## ECON 4910 Environmental Economics, Spring 2011

## Seminar 1

## Problem 1

Assume that a profit maximizing factory is using a fixed amount of labor in its production. The production function is given by
$y=F(L) \quad$ where $y$ is the production level, and $L$ is the fixed amount of labor input.

The wage per unit of labor is $w$, while the price per unit of output is $p$.

When the factory is run with the fixed level of labor input L , and no cleaning equipment is installed, it emits an amount $\mathrm{m}^{0}$ of a polluting substance into the air. If production is unprofitable, the factory will close down and emissions will be zero.

Assume that there exists cleaning equipment which can be bought at a cost and installed in the factory. Assume, moreover, that the better equipment one purchases, the more pollution is cleaned; however, better equipment is more costly. Let the cost of cleaning equipment (the cleaning cost, or abatement cost) be given by $c(A)$, where
$A=m^{0}-m \quad$ denotes abatement.
Assume that $c(A)$ is twice continously differentiable and that $c^{\prime}(A)>0$ for all $A>0, c^{\prime \prime}(A)>0$, and $c^{\prime}(0)=0$.
a) Write down the firm's profit function. If there is no environmental policy, what will the emissions from the factory be? Characterize the solution (specify what we know about it) and interpret it (explain what your characterization means in terms of economic intuition). Will the unit cost of labor $w$ matter?
b) If there is a tax $t$ per unit of emissions, where $t>0$, what will the emissions from the factory be? Characterize and interpret the solution. Will there necessarily be a strictly positive level of abatement? Will the unit cost of labor w matter?
c) If there is a subsidy $s$ per unit of abatement, where $s>0$, what will the emissions from the factory be? Characterize and interpret the solution. Does your answer differ from your answer to 1b)? If yes, how? Does the cost of labor matter for this?

## Problem 2

Consider an economy with one private good $x$ and a pure public good $E$, which we may think of as environmental quality. There are $n$ consumers. The preferences of consumer $i$ are as follows (where $u_{i}$ is quasiconcave and increasing in both arguments, and $x_{i}$ is $i$ 's private consumption of $x$ ):
$U_{i}=u_{i}\left(x_{i}, E\right)$

The private good $x$ is produced by $K$ producers with production functions
$y_{j}=f_{j}\left(m_{j}\right)$
where $y_{j}=$ firm j's production of $x, m_{j}=$ firm j's emissions of a uniformly mixing pollutant, $f_{j}^{\prime}>0$ and $f_{j}^{\prime \prime}<0$, and where $\sum_{i=1}{ }^{n} x_{i}=\sum_{j=1}{ }^{k} f_{j}\left(m_{j}\right)$ (total consumption of the private good equals total production).

The public good E is affected by pollution in the following way:
$\mathrm{E}=\mathrm{E}^{0}-\mathrm{z}(M) \quad \mathrm{z}^{\prime}>0$
where $E^{0}$ is an exogenously given level and $M=\sum_{j=1}{ }^{k} m_{j}$.
a) Define: What does it mean that a resource allocation is Pareto optimal?
b) Derive the first order conditions for Pareto optimality in this economy, and explain their economic interpretation. (Hints: Use the definition to specify the problem; optimize with respect to every individual's consumption and every firm's emissions; be careful about which sum is over which group of consumers or firms. To simplify, you may assume that $n=K=2$ ).

## Problem 3

Consider now the same economy as in Problem 2, except that now we will assume that the production function for every producer $j$ is given by
$y_{j}=f_{j}\left(m_{j}\right)=30 \ln \left(m_{j}\right)-m_{j}$.
Assume that the price of $x$ is 1 , and that the number of firms $(K)$ equals 100.
Moreover, let $E=E^{0}-z(M)=1000-M / 500$. Each consumer $i$ has the same income, $F$, which he/she considers exogenously given. Assume further that all consumers have identical preferences.
a) If consumers cannot bargain with producers about emission levels, and there is no environmental regulation, how large will total emissions $M$ be? What will the level of $E$ be?
b) Assume now that consumers can bargain with producers about emission levels. Let one consumer, call him 1, discovers this first, and let him assume that no-one else will act. Assume, for simplicity, that he is approached by a firm offering him to pay them in return for abatement, at a fixed abatement unit price $P>0$. Will he purchase (pay for) a strictly positive level of abatement? Explain, and characterize the solution. Can we know for sure that there exists a strictly positive price $P$ that will make him purchase a strictly positive level of abatement? (Hint: Start by finding an expression for M from consumer 1's point of view.)
c) Assume that next, the other n-1 consumers hear about the first consumers' contribution and now understand that bargaining is possible. How much will each of these other consumers offer to pay firms in order to reduce emissions, if each of them take others' contribution as exogenously given? Will the resulting situation be Pareto optimal? (A brief, intuitive answer is sufficient.)

