Wage distributions under firm wage posting

and directed worker search

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Abstract

We derive endogenous and continuous wage distributions under directed nonsequential search where search messages are allocated to firms with "few" messages, and firms post a given job offer to only one worker. The solution exhibits multiple internal equilibria, the Diamond equilibrium always exists, and equilibrium may entail workers searching several firms each. The model gives additional realism to the standard non-sequential search framework.

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1. Introduction

This paper presents a new model of equilibrium wage distributions for identical workers, similar to Burdett and Judd (1983) and Acemoglu and Shimer (2000) (AS) but under some different assumptions: Workers' search messages are not allocated at random among firms but go to firms with few messages; and firms post wages but are allowed to offer a given job to only one worker. We study two separate cases, a and b ("high" and "low" firm establishment costs respectively), where there are more (fewer) search messages than firms. In both cases we derive continuous market wage distributions. Under case b, a Diamond (1971) equilibrium always exists, while there may be (single or multiple) dispersion equilibria under both cases a and b. We argue that our assumptions are in some respects more realistic than those of AS, and lead to the new realistic result that workers search out many firms when search costs are small. Our derived wage distribution also has a simpler analytical form than others derived from similar assumptions. This may be helpful e.g. for empirical investigation.

2. Wage distributions when workers are informed about firm location

Assume a single-period labor market where firms initially sink a cost C per job with unit output capacity. Firms post (binding) wage offers prior to engaging workers, initially unknown to workers, who are identical with zero reservation wages. Workers submit one or more search messages to a "clearing central" which allocates messages evenly across firms (if all jobs have received n search messages except firm i which has received n-1 messages, a further message goes to firm i). Normalize the measure of workers to unity, and their average number of search messages sent is N. M firms are established. A firm receiving at least one message offers a job to one of these workers drawn at random.[1] This offer contains a (binding) wage quote, and a binding promise to employ this worker. A worker who receives only one offer accepts this offer. A worker who receives more than one offer, accepts the highest wage offered.[2] These assumptions are non-standard but may arguably be more realistic than those of standard nonsequential random search models, notably AS.

Two cases are discussed: a) N ≥ M, where all firms receive at least one search message, and all send exactly one job offer back; and b) N < M, some firms receive no message.

2.1 Case a: N≥M

Define m = M/N (≤ 1) = the probability that a given search message is answered by a firm, all messages going to different firms. Fractions z and 1-z of workers send one and two messages respectively.[3] With a unit measure of workers, N = 2-z. Fractions z/(2-z) = 1-γ and 2(1-z)/(2-z) = γ of all messages are sent from workers who send one and two messages respectively. A worker sending two messages has probabilities $(1-m)^2$, 2m(1-m) and m^2, of receiving zero, one and two job offers. Expected offers are 2m(1-m) from one offer, and $2m^2$ from two offers. Total offers are M = (2-z)m. The probability that a firm employs a worker, as a function of the wage offer w, is

(1) $$\tau(w) = 1 - \frac{2(1-z)}{2-z}m + \frac{2(1-z)}{2-z}mF(w) = 1 - \gamma m + \gamma mF(w).$$

[1] The firm thus cannot tell how many messages a given worker has sent out; such information would affect which worker ought to receive the wage offer.
[2] In contrast to AS, it is here always optimal for a worker to accept the highest wage offer, since the worker always gets the job for certain.
[3]The case of more than two searches, left for future work, is not modelled explicitly. It is relevant when workers' marginal search costs are low.

F(w) and f(w) denote the market distribution and density of wage offers. γm is the probability that the worker receiving a given offer, receives another offer. The zero-profit (or free-entry) conditions for firms are

(2) $$\pi(w) = (1-\gamma m + \gamma m F(w))(1-w) - C = 0\,,$$

for all w in the support $[w_0, w_m]$ of the wage distribution. First-order conditions for all $w \in [w_0, w_m]$ are found differentiating (2) with respect to w:

(3) $$\gamma m(1-w) f(w) - 1 + \gamma m - \gamma m F(w) = 0\,.$$

$F(w_0) = 0$, $F(w_m) = 1$, while $w_0 = 0$ (the reservation wage).[4] The zero-profit condition yields, using (1),

(4) $$\gamma m = w_m = 1\text{-}C.$$

Note that

(5)-(6) $$\gamma m = \frac{2(1-z)}{(2-z)^2} M = \gamma \frac{2-z}{2(1-z)}(1-C)\,,$$

where $m \leq 1$. A "Diamond equilibrium" (characterized by z=1) is not possible in this case. In the limit case of m = 1 (each firm receives exactly one message),

(7) $$1 - z = \frac{1-C}{1+C}.$$

From (3) and (4), the distribution and density of wage offers are

(8) $$F(w) = \frac{C}{1-C}\frac{w}{1-w}$$

(9) $$f(w) = \frac{C}{1-C}\left(\frac{1}{1-w}\right)^2.$$

The density of realized market wages is proportional to the density of wage offers, f(w) and to the density of worker job acceptance for a given wage offer, $\tau(w)$, and

[4] A firm offering a wage at the bottom of the wage distribution offers the lowest feasible (or reservation) wage, since this firm is not competing with any other firm for the worker.

takes the form g(w) = B τ(w)f(w), where B is a constant. The market wage distribution G(w) takes the form

(10) $$G(w) = B\frac{C^2}{1-C}\frac{1}{2}\left(\frac{1}{1-w}\right)^2 + B_0,$$

where B_0 is another constant. Using that G(0) = 0 and G(A) = 1,

(11) $$B_0 = -\frac{B}{2}\frac{C^2}{1-C}, B = \frac{2}{1+C}.$$

(12) $$G(w) = \frac{C^2}{1-C^2}\left(\left(\frac{1}{1-w}\right)^2 - 1\right)$$

(13) $$g(w) = \frac{2C^2}{1-C^2}\left(\frac{1}{1-w}\right)^3.$$

The expected realized wage is

(14) $$Ew = \int_{w=0}^{1-C} wg(w)dw = \frac{1-C}{1+C}.$$

This model cannot represent equilibrium when C is very small, since m then exceeds unity and we are in case b studied below.

