Radiation stress and the drift in gravity waves with Rayleigh friction

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ABSTRACT

By utilizing a Rayleigh friction formulation to model the dissipative processes in the ocean, a Lagrangian analysis for the mean drift due to surface gravity waves is performed. The waves have amplitudes that vary slowly in time and space, and can be forced by a prescribed wind-stress distribution normal to the free surface. The ocean depth is constant, and the analysis is valid for arbitrary wavelength/depth ratios. The derived equations for the horizontal Lagrangian mean drift contain depth-dependent forcing terms that are proportional to the Stokes drift, expressing the external forcing of the waves. There are no depth-varying radiation stress-like terms in these equations. For waves along the \textit{x}-axis (1-axis), the mean drift equations contain an additional forcing from a depth-independent pressure gradient identical to the divergence of the radiation-stress tensor component $\mathbf{S}_{11}$ of Longuet-Higgins and Stewart (1960). From the surface-boundary condition for the wave field, a new relation is derived for the conservation of mean wave momentum. Applying this condition, and integrating the mean Lagrangian drift equations in the vertical from the bottom to the free surface, the only wave-forcing term in the derived mass transport equations is the divergence of the radiation-stress tensor component $\mathbf{S}_{11}$ of Longuet-Higgins and Stewart, as one would expect from an Eulerian analysis.

KEY WORDS: Gravity waves, Lagrangian drift; radiation stress, Rayleigh friction.

INTRODUCTION

A considerable interest has developed in formulating equations for oceanic circulation that take into account the effect of surface gravity waves. In the traditional Eulerian description the mean wave momentum is confined between the crests and the troughs of the wave, since the fluid motion is purely periodic below the troughs. However, individual fluid particles in the waves do have a slow drift, and hence there is a mean wave momentum in the wave propagation direction; the so-called Stokes drift (Stokes, 1847). The Stokes drift is inherent in the waves, and whatever causes a change in a wave property such as amplitude, wavelength or frequency, thus causes a change in the mean wave momentum. Our definition here of the mean wave momentum as represented by the Stokes drift coincides with the wave pseudomomentum defined by Andrews and McIntyre (1978). However, the total momentum in the fluid is not necessarily distributed in space or time in the same way as the waves, e.g. McIntyre (1981).

Stokes considered waves in inviscid fluids. Applications to real fluids had to wait for Longuet-Higgins (1953), who took into account the effect of viscosity on the wave drift. Especially for shallow-water waves this has important consequences, since a no-slip bottom creates secondary mean momentum that through viscous diffusion eventually changes the drift velocity in the entire fluid layer. In the case of vertical integration of the Eulerian equations from the bottom to the undulating surface, the derived volume fluxes do include the effect of waves, e.g. Phillips (1977). For deep-water waves the wave-induced fluxes from the traditional Eulerian approach becomes identical to those derived from a direct Lagrangian approach (Weber et al., 2006).

For waves with amplitudes that vary in space, a concept called radiation stress, which acts to force the mean wave-induced fluxes, appears in the vertically integrated Eulerian momentum equations (Longuet-Higgins and Stewart 1960). Recently, much effort has been spent by transforming the Eulerian equations in various ways to obtain the anticipated forcing from the depth-dependent radiation-stress terms, by analogy with the traditional vertical variation of the viscous stress, the turbulent Reynolds stress, or the wave-induced Reynolds stress. Most notably in this respect are the papers by Mellor (2003, 2005). In the present paper we investigate this question in detail, utilizing a Lagrangian description of motion. We consider drift due to waves with amplitudes that vary slowly in time and space. The wind stress, through a prescribed variation normal to the free
surface, may act to sustain the wave amplitude. The dissipative process in the fluid, promoting amplitude decay, is modelled by a linear friction, or a Rayleigh friction. We refer to Lamb (1932) for a discussion of Rayleigh’s work on waves on a running stream. In short, the viscous force per unit mass \( \nu \dot{\mathbf{v}} \), where \( \nu \) is the kinematic viscosity, is replaced by \( -\mathbf{r} \dot{\mathbf{v}} \). Here \( \mathbf{r} \) is a constant friction coefficient. This procedure has been applied to weak friction in water waves by Miles (1967), Lake et al (1977), and more recently by Mei and Hancock (2003), and Segur et al. (2005). The present investigation is valid for surface gravity waves with arbitrary wavelength/depth ratio.

