two job offers, some will receive one, and some none. Among those who send one search message, some will receive one offer and the rest none. In case b, not all firms receive a message, and all messages are answered. Thus workers who send two search messages receive two job offers, while those who send one message receive one job offer. This is as in section 2. An important difference is however that now any job offer made by a firm, and accepted by a worker, results in the worker becoming employed. In section 2, by contrast, firms could make several job offers for each vacancy, and the worker would be uncertain about actually getting a given offered job.

An important technical difference from the first model is that now the value to the worker of a job offer is always strictly increasing in the (implicit) wage offered, since job offers are certain. This can be shown to lead to a perfectly continuous equilibrium distribution of firm productivities, thus avoiding the technical problem in AS, that there may exist a mass point at this distribution’s upper support. Another difference is that whenever the probability p by which a search message is manifested in the form of a subsequent job offer, is less than one in the present case, a worker may have incentives to search more than one job even when $\Phi(w)$ is degenerate, since increasing the number of search messages may increase the probability of obtaining at least one job offer and thus a job. In section 2 the probability of obtaining job offers was by contrast assumed to be independent of the number of job messages sent (such

accepted whenever applying).

that this number only affected the quality of the best job offer obtained).

Case a: N≥M

Here all firms receive at least one search message, and all send exactly one job offer back. Define m = M/N as the probability that any given search message will be answered by a firm, since all messages go to different firms. When a fraction z of workers send one message and a fraction 1-z send two messages, and the measure of workers is unity, N = 2-z. Workers who send one message will then represent a fraction z/(2-z) of the total number of messages, which is then the probability that a firm's job offer will go to such a worker. Workers who send two messages represent a fraction 2(1-z)/(2-z) of total messages, which is the probability that a given job offer goes to such a worker. For a worker who sends two messages, the probabilities that such a worker receives zero, one and two job offers are $(1-m)^2$, 2m(1-m) and m^2, respectively. Their ex ante expected numbers of job offers are then 2m(1-m) from receiving one offer, and $2m^2$ from receiving two offers. The aggregate number of job offers is M = (2-z)m. The probability that a given offer by a firm goes to a worker who gets only one offer, and two offers are then 1-m and m respectively. Denote the probability that a firm employs a worker, as a function of the resulting capital choice k, by τ(k). We then find

$$\tau(k) = 1 - \frac{2(1-z)}{2-z}m + \frac{2(1-z)}{2-z}mG(k) = 1 - \gamma m + \gamma m G(k). \qquad (14)$$

γ=2(1-z)/(2-z) is the fraction of overall search messages that come from worker searching two firms each, where the distribution function G(k) (with its corresponding density function g(k)) is indexed by k instead of by w (as the relationship between w and k is monotonously increasing). The probability that a worker who has issued two search messages, will have the other message answered, is γ. Thus γm is the probability that the worker to whom the firm has given a wage offer, will actually have another offer on hand. The firm zero-profit (or free-entry) condition is

$$\pi(k) = (1-\gamma m + \gamma m G(k))(1-\beta) f(k) - k - C = 0 . \qquad (15)$$

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The first-order condition with respect to k is found differentiating (15), as

$$(1-\beta)(\gamma m g(k) f(k) + (1-\gamma m + \gamma m G(k)) f'(k)) - 1 = 0 . \qquad (16)$$

Such differentiation is now straightforward since, as noted, G will be perfectly continuous on its domain. Under the Cobb-Douglas specification, expressions for k_0 and k_m are as follows:

$$(1-\beta)(1-\gamma m) k_0^{\alpha} = k_0 + C \qquad (17)$$

$$(1-\beta) k_m^{\alpha} = k_m + C . \qquad (18)$$

Also here $g(k_0) = 0$, and k_0 still given by (9) (at the lower support, unconditional profit maximization with respect to k must yield zero profits). (9) and (15)-(18) yield closed-form solutions for the distribution of k including the supports k_0 and k_m, and in addition γm. The solution for G can be written as follows:

$$G(k) = \frac{1}{(1-\beta)\gamma m}(k+C)k^{-\alpha} - \frac{1-\gamma m}{\gamma m}, k \in \left[k_0, k_m\right]. \qquad (19)$$

(16) embeds a necessary condition $g(k) \geq 0$ for all $k \in [k_0, k_m]$, for existence of a solution.