When $\gamma \in (0,1)$, workers are indifferent between sampling one and sampling two jobs. When all workers strictly prefer to sample two jobs, z = 0, γ = 1, and M = 2m. (All workers sampling only one job would require γ = 0, and is precluded by the condition γm = 1-C > 0.) The probability of employment for a worker is m when sampling only one job, and 2m(1-m) and m^2 the probabilities of obtaining one and two offers when sampling two jobs. The expected wage when sampling two jobs is Emax (w_1,w_2), w_1 and w_2 being two random wage offers. The return to searching two jobs instead of one is

(15) $$\Delta ES = m(1-2m)Ew + m^2 E\max(w_1,w_2) .$$

Expectation of the highest of two wage offers is

(16)

$$E\max(w_1, w_2) = 2wf(w)F(w) = \frac{2C^2}{(1-C)^2}\int_{w=0}^{1-C}\frac{w}{(1-w)^3}dw = 1-2\frac{C}{1-C}-2\frac{C^2}{(1-C)^2}\ln C.$$

The expected wage given only one offer is

(17) $$E(w_1) = \frac{C}{1-C}\int_{w=0}^{1-C}\frac{w}{(1-w)^2}dw = 1+\frac{C}{1-C}\ln C.$$

An internal solution for z entails ΔES = c. For small positive c, ΔES–c ≥ 0, and solutions where at least some workers search more than one firm then exist.

In a possible corner solution z = 0, M = 2(1-C). For small c, then ΔES > c. All workers then search (at least) twice. This requres, from m ≤ 1 and (9):

(18) $$z \leq \frac{2C}{1+C}.$$

An increase in the fraction 1-z of workers who search two instead of one firm spreads out the wage distribution by increasing w_m, and reduces firm profitability and entry. The first factor increases and the second reduces worker search, with uncertain net effect. Numerical simulations show that, for particular levels of C, the model exhibits multiple (two) wage distribution equilibria.[5]

2.2 Case b: N<M

Now there are more firms than messages, and some firms receive no message. When shares z and 1-z of workers send one and two messages, a fraction 1/m = N/M = (2-z)/M of firms receive one message, none more than one, all messages result in job offers, and all workers are employed. When z<1 not all firms making job offers employ a worker. A fraction 1/M of firms operate. For a firm receiving a search

[5] For C = 2/3 we find two internal equilibria with z ∈ [0,1] over the range of approximately [0.05, 0.07] for c. In particular, for c = 0.05, both z = 0 and z ≅ 0.8 are equilibrium values for z.

message, the probability $\tau(w)$ of employing a worker is the same as under case a except that m=1. The zero-profit condition is now, for all $w \in [w_0, w_m]$:

(19) $$\pi(w) = \frac{1}{m}(1-\gamma+\gamma F(w))(1-w) - C = 0\,.$$

Maximizing (22) with respect to k yields the first-order conditions for firms:

(20) $$\frac{1}{m}\left[\gamma(1-w)f(w) - 1 + \gamma - \gamma F(w)\right] = 0\,.$$

Applying the zero-profit condition,

(21) $$w_m = \gamma = 1 - mC.$$

Here $\gamma \in [0,1)$, and $z \in (0,1]$. Some workers search one firm and others two, and none more than two, in contrasts case a above. Note that z=0 would create perfect Bertrand competition (as in AS).

In the F and G functions, we now replace C in (8)-(9) and (12)-(13) by mC. The expected realized wage is

(22) $$Ew = \int_{w=0}^{1-mC} wg(w)dw = \frac{1-mC}{1+mC}.$$

When $\gamma \in (0,1)$, workers are indifferent between sampling one and two jobs. All workers get binding job offers; thus all are employed. The expected wage when sampling one job is Ew taking the expectation over F(w). The distribution function for the best of two offers is $(F(w))^2$ with density 2f(w)F(w). The expected realized wages with one and two wage offers are now

(23) $$E(w) = 1 + \frac{mC}{1-mC}\ln(mC)$$

(24) $$E\max(w_1, w_2) = 2wf(w)F(w) = 1 - 2\frac{mC}{1-mC} - 2\frac{(mC)^2}{(1-mC)^2}\ln(mC).$$

Indifference entails the condition

(25) $$E \max (w_1,w_2) - E(w) = c.$$

(25) determines m from (23)-(24), which in turn determines γ from (21).

A Diamond equilibrium here exists. It requires $\gamma = 0$, and $mC = 1$ from (21). All surplus is then exhausted through excessive firm entry. The wage distribution collapses to a mass point at zero, all workers search once (given one free search), all find a job immediately, and only a fraction 1/m of firms fill their jobs.

3. Extensions

Our model can and should be extended in several directions. First, firms' productivity choices may be endogenized and joint wage-productivity distributions derived, as in AS or in Strand and Yin (2003), where we assume ex post wage bargaining. Other fruitful extensions may be to cases where workers observe firm's capital choices prior to submitting search messages, and by embedding the assumption that workers know firms' wage offers when sending their search messages.

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