Simulating the Spatial Evolution of a Measured Time Series of a Freak Wave

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Abstract. The measured time history of the "New Year" freak wave that hit the "Draupner" platform is simulated forward and backward in space to find the time histories at neighboring locations. The impression that could have been seen by an observer is reconstructed. The freak wave likely did not suddenly appear from nowhere. Instead, the simulation suggests that a short and tall wave group approached the platform for at least a minute or so.

1 Introduction

In many cases it is more useful to perform space-domain simulation than time-domain simulation. Conventional methods for quantitative field observations usually yield time series of the surface displacement at one or a few selected locations. Spatial evolution is implied between the selected locations, or between the instrumented locations and other locations of interest. A time-domain simulation tool would likely not be very useful for application to such data, or at best it would be quite difficult to initialize since knowledge of the entire sea surface at initial time is not known. A better approach is to interchange the role of space and time in the evolution equations in order to obtain a space-domain simulator; initialization with the measured time history at a point then becomes trivial.

We have cast the modified nonlinear Schrödinger equation as a space domain simulator, and have "initialized" it with the measured time series of the freak wave that hit the Statoil operated "Draupner" platform, January 1, 1995 at 15:20. Figure 1 shows a 20 minute wave recording measured by a down-looking laser device (see Haver & Andersen 2000). The measured response of the platform was found to be unidirectional (Karunakaran et al. 1997), therefore it is reasonable to assume that the waves were long-crested. The sea floor is flat at 70 m depth.

The desired model should obey the empirical scaling laws that are observed in the field. We have earlier reported (Trulsen & Dysthe 1997) that characteristic values for steepness and bandwidth for this wave train is $k_c a_c \sim 0.12$, $\Delta\omega/\omega_c \sim 0.24$ and $\Delta k/k_c \sim 0.4$, where k_c, a_c, ω_c, $\Delta\omega$ and Δk are the characteristic wavenumber, amplitude, frequency, modulation frequency and modulation wavenumber, respectively. Therefore, we have previously argued that the modulation should be scaled as the square root of the steepness; we thus derived

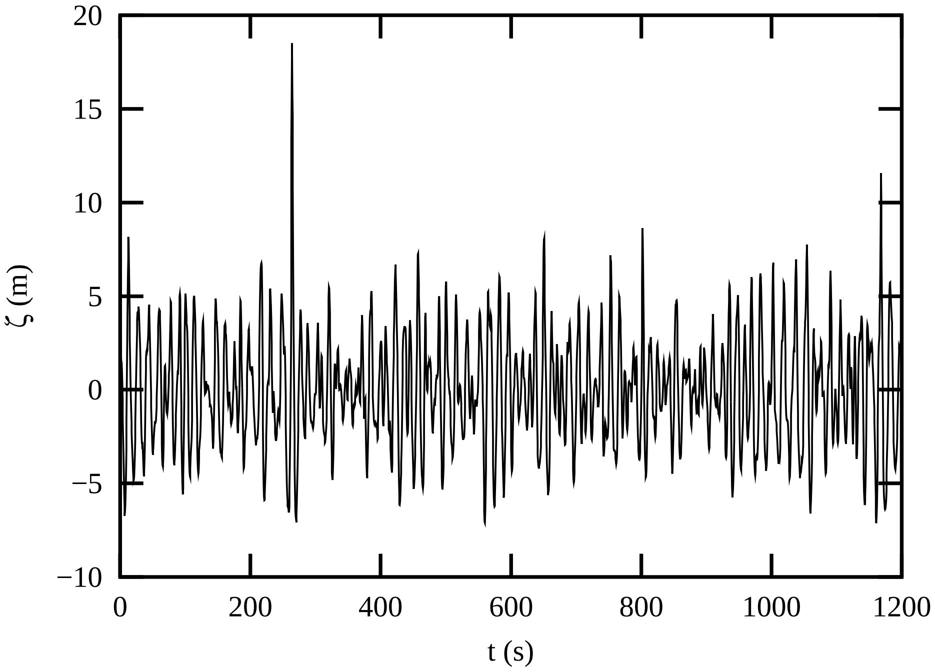

Fig. 1. Freak wave measured at the Statoil Draupner platform, January 1, 1995 at 15:20.

a modified nonlinear Schrödinger equation for this purpose (Trulsen & Dysthe 1996). Recently, we have further taken the consequence of the importance of linear dispersion by enhancing the modified nonlinear Schrödinger equation with exact linear dispersion (Trulsen et al. 2000). Our approach is based on the assumption that the spectrum to leading order of approximation is narrow-banded. The remaining part of the spectrum is reconstructed only to the extent that it is nonlinearly forced by, and thus coherent with, the linear waves near the spectral peak. Special care must be taken for proper initialization to distinguish between linear free waves and nonlinearly forced waves. To this end we have developed an iterative technique by which the extracted spectrum of linear free waves is refined until exact reconstruction of the most energetic part of the measured complex spectrum has been achieved.