In the following we derive equations for the total momentum of the fluid, and no explicit distinction needs to be made between ‘wave momentum’ and ‘mean flow momentum’. By using a regular perturbation of the Lagrangian equations, the first order solution will naturally correspond to the waves, while the second order correction to this solution may have a nonzero mean value, and then correspond to the wave drift. Changes in the second order momentum can in general be traced back to either of three distinct physical mechanisms; (i) frictional decay, (ii) atmospheric forcing, and (iii) barotropic pressure forces. All three mechanisms can impact on the mean drift both directly and through the waves.

The main objective of this paper is to investigate whether the radiation stresses are dependent of depth or not for mean horizontal wave-induced particle drift that varies in the vertical. The paper is organized as follows: First we state the governing equations in Lagrangian form. Then we discuss the primary wave field before we derive the equations for the wave-induced drift. After that we discuss the solutions for a quasi-steady state, and finally we present a short summary and some concluding remarks.

MATHEMATICAL FORMULATION

We consider plane waves that propagate along the \( x \)-axis in a Cartesian coordinate system, where the \( x \)- and \( y \)-axis are situated at the undisturbed sea surface. The \( z \)-axis is vertical, and directed upwards. The corresponding unit vectors are \((\hat{i}, \hat{j}, \hat{k})\), respectively. Our system rotates with constant angular velocity \( \omega / 2 \) about the \( z \)-axis, where \( \omega \) is the constant Coriolis parameter. The undisturbed ocean depth is constant, and given by \( z = -H \).

Our study is based on a Lagrangian description of motion. In this formulation a fluid particle is associated with its Lagrangian coordinates \((a, b, c)\). The initial position of the particle is \((X_0, Y_0, Z_0)\). The particle position \((X, Y, Z)\) at later times and the pressure \( P \) will then be functions of \(a, b, c\) and time \( t \). Velocity components and accelerations are given by \((X_0, Y_0, Z_0)\) and \((X_0, Y_0, Z_0)\), respectively, where subscripts denote partial differentiation. For plane waves along the \( x \)-axis we then may write

\[
X = a + x(a, c, t), \quad Y = b + y(a, c, t), \quad Z = c + z(a, c, t),
\]

\[
P = P_s - \rho g c + pt(a, c, t),
\]

where the deviations \((x, y, z, p)\) do not depend on \(b\).

Furthermore, \( \rho \) is the constant density, \( P_s \) is a constant pressure, and \( g \) is the acceleration due to gravity. By including the effect of the earth’s rotation, the equations for the conservation of momentum and mass can be obtained from Lamb (1932).

In order to study the radiation-stress problem as simple as possible, we replace our viscous force with a friction force that is directly proportional to the fluid velocity, i.e. a Rayleigh friction. This simplification preserves the dissipation of wave momentum. In this paper we consider waves with an amplitude of \( O(\varepsilon) \), where \( \varepsilon \) is a small parameter proportional to the wave steepness. It is defined later in the paper by (13). It is easy to show for periodic waves that \( J(X, Z) = 1 + O(\varepsilon^2) \), where \( J(A, B) = A B - A B \) is the Jacobian (Høydalsvik and Weber, 2003). The momentum equations, correct to \( O(\varepsilon^2) \), then become

\[
x_x + f_y - f_x = -\frac{1}{\rho} (p + \rho g z), \quad -\frac{1}{\rho} J(p, z),
\]

\[
y_y + f_x + f_y = 0,
\]

\[
z_z - f_x = -\frac{1}{\rho} (p + \rho g z), \quad -\frac{1}{\rho} J(x, p) - g J(x, z).
\]

Finally, the conservation of mass (here volume), which can be expressed as \( J(X, Z) = J(X_0, Z_0) \), leads to

\[
x_x + z_z + J(x, z) = 0.
\]

In the Lagrangian formulation the free material surface is given by \( c = 0 \), and the bottom by \( c = -H \).