Combining (9) and (17) also implies the following solution for γm:

$$(1-\gamma m)=\frac{1}{(1-\beta)}\left(\frac{C}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\equiv 1-\phi. \qquad (20)$$

From (20), $\gamma m \equiv \phi$ is e.g. independent of γ. The present case requires the constraint $m\leq 1$ to be observed, with several implications. First, γ now cannot be close to zero, since this would in general imply $m > 1$, which precludes the current case. Secondly, an LWE (which would imply $z=1$ and $\gamma=0$) does not exist in this case. Both k_0 and k_m have fixed values, and γ close to zero would yield $m>1$, in contradiction to assumptions under this case. Thirdly, the entire wage and productivity distributions will be independent of γ and m separately. This distribution will however depend on the bargaining parameter β. We find, using (9):

$$\frac{dk_m}{d(1-\beta)}\frac{1-\beta}{k_m}=\frac{k_m+C}{(1-\alpha)(k_m-k_0)}>0. \qquad (21)$$

Thus a greater bargaining power to the firm raises the maximum capital and wage levels, and moves the entire distribution to the "right".

Solutions where $\gamma\in(0,1]$ and "sufficiently large" are feasible as long as the right-hand side of (20) is less than one. When γ is close to one, $m<1$ from (20). In the (limit) case of $C=0$ (no fixed establishment cost for firms), $k_0 = 0$.

Consider now worker behavior. $\gamma\in(0,1)$ implies that workers are indifferent between sampling one and two jobs. The probability of obtaining a job offer (and thus obtaining employment) for a worker sampling only one job equals m, while the probabilities of obtaining one and two offers when sampling two jobs are $2m(1-m)$ and m^2 respectively. The expected wage when sampling two jobs is Emax (w_1,w_2), where w_1 and w_2 are the two wage offers received. This expression is simply ES(2) from (12) where $\rho(w) \equiv 1$ (since here each firm only makes a job offer to one worker). The increase in value from searching two instead of one job, denoted ΔES, is then given by

$$\Delta ES = m(1\text{-}2m)Ew + m^2 E\max(w_1,w_2)\ . \tag{22}$$

From the analysis above, Ew and $E\max(w_1, w_2)$ are constants (and independent e.g. of γm) at the equilibrium solution for firms. m is taken as given by each worker in choosing the number of searches. An internal solution for z entails $\Delta ES = c$ (i.e., (13)). A question is whether such a solution exists. Note then first that solutions for m can always be found where $\Delta ES > 0$. Thus for some sufficiently small positive c, $\Delta ES - c \geq 0$, and solutions where at least some workers search more than one firm exist. Consider the possibility of a corner solution $z = 0$. Then m is given from (13) and (22) inserting $\gamma = 1$. For sufficiently small c, $\Delta ES > c$ at such a solution. All workers then search (at least) twice.

The analytical solution to the model for the case of more than two searches is not derived explicitly. Note however that the $\tau(k)$ function in (14) then would need to be altered. Intuitively, three or more searches are always optimal given that c is small enough. The reason is that the expected return for each individual worker to additional search is always positive since it raises the probability of obtaining (at least) one job offer (regardless of the wage distribution); in addition it increases the expected best offer when the wage distribution is spread out. Here two searches or more for each worker does not induce perfect competition since not all workers receive more than one job offer.

We formulate the following result:

Proposition 3: Under our altenative search technology, for a particular range of search costs *for workers there exists an equilibrium where $z \in (0,1)$. M and the distributions for w and k are given by (9) and (15)-(18) and the lower of the two values of z solving $\Delta ES = c$ in (22).*

Consider a simple numerical example with $\alpha = \beta = ½$, and C = 0. This implies $k_0 = w_0 = 0$, $k_m=w_m=¼$, and $\gamma=m=1$, i.e. we have a special limiting case where $\gamma m=1$, all workers send two search messages each, and each established firm receives exactly one search message.[12] This example implies perfect Bertrand competition for workers among the firms; it is still a feasible solution in our example by virtue of C=0. All workers search exactly twice, which implies a particular range for the search cost c. There is no unemployment (since all workers receive binding job offers), while the number of firms is exactly twice the number of workers. This results in half of all firms being idle in equilibrium. The wage offer distribution is uniform on [0, ¼], with expectation 1/8, which would be the expected wage if searching just once. The realized wage distribution given that all workers search twice is however not uniform (it has a thicker upper tail) and has expectation 1/6. The expected gain to workers from searching twice instead of once is then 1/6 – 1/8 = 1/24. Thus whenever $c \leq 1/24$, all workers search (at least) twice. There is then a range for c below 1/24, such that all workers search exactly twice (the gain from searching a third time is strictly less than 1/24). Whenever c is sufficiently small, it is instead optimal for workers to search more than twice. Note also that net welfare assuming that workers search exactly twice (net of search costs, where the first search is considered free as before) is given by $1/6 – c \in [W(2), 1/8]$, where $W(2) \in (1/6, 1/8)$ is an upper bound on welfare.