Early attempts to simulate the ocean surface in two horizontal dimensions with the nonlinear Schrödinger equation were only partially successful due to energy leakage that broadened an initially narrow spectrum such that the model eventually violated its own bandwidth constraint (Martin & Yuen 1980). The higher-order modified nonlinear Schrödinger equation reduced the leakage such that 2D simulations became feasible. As of the new equation with exact linear dispersion, the leakage is completely eliminated (Trulsen et al. 2000). Numerical integration can be done as efficiently as for the conventional nonlinear

Schrödinger equation through operator splitting methods. In this paper we limit consideration to simulation in one horizontal dimension.

The natural spatial scale of nonlinear modulation is $\eta = \epsilon^2 k_c x$, where $\epsilon = k_c a_c$ is the wave steepness, k_c and a_c are characteristic scales for wavenumber and amplitude, and x is the fetch. Results presented elsewhere (Trulsen & Stansberg 2001) comparing simulations with laboratory experiments suggest that the modified nonlinear Schrödinger equation predicts the evolution of individual wave crests quite well at least up to $\eta = 3$, while it becomes poor for $\eta > 5$. In the present paper we present simulations over 500 m, which corresponds to $\eta = 0.26$.

Lo & Mei (1985) first presented comparisons between experiments and the space evolution predicted by the modified nonlinear Schrödinger equation, and obtained good results. Similar work with the cubic nonlinear Schrödinger equation was done by Shemer et al. (1998) for deep water and with the Korteweg de-Vries equation for shallow water by Kit et al. (2000). The Zakharov equation, which in its original form is a time-domain equation, has been discretized for application to measurements (Rasmussen & Stiassnie 1999); it can likely be cast as a space domain simulator and be used for the same purpose as in the present paper.

2 Mathematical Model for Space-Domain Simulation

Starting from the inviscid equations for potential flow, normalized with the characteristic wavenumber k_c and frequency ω_c, we make an assumption that the velocity potential ϕ and surface displacement ζ of the wave field can be expanded in harmonic expansions

$$\phi = \bar{\phi} + \frac{1}{2}\left(A\mathrm{e}^{\mathrm{i}(x-t)+z} + A_2\mathrm{e}^{2\mathrm{i}(x-t)+2z} + A_3\mathrm{e}^{3\mathrm{i}(x-t)+3z} + \cdots + \mathrm{c.c.}\right), \tag{1}$$

$$\zeta = \bar{\zeta} + \frac{1}{2}\left(B\mathrm{e}^{\mathrm{i}(x-t)} + B_2\mathrm{e}^{2\mathrm{i}(x-t)} + B_3\mathrm{e}^{3\mathrm{i}(x-t)} + \cdots + \mathrm{c.c.}\right). \tag{2}$$

Here x and y are horizontal coordinates, z is the vertical coordinate, t is time, $\bar{\phi}$ and $\bar{\zeta}$ are the mean fields (zeroth harmonic), A and B are the linear first harmonics, and A_n and B_n are the higher order nonlinear harmonics. We limit consideration to a constant depth h and assume that the linear dispersion relation can be approximated by the deep water assumption, while the induced flow will be affected by the depth.