THE PRIMARY WAVE FIELD

We assume that our primary wave (marked by a tilde) has a real frequency \( \omega \) that is much larger than the inertial frequency \( f \). Then we can disregard the effect of the earth’s rotation on the primary wave field, which means that we have no velocity along the \( y \)-axis to this order. In general, for a viscous fluid, we may write the linear horizontal and vertical wave velocity as a sum of a potential part \( \varphi \) and a vorticity part \( \psi \) (Lamb, 1932). In the case of Rayleigh friction, the vorticity part vanishes identically, and the wave solutions can be expressed as

\[
\tilde{x} = -\varphi_x, \quad \tilde{z} = -\varphi_z,
\]

\[
\tilde{p} / \rho = \varphi_x + r \varphi + g \varphi_z.
\]

Here

\[
\varphi = \varepsilon \cos(k(x + H)) \exp(\varepsilon \alpha + n t),
\]

where \( \varepsilon \) is the aforementioned small ordering parameter. In (7) we have defined

\[
k = k + i \alpha, \quad n = -\beta - i \omega,
\]

where \( k, \omega \) are the real and positive wave number and wave frequency, respectively, and \( \alpha, \beta \) are the small real growth/attenuation rates in space and time.

The waves studied here can be supported by a suitably arranged small normal wind stress \( \tilde{\tau}_w \) at the surface. We assume that

\[
\tilde{\tau}_w / \rho = -\varepsilon \delta \cos(kH) \exp(\varepsilon \alpha + n t),
\]
where $\delta$ is a small amplitude factor. The real and imaginary parts of the boundary condition $\tilde{r}_s = -\tilde{p}$ at $c = 0$ yields:

$$\omega = (gk \tanh(kH))^2,$$

and

$$\delta = -r - 2\beta - 2C_\alpha \alpha.$$

Here $C_\alpha = \frac{d\omega}{dk}$ is the group velocity. It is related to the phase speed $C = \omega / k$ by the formula

$$C_\alpha = \frac{C}{2}\left(1 + \frac{2kH}{\sinh(2kH)}\right).$$

(12)

In the calculations above we have utilized that $\alpha \ll k, \beta \ll \omega$, i.e. the wave amplitude varies slowly in space compared to the wavelength, and slowly in time compared to the wave period. Furthermore, we have assumed that $\cos(\alpha h) \approx 1$, and $\sin(\alpha h) \approx \alpha h$. From (11) we note that a suitably arranged normal wind stress along the undulating surface may sustain the wave against dissipation. This relation is analogous to the one obtained by Jenkins (1986) for deep-water waves with viscosity. Negative values of $\alpha, \beta$ means wave growth by our definition, and that is possible if the normal stress in phase with the wave slope has an amplitude $\delta > r$. Without any influence from the wind, and purely spatial decay ($\delta = \alpha = 0$), the relation (13) yields the spatial damping rate $a_c = r / (2C_\alpha)$. For purely temporal decay ($\delta = \alpha = 0$), we find $\beta_c = r / 2$. As first pointed out by Gaster (1962) for such problems in general, we must have that $\beta_0 = C_\alpha a_c$ for the two separate cases. If the amplitude of the non-modulated surface wave is $\zeta_0$, we find from (6) that our ordering parameter $\varepsilon$ is related to the wave steepness through

$$\varepsilon = \frac{\omega}{k} \sinh(kH)(k \zeta_0).$$

(13)

NONLINEAR RESULTS

The relation (11) leads to an interesting nonlinear result for the conservation of wave momentum. We define the form drag $r_\alpha$ at the surface as

$$r_\alpha = -\tilde{r}_s \tilde{z}_s, \quad c = 0,$$

(14)

where we insert real parts of the normal stress and the surface slope, and the over-bar denotes averaging over one wave cycle. Utilizing (13), we then find

$$r_\alpha = \frac{\delta}{\rho} \left(\frac{\zeta_0^2}{2\tanh(kH)}\right) \exp(-2(\alpha a + \beta t)).$$

(15)

The Stokes drift $u_s$ for this problem can be written

$$u_s = \frac{\zeta_0}{k} \frac{\cosh(2k(c + H))}{\sinh^2(kH)} \exp(-2(\alpha a + \beta t)).$$

(16)

The total wave momentum $U_s$ per unit density thus becomes

$$U_s = \int_{-\infty}^{\infty} u_s dc = \frac{\zeta_0}{k} \frac{\omega}{2\tanh(kH)} \exp(-2(\alpha a + \beta t)) = E/C,$$

(17)

where $E$ is the total wave energy per unit density. From (15) and (17) we find for the form drag that

$$r_\alpha = \frac{\delta}{\rho} U_s.$$

(18)

Multiplying (11) by $U_s$, we obtain

$$r_\alpha = \frac{\tau_\rho}{\rho} + \frac{\partial U_s}{\partial t} \frac{\partial (C_U U_s)}{\partial t}.$$

(19)