Case b: N<M, some workers send one and others two messages

[12] Note that (23) does not hold under this particular example. This is however not crucial since it is a border-case solution for z (=0); the stability condition only concerns internal solutions

In this case the number of firms exceeds the number of messages sent. We here first study the properties of possible solutions where fractions z and 1-z of workers send one and two messages respectively, while none send more than two messages. Given z∈(0,1], the number of search messages sent, N, equals z + 2(1-z) = 2-z. Fractions z/(2-z) and 2(1-z)/(2-z) of these are sent by workers who send one and two messages respectively. Given our assumption of fully directed search (whereby any additional search message is always sent to that firm, or one of those firms, that has initially received the fewest search messages; here this number equals zero), a fraction 1/m = (2-z)/M of firms receive one message, no firm receives more than one message, and each message results in a job offer from the firm receiving the message. Since no firm receives more than one message, no firm makes more than one offer, and all workers will be employed. This version of the model thus leads to perfect matching in this particular sense. When z<1 not all firms making job offers employ a worker, and the fraction of firms actually employing a worker equals 1/M =1/[(2-z)m]. The probability that a job will be filled given that the firm receives a search message is now the same as under case a, except that we there had m=1 (since in that case, all firms received at least one search message). The condition of zero ex ante expected profit can now be written as

$$\pi(k) = \frac{1}{m}(1-\gamma+\gamma G(k))(1-\beta) f(k) - k - C = 0, \qquad (23)$$

where as before, γ = 2(1-z)/(2-z) is the number of search messages sent by workers who send two messages, while 1-γ is the fraction sent by workers who send one message only. In the expression (23), the first main term on the right-hand side is ex ante expected profits from establishing, which equals current expected profits as a function of k (which takes into

for z ∈ (0,1).

consideration that a higher k increases the probability of operating), multiplied by 1/m = the probability that the firm at all receives an application. The overall probability of operating, over all firms (and capital levels), here equals 1/M = 1/[(2-z)m], which is less than 1/m whenever z<1. Maximizing (23) with respect to k yields the following set of first-order conditions for firms:

$$\frac{1}{m}(1-\beta)(\gamma g(k)f(k)+(1-\gamma+\gamma G(k))f'(k))-1=0\,. \tag{24}$$

Given Cobb-Douglas technology, k_0 and k_m can now be found as follows:

$$\frac{1}{m}(1-\beta)(1-\gamma)k_0^{\ \alpha}=k_0+C \tag{25}$$

$$\frac{1}{m}(1-\beta)k_m^{\ \alpha}=k_m+C\,. \tag{26}$$

For given z, (9) and (23)-(26) solve for the distribution of k including supports k_0 and k_m, and for m. Integrating (24) yields the following closed-form solution for G(k):

$$G(k)=\frac{m}{(1-\beta)\gamma}(k+C)k^{-\alpha}-\frac{m-\gamma}{m\gamma},k\in\left[k_0,k_m\right]. \tag{27}$$

k_0 and k_m and z/M are now given from (9), (26) and (27). The equilibrium level of z/M is

$$\frac{z}{M}=\frac{1}{(1-\beta)}\left(\frac{C}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}. \tag{28}$$

In (28), both z and M are endogenous variables. M can here be viewed as determined from (28), by firms, for given z (which in turn is determined through worker behavior to be described below). (28) yields a unique ratio z/M. This implies that equilibrium firm entry, M, is proportional to the share of workers who make only one search (given that no worker makes more than two searches). Note also that the condition $m > 1$, which translates into $M > 2-z$, is required under this case. Calling the right-hand side of (28) K, we find the necessary condition for case b to hold:

$$z > \frac{2K}{1+K}. \tag{29}$$

Since $z/M < 1$ (as z cannot exceed one, while $M > 1$), $0<K<1$, and $0<z<1$. This in turn implies that a possible equilibrium in this case must imply that some workers search one firm, others two, and none more than two (since workers must be indifferent between searching once and twice, none will search three times due to diminishing returns to additional search). This contrasts case a above, where an equilibrium could imply workers searching more than two firms.

Since $w_0 = [\alpha/(1-\alpha)]C$, from (9), it is straightforward to show that k_m increases strictly in γ, from (25)-(26). This implies that the productivity (and wage) distribution is more dispersed when γ (the fraction of search messages coming from workers who search two firms) is greater (and z smaller).