Trulsen et al. (2000) enhanced the modified nonlinear Schrödinger equation with exact linear dispersion by introducing a pseudo-differential operator for the linear part. In two horizontal dimensions it reads

$$\frac{\partial B}{\partial t} + L(\partial_x, \partial_y)B + \frac{\mathrm{i}}{2}|B|^2 B + \frac{3}{2}|B|^2\frac{\partial B}{\partial x} + \frac{1}{4}B^2\frac{\partial B^*}{\partial x} + \mathrm{i}\frac{\partial \bar{\phi}}{\partial x}B = 0 \quad \text{at} \quad z = 0, \tag{3}$$

$$\frac{\partial \bar{\phi}}{\partial z} = \frac{1}{2}\frac{\partial}{\partial x}|B|^2 \quad \text{at} \quad z = 0, \tag{4}$$

$$\nabla^2 \bar{\phi} = 0 \quad \text{for} \quad -h < z < 0, \tag{5}$$

$$\frac{\partial \bar{\phi}}{\partial z} = 0 \quad \text{at} \quad z = -h. \tag{6}$$

The pseudo-differential operator L is

$$L(\partial_x, \partial_y) = \mathrm{i} \left\{ \left[(1 - \mathrm{i}\partial_x)^2 - \partial_y^2 \right]^{1/4} - 1 \right\}. \tag{7}$$

These equations can be inverted with respect to space and time to yield a space-domain formulation

$$\frac{\partial B}{\partial x} + \mathcal{L}(\partial_t, \partial_y) B + \mathrm{i}|B|^2 B - 8|B|^2 \frac{\partial B}{\partial t} - 2B^2 \frac{\partial B^*}{\partial t} - 4\mathrm{i} \frac{\partial \bar{\phi}}{\partial t} B = 0 \quad \text{at} \quad z = 0, \tag{8}$$

$$\frac{\partial \bar{\phi}}{\partial z} = -\frac{\partial}{\partial t} |B|^2 \quad \text{at} \quad z = 0, \tag{9}$$

$$4 \frac{\partial^2 \bar{\phi}}{\partial t^2} + \frac{\partial^2 \bar{\phi}}{\partial y^2} + \frac{\partial^2 \bar{\phi}}{\partial z^2} = 0 \quad \text{for} \quad -h < z < 0, \tag{10}$$

$$\frac{\partial \bar{\phi}}{\partial z} = 0 \quad \text{at} \quad z = -h. \tag{11}$$

Here we have used the fact that $\partial \bar{\phi}/\partial x = -2 \partial \bar{\phi}/\partial t$ to the leading order. The pseudo-differential operator becomes

$$\mathcal{L}(\partial_t, \partial_y) = -\mathrm{i} \left\{ \left[(1 + \mathrm{i}\partial_t)^4 + \partial_y^2 \right]^{1/2} - 1 \right\}. \tag{12}$$

By expanding the linear pseudo-differential operators L or $\mathcal{L}$ in power series expansions and truncating at appropriate orders, we recover the previous modified nonlinear Schrödinger equation of Dysthe (1979) and the broader bandwidth equation of Trulsen & Dysthe (1996). Furthermore by truncating the nonlinear part to retain only the leading cubic nonlinear term, we recover the standard cubic nonlinear Schrödinger equation.