Here $r_\alpha$ is defined by

$$r_\alpha = \rho \bar{U_s},$$

(20)

which we refer to as the virtual wave stress. The relation (19) constitutes a conservation equation for the mean wave momentum in a single wave. An exact analogue for deep-water waves with viscosity was derived by Weber et al. (2006), their equation (45). The definition of the virtual wave stress (20) resembles Longuet-Higgins’ (1969) definition for deep-water waves. In a viscous fluid the role of $r_\alpha$ is to redistribute the mean wave momentum lost by dissipation into a mean Eulerian current. Here, in the Rayleigh-friction formulation, there are no diffusive mechanisms, and accordingly no such redistribution. In this case we only have a sink of mean momentum in the fluid.

The horizontal mean drift velocities are obtained from (2) and (3). Averaging over one wave cycle, we find that $\tilde{x}_s + r \tilde{x}_s - f \tilde{y}_s = -\tilde{p} / \rho + g \tilde{z}_s - \tilde{J}(\tilde{p} / \rho, \tilde{z}_s), \quad \tilde{y}_s + r \tilde{y}_s + f \tilde{x}_s = 0.$

(21)

The acceleration and the friction force can be neglected for the mean vertical motion to second order. Hence (4) reduces to $\tilde{J}(\tilde{p} / \rho + g \tilde{z}_s) = -\tilde{J}(\tilde{p} / \rho, \tilde{z}_s) - g \tilde{x}(\tilde{x}, \tilde{z}_s)$. (22)

The validity of this approximation has been checked against the results obtained by eliminating the mean pressure from (2) and (4) by operating the curl, and subsequently using (5).

The continuity equation (5) for the mean motion becomes $\tilde{x}_s + \tilde{z}_s = \tilde{J}(\tilde{x}, \tilde{z}_s)$. (23)

We insert the real parts of $\tilde{x}, \tilde{z}$ and $\tilde{p}$ of the primary wave field, and define for convenience $\tilde{J}_s = \tilde{J}(\tilde{p} / \rho, \tilde{x}_s), \quad \tilde{J}_s = \tilde{J}(\tilde{p} / \rho, \tilde{z}_s), \quad \tilde{J}_s = \tilde{J}(\tilde{x}, \tilde{z}_s).$ (24)

From (22) we then obtain $\tilde{p} / \rho + g \tilde{z}_s = \int (\tilde{J}_s - g \tilde{T}_s) dc + F(a, t),$ (25)

where $F$ does not depend on $c$. The dynamic boundary condition at the free surface requires that $\tilde{p}(c = 0) = 0$. Hence, from (25):

$$F(a, t) = \frac{1}{h} \int (\tilde{J}_s - g \tilde{T}_s) dc,$$

(26)

where $h = \tilde{z}(c = 0)$ is the mean second order change in surface level. By integrating (23) from the flat bottom to the free surface, and utilizing that $\tilde{z}(c = -H) = 0$, we find that

$$h = \int_{-\infty}^{\infty} \tilde{x}_s dc - \int_{-\infty}^{\infty} \tilde{J}_s dc.$$

(27)

The last term on the right-hand side represents the “divergence effect” (McIntyre, 1988), i.e. a change in the mean surface level due progressive surface waves. Finally, by combining (26) and (27), (25) yields for the mean horizontal dynamic pressure gradient
we obtain an
\begin{equation}
\rho \frac{\partial}{\partial t} \left( \frac{\vec{v}}{\rho + g\vec{z}} \right) = -g \left( \int_{\alpha}^{\beta} \vec{J} \, dc \right) \quad + g \left( \int_{\alpha}^{\beta} \vec{J} \, dc \right).
\end{equation}

Insertion of (28) into (21) yields
\begin{equation}
\vec{x}_6 = -g \left( \int_{\alpha}^{\beta} \vec{J} \, dc \right) + g \left( \int_{\alpha}^{\beta} \vec{J} \, dc \right),
\end{equation}

or alternatively:
\begin{equation}
\vec{x}_6 + r \vec{x}_4 = 0.
\end{equation}

The nonlinear terms on the right-hand side of (29) are easily calculated. By introducing the Stokes drift \( U_x \) from (16) and the Stokes flux \( U_x \) from (17), we find by neglecting higher powers than the first in the small quantities \( \alpha, \beta, r \):
\begin{equation}
\vec{x}_6 + r \vec{x}_4 = \frac{(r - 2\beta)U_x + 2\alpha}{H} \left( \frac{C}{C} - \frac{1}{2} \right) \left( C U_x \right),
\end{equation}