When instead all workers search only once ($z=1$, and $\gamma=0$), the distribution again collapses to an LWE with a single mass point. To check existence of such an equilibrium, note that the productivity distribution then, from (25)-(26), collapses to the single point $k_0 = [\alpha/(1-\alpha)]C$, from (9). With a corresponding point wage distribution no worker searches more than once, since all searches result in job offers. Consider next the firm entry condition in this case. An equilibrium here requires that we can find a permissible solution for firm entry such that (28) holds for z =1. Such a solution must fulfil our basic assumption under this case, that m = N/M < 1. Since N = 2-z = 2-1 = 1 in this case, this assumption translates into the condition that 1/M < 1, i.e. z/M < 1. This implies, from (28),

$$\frac{1}{(1-\beta)}\left(\frac{C}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}<1. \tag{30}$$

Thus we may state the following result:

Proposition 4: Under our alternative search technology, there exists a low-wage equilibrium where the productivity distribution collapses to (9), given that (30) holds.

Consider next the possibility of a solution where z $\in$ (0,1) in this case, and consequently some workers search one and others search two jobs. Workers must then be indifferent between sampling 1 and 2 jobs. The probability of employment when sampling (at least) one job is always one. The expected wage when sampling two jobs with wage w_1 and w_2 is Emax (w_1, w_2), given by

$$E\max(w_1,w_2)=\int_{w_0}^{w_m}2w\Phi(w)\varphi(w)dw, \tag{31}$$

where Φ(w) and φ(w) are respectively the cumulative distribution and density of wages implied by the distribution over firms' capital choices k, from (27). (31) corresponds to (12) for ρ(w) ≡ 1 (all job offers real). The equilibrium condition for workers is now simply

$$E\max(w_1, w_2) - Ew = c, \tag{32}$$

where Ew is the expected wage using the distributions of k from (27), and where E max (w_1,w_2) is given from (31). In principle the expression (32) can be given in closed form, but it will be complicated. We may then state the following conclusion:

Proposition 5: Assume our alternative search technology, and that we can find a solution where (25)-(28) and (32) hold, for solution values of type N<M and 0<z<1. Then we have at least one wage dispersion equilibrium under case b, in which all workers are employed, but all generally earn different wages.

The intuition behind this solution is similar to that that invoked for the parallel solution in case a. Also here it is the case that greater search activity among workers leads to competition between firms for workers. This competition leads to dispersed wages, which in turn justifies the increased search. Conceivably, there may be more than one such equilibrium, although this issue is not fully analysed here.

4. Welfare under cases a and b

We will now briefly consider welfare properties of the dispersion solution under case a (or more precisely, under a "borderline" case between a and b), and the LWE solution under case b, and compare these to the socially optimal solution. Note that the social optimum is very simple when search is directed as we here assume. It entails zero unemployment, all workers search once, k is determined maximizing social surplus per job created, f(k)-k-C, and the number of jobs established equals the number of workers. The welfare optimal solution for k is, given our specified Cobb-Douglas technology,

$$k_w = \alpha^{\frac{1}{1-\alpha}}. \tag{33}$$

Since C is a fixed establishment cost, k_w is independent of C. We find that $k_0 < k_w$, while the relationship between k_m and k_w is ambiguous, from (17)-(18). k_m is found to decrease in β and C. For sufficiently low C and β, $k_m > k_w$, while the opposite inequality holds for other cases. For "high" C and β, the entire market distribution for k is below k_w, while for lower C and β, k_w lies strictly inside the distribution for k. Note that k_m can exceed k_w only if $1-\beta > \alpha$ (and in the marginal case, only when C = 0), i.e., workers' bargaining power must be small. Search will be overoptimal, since (at least some) workers search more than once.

Consider again the numerical example used for the wage dispersion equilibrium in section 3, where $\beta = \alpha = ½$, and C = 0. Now maximal welfare equals maximum aggregate production surplus less aggregate search cost (assuming still that one search is free for all workers). In this case, $k_w = ¼$, $f(k_w) = ½$, and maximal welfare, U(w), equals ¼. In the market solution, welfare, U(m), can be represented by the aggregate of workers' wages less excessive search costs. Welfare in the market solution, U(m), lies between 1/6 and 1/8, and close to the latter figure when c is close to 1/24. Thus close to half of the social surplus is here dissipated through inefficiently high entry (half of all firms are vacant), inefficiently low capital per firm, and excessive worker search. Note that $k_m = ¼ = k_m$, i.e., efficient, but $k_0 = 0$, and average k is 1/8, i.e., half the efficient level. Inefficiency is reduced by the property that firms with gradually higher (and more nearly efficient) capital stocks have higher employment probabilities (equalling one for the, efficient, firm at the upper support). Moreover, the inefficiency due to excessive entry is limited by the fact that firms on average have low capital, and capital is particularly low in firms with low employment probabilities.