The reconstruction of the surface displacement (2) is achieved by the formulas

$$\bar{\zeta} = -\frac{\partial \bar{\phi}}{\partial t}, \tag{13}$$

$$B_2 = \frac{1}{2} B^2 + \mathrm{i} B \frac{\partial B}{\partial t}, \tag{14}$$

and

$$B_3 = \frac{3}{8} B^3. \tag{15}$$

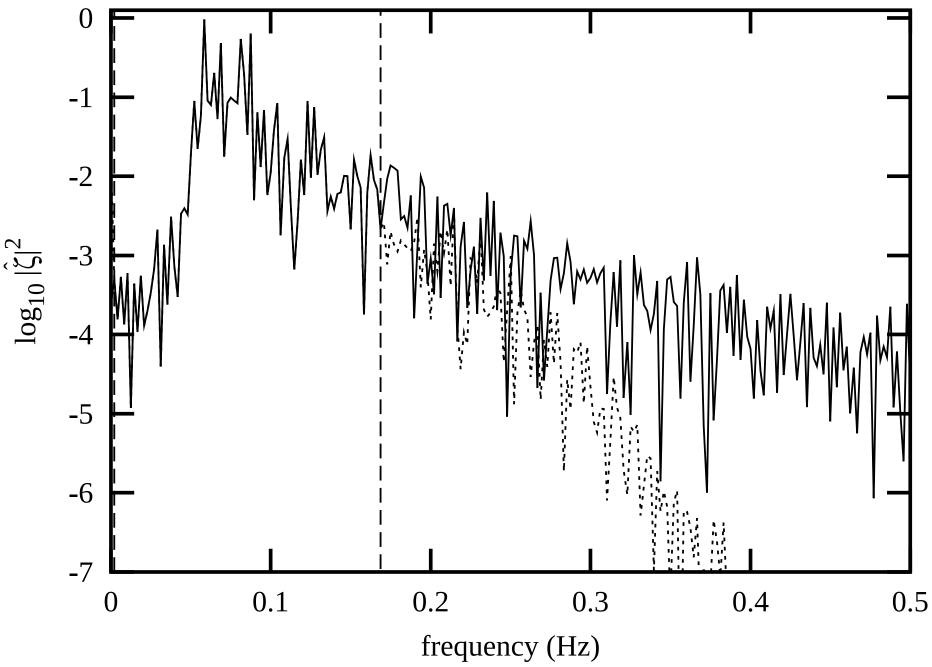

Fig. 2. Power spectrum: —, measured; $\cdots$, reconstructed. The vertical dashed lines indicate the bandpass filter used to define the domain of the first harmonic.

3 Initialization

The complex spectrum of the first harmonic B is first tentatively assigned by bandpassing the most energetic part of the desired complex spectrum of ζ. Then the first harmonic B is adjusted in an iterative manner until the desired spectrum of ζ is exactly reconstructed within the bandpass part of the frequency domain.

Figure 2 shows the measured and the reconstructed power spectrum of the surface displacement ζ. The full complex spectrum is reconstructed exactly within the bandpass region bounded by the vertical dashed lines (the figure only shows the reconstruction of the power spectrum). The reconstruction of the high frequency tail includes contributions that are coherent with the energetic part near the peak. The mismatch in reconstruction in the tail is to a large extent due to uncorrelated noise and incoherent waves, which are assumed to be unimportant for the evolution of the main features of the wave train.

The reconstruction of the freak wave is compared with the desired surface displacement in figure 3. The consequence of the mismatch in the spectrum seen in the previous figure is that the most rapid wave disturbances are smoothed out.

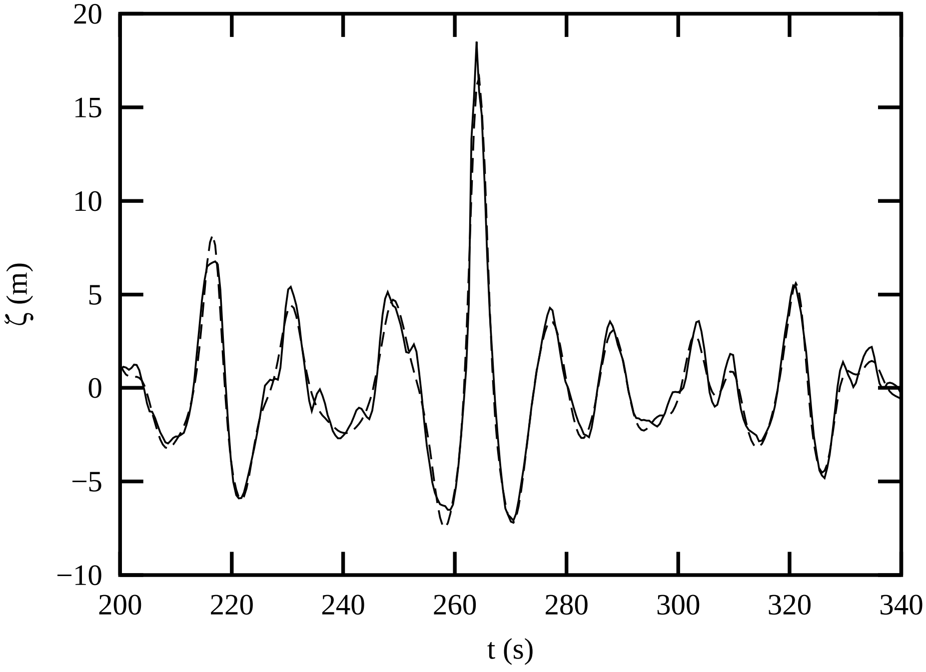

Fig. 3. Measured (—) and reconstructed (- - -) surface displacement.