or alternatively:
\begin{equation}
\vec{x}_6 + r \vec{x}_4 = \theta \left( \frac{C}{C} - \frac{1}{2} \right) \left( C U_x \right),
\end{equation}

\begin{equation}
\vec{x}_6 + r \vec{x}_4 = 0.
\end{equation}

Since the total wave energy \( E \) per unit density is given by \( E = C U_x \), we realize that the third term on the right-hand side is equal to the radiation stress component \( S_3 \) per unit depth for waves along the 1-axis of Longuet-Higgins and Stewart (1960). The first two forcing terms, which are the only forcing terms that vary with depth, reflect the change in the total mean momentum caused by friction and temporal variation of the waves. We note from (30) or (31) that the term corresponding to \( S_3 \) does not vary in the vertical. This is in fact quite interesting, since the first and third terms on the right-hand side of (29) do contain depth-varying radiation stress-like terms. More specifically, the nonlinear pressure term \( J = J(\vec{P}/\rho, \vec{z}) \) yields a contribution
\begin{equation}
I_3 = \frac{\alpha}{\partial t} \left( \frac{1}{2} C U_x \right),
\end{equation}

where \( U_x \) is the vertically varying Stokes drift given by (16). From the mean pressure gradient \(-\vec{P}/\rho + g\vec{z}\), we obtain an exactly compensating term
\begin{equation}
I_3 = \frac{\alpha}{\partial t} \left( \frac{1}{2} C U_x \right).
\end{equation}

Accordingly, the only radiation stress-like term in (31) for the wave-induced drift arises from the depth-independent surface contribution (26) to the mean pressure.

The total mean horizontal displacements are defined as
\begin{equation}
\vec{q}_6 = \int_{\alpha}^{\beta} \vec{x} \, dc, \quad \vec{q}_6 = \int_{\alpha}^{\beta} \vec{x} \, dc.
\end{equation}

From (31) we then obtain
\begin{equation}
\vec{q}_6 + r \vec{q}_4 = \vec{q}_6 = \frac{r}{C} \left( C U_x \right),
\end{equation}

or alternatively:
\begin{equation}
\vec{q}_6 + r \vec{q}_4 = 0,
\end{equation}

where \( C_0 = gH \). Utilizing the boundary condition (19), (35) becomes:
\begin{equation}
\vec{q}_6 + r \vec{q}_4 = \vec{q}_6 = \frac{r}{C} \left( C U_x \right),
\end{equation}

Finally, we obtain from (27)
\begin{equation}
\vec{h} = \vec{q}_6 = \frac{U_x}{C}.
\end{equation}

We observe that the second term on the right-hand side of (36) is equal to the radiation stress component \( S_3 \) for waves along the 1-axis by Longuet-Higgins and Stewart (1960). This is exactly what we would expect from an Eulerian analysis, e.g. Phillips (1977). When comparing with Phillips’ result we should keep in mind that the nonlinear displacement of the mean Lagrangian water level (37) is different from the nonlinear mean Eulerian water level (Longuet-Higgins, 1986). Denoting the latter by \( \vec{h}_L \), we obtain by integrating the continuity equation \( u_x + w_x = 0 \) in the vertical from the bottom to the undulating surface, that
\begin{equation}
\vec{h}_L = \vec{h}_L = \frac{U_x}{C},
\end{equation}

correct to second order. Hence, by inserting into (37), and integrating in time, we find that
\begin{equation}
\vec{h} = \vec{h}_L = \frac{U_x}{C}.
\end{equation}

For deep water we have that \( U_x / C = C^{-k}/2 \), and we see that (39) is identical to Longuet-Higgins (1986), his equation (3.7). We shall apply (39) in the next section when we compare our results with those of Phillips (1977).