Under the LWE, in case b (N<M), welfare goes to zero as $C \rightarrow 0$, since the distribution for k collapses to a mass point in zero, resulting in zero wages. When C > 0, the LWE however implies positive welfare. We have a single mass point $k_0 = C > 0$ for k. In this case U(w) = ¼ - C, while U(m) = $\frac{1}{2}C^{1/2}$. From (29), feasibility also here requires $C \leq 1/16$ (as z = 1 and $M \geq 1$ in this case). In the special case of C = 1/16, U(w) = 3/16, while U(m) = 1/8. Thus, apparently, the LWE in such a case cannot fare much worse than the wage dispersion equilibrium, possibly better. This is partly due to less worker rent-seeking in the form of less search, as all firms set the same wages that makes excess search privately inefficient. Such a possibility could not arise in AS, where an LWE always leads to a zero wage.

Consider finally what happens in the model as the bargaining power of workers, β, increases. When β = 0 and we are in case b, there is a (genuine) LWE with zero welfare. In case a we have now instead a wage dispersion equilibrium which resembles (but is not the same as) that in AS, and characterized by (9) and (18). From (18) we find under our example that $k_m = (1-\beta)^2$ given $\gamma m = 1$. The corresponding expression for the maximal wage is $w_m = \beta(1-\beta)$, which is maximized for β = ½.. As now β increases beyond its equilibrium value in the example (= ½), firm entry will drop, and m drop correspondingly, below its initial equilibrium value of 1. Employment will then also drop (below the initial full-employment level). This will effectively preclude an equilibrium of the type derived given C = 0 (since the probability of hiring at the lower support of the wage distribution must now be strictly positive, which is incompatible with a lower-support capital choice of zero). The expression for w however indicates that w is maximized at ½ in this case.

5. Conclusions

We have considered (Nash) wage bargaining in a model of non-sequential worker search where firms choose technologies before matching with workers. We have studied two different versions of the model, corresponding to different matching technologies available to workers and firms in the market. When workers' search costs are "not too high", there exist, under both search technologies, equilibria with nondegenerate capital and wage distributions, where at least some workers search two firms or more. Moreover, we cannot generally rule out the existence of two or more such equilibria, at least not under our "alternative" search technology in section 3. Under both search technologies we also find one "low-wage equilibrium" (LWE) (similar to the "Diamond equilibrium" in AS) where the wage and productivity distributions collapse to single mass points and all workers search only one firm, but where workers earn rents by virtue of their bargaining strength. Other results are that the lower support of the wage distribution under wage dispersion always coincides with the LWE wage, and that the wage distribution is generally more spread out when the share of workers searching two firms (or more) is greater. Under the search technology applied by AS (treated in section 2), a wage dispersion equilibrium implies that all workers search either one or two firms, as under wage posting. Under the alternative technology (in case a) some or all workers may instead search more than two firms at equilibrium. This principle is illustrated in a numerical example where all workers searching twice (or more) is shown to be an equilibrium, under realistic parametric assumptions. Another feature of our alternative search technology is that the LWE here always implies full employment, while the alternative wage dispersion equilibrium may or may not embed unemployment, higher average search costs, and higher average wages to employed workers. Welfare typically increases with the bargaining power of workers, at least up to a point. This is unambiguously true in the LWE where net welfare simply is the sum of wages, there is full employment, and firm size is independent of bargaining strengths In the wage dispersion equilibrium things are more complicated, since high wages then may imply higher unemployment. A welfare analysis must involve a comparison of gains and losses, for employed and unemployed workers respectively, and it is not obvious that the LWE is the inferior one.

Several elements need to be addressed in extensions of the current framework. First, our alternative search technology, while representing an interesting (at least radically different) alternative to the standard random-matching model represented by AS, is in itself unrealistic, and the exact results derived from this technology are thus questionable. More realistic "middle grounds" thus ought to be explored in future work. Secondly, efficiency properties of the model ought to be studied more explicitly. And thirdly, various dynamic issues ought to be addressed in the context of our search-theoretic framework. Possible such issues are on-the-job search and continuous matching with nonsequential search, which to our knowledge have not been studied so far in the literature. We intend to come back to such issues in future work.

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