**DISCUSSION OF THE SOLUTION FOR THE MEAN DRIFT**

The solution of the homogeneous part of (36) yields the free wave modes that accompany the adjustment of the mean displacements towards a steady, or quasi-steady state. At high frequencies, the adjustment occurs via the propagation of amplitude-modulated shallow-water gravity waves along the x-axis. Taking \( f = 0 \), we obtain from (36) and (37) for a time-independent wave field
\[ \vec{h}_s + r \vec{h} - C_i \vec{h}_o = -\left( \frac{\tau_o}{\rho} \right) - \left[ \left( \frac{C_i}{C} - \frac{2C_e}{C} + \frac{1}{2} \frac{C_U}{C} \right) \right] \cdot. \] (40)

The homogeneous part of this solution corresponds to long-damped gravity waves which radiate out from the generation area in the early stage of wave formation. Finally, the surface slope adjusts to

\[ \vec{h}_s = \left( \frac{\tau_o}{\rho g H} \right) + \frac{1}{C^2} \left[ \left( \frac{C_i}{C} - \frac{2C_e}{C} + \frac{1}{2} \frac{C_U}{C} \right) \right] \cdot. \] (41)

To check our result, we take \( \tau_o = 0 \) as in Phillips’ analysis. Introducing the mean Eulerian elevation from (39), we obtain from (41):

\[ \vec{h}_s = -\frac{1}{C_i} \left( \frac{2C_e}{C} - 1 \right) \cdot \vec{U} \cdot, \]

which is equation (3.7.5) in Phillips’ book.

We do not consider the initial value problem here, but discuss the drift solution for a steady state. The amplitude of the wave field is taken not to vary in time, i.e. \( \beta = 0 \). Furthermore, we disregard any effects of the wind, which means that \( \delta = 0 \) in (11). Then, \( r = 2C_i \alpha \) for this problem. In a balanced state we have that \( \vec{h}_s \) is given by (41). Applying (37), we readily find from the steady drift equations (31) for the mean Lagrangian drift components in the \( x \)- and \( y \)-direction, respectively:

\[ \vec{x} = \frac{r^2}{r^2 + f^2} \left( u_s - \frac{1}{H} U_x \right), \]

\[ \vec{y} = -\frac{r f}{r^2 + f^2} \left( u_s - \frac{1}{H} U_x \right). \] (43)

We note that we have an along-wave drift current near the surface and a return current in the lower part of the water column. Due to the earth’s rotation, the drift velocity is deflected somewhat to the right of the waves (when \( f > 0 \)) in the upper part of the water column, and oppositely in the lower part. The integrated transport is zero. In the Rayleigh formulation the effect of friction is felt in the entire water column, and we have free slip at the boundaries. In a viscous ocean, with a no-slip bottom, a proper Ekman layer would develop (Xu and Bowen, 1994; Weber and Høydalsvik, 2000).

SUMMARY AND CONCLUDING REMARKS

In the present paper we consider drift due to waves with amplitudes that vary slowly in time and space utilizing a Lagrangian description of motion. The wind stress, through a prescribed variation normal to the free surface, may act to sustain the wave amplitude. The dissipative process in the fluid, promoting amplitude decay, is modelled by a linear friction (Rayleigh friction). In particular, we looked for forcing terms for the Lagrangian mean velocity that are depth-varying, and not directly proportional to the Stokes drift. This has been motivated by the results of Mellor (2003, 2005), where he applies a coordinate transformation for the vertical coordinate, and finds vertically-varying radiation stress terms in the Eulerian equations for the mean flow. We find no such terms in the equations for the Lagrangian mean flow. In our approach, the radiation stress-like term in the equation for the mean drift is depth-independent, while another radiation stress-like term appears in the dynamic boundary condition at the free surface. When integrating in the vertical, and applying our boundary condition, we reproduce the radiation stress forcing term of Longuet-Higgins and Stewart (1960) for the integrated flow. Since our Lagrangian coordinates are particle following, the forcing terms we derive for the horizontal mean drift should make sense from a Newtonian point of view. Our calculations demonstrate that these forcing terms, apart from those proportional to the Stokes drift, are barotropic. The focus of this paper has been on the role of the radiation stresses in a three-dimensional model for the wave-induced drift velocities. For that purpose we have simplified the model (Rayleigh friction, constant depth) so that the analysis should be easy to follow. However, the model is realistic enough to describe basic processes in a wave-driven ocean circulation model. By using prognostic wave models for the surface wave spectrum like WAM (Komen et al., 1994), or SWAN (Booij et al., 1999), the Stokes drift terms and the barotropic radiation stress term in (31) can be obtained in spectral form (Jenkins, 1989; Weber et al., 2006). The friction parameter \( r \) should in shallow water model the effect of bottom friction, while in deep water \( r \) would basically be related to wave breaking. However, varying bottom topography induces effects that cannot be resolved in the present framework. Work in progress on a quasi-Lagrangian quasi-Eulerian approach will hopefully bring us closer to a solution for the wave-induced circulation in the coastal zone.

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