4 Forward and Backward Propagation in Space

The predicted time histories at 50 meter intervals upstream and downstream are shown in figures 4 and 5, respectively. At 500 m upstream of "Draupner" there appears to be a large wave group passing by about one minute before the freak wave hits the platform. This wave group appears to split up into a short and large leading group that runs away from a longer and smaller trailing group that becomes rather diffuse as it approaches the platform. The freak wave that hits the platform is in the middle of the short leading group. After the impact with the platform, this wave group broadens, becomes less steep, and slows down slightly. A large trough (a "hole" in the ocean) can be observed slightly upstream of the platform.

Since the platform is of jacket type, it is not expected to have modified the waves.

5 Conclusion

We have presented a model for weakly nonlinear spatial evolution of waves, and have shown how a measured time history of a freak wave can be used for initialization and be propagated forward and backward in space.

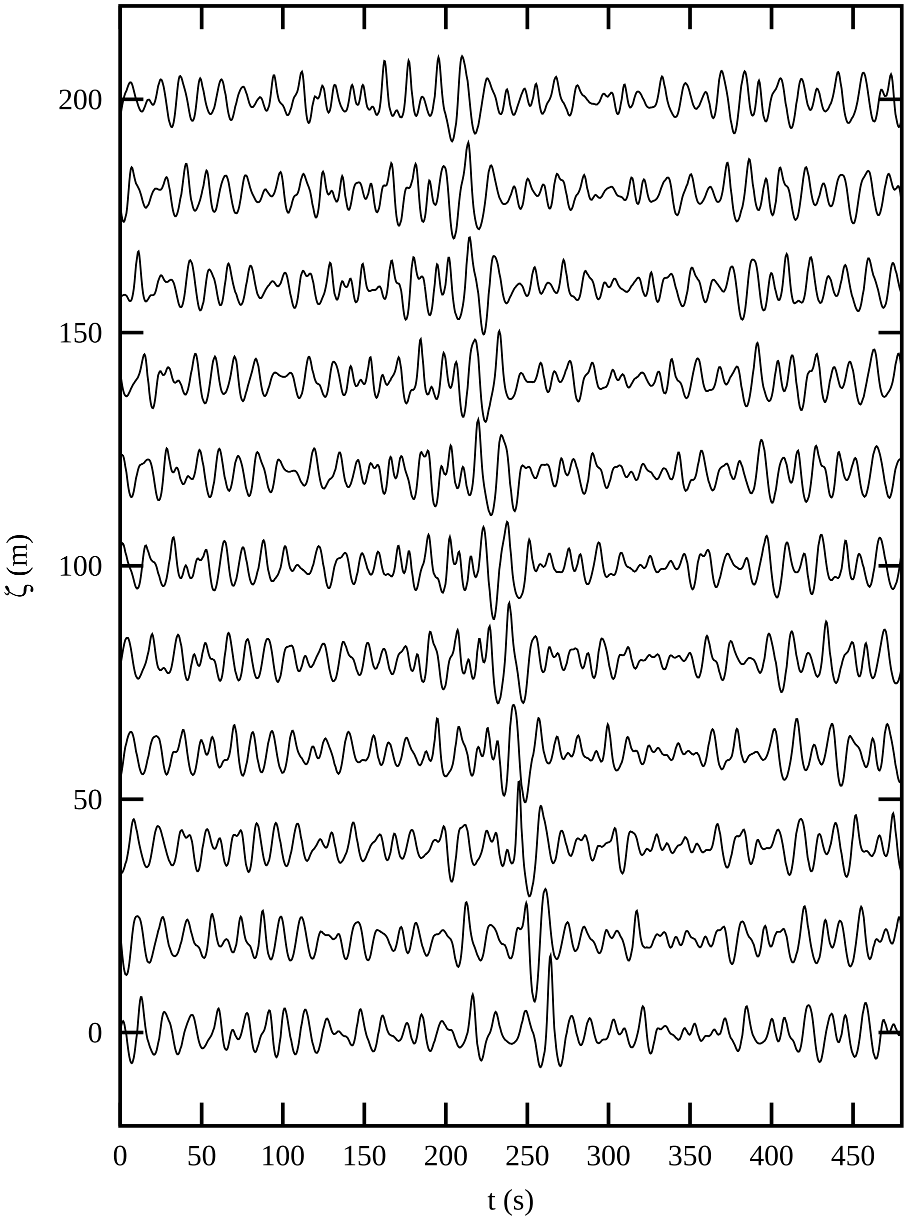

Fig. 4. Time histories at 50 meters intervals upstream (bottom to top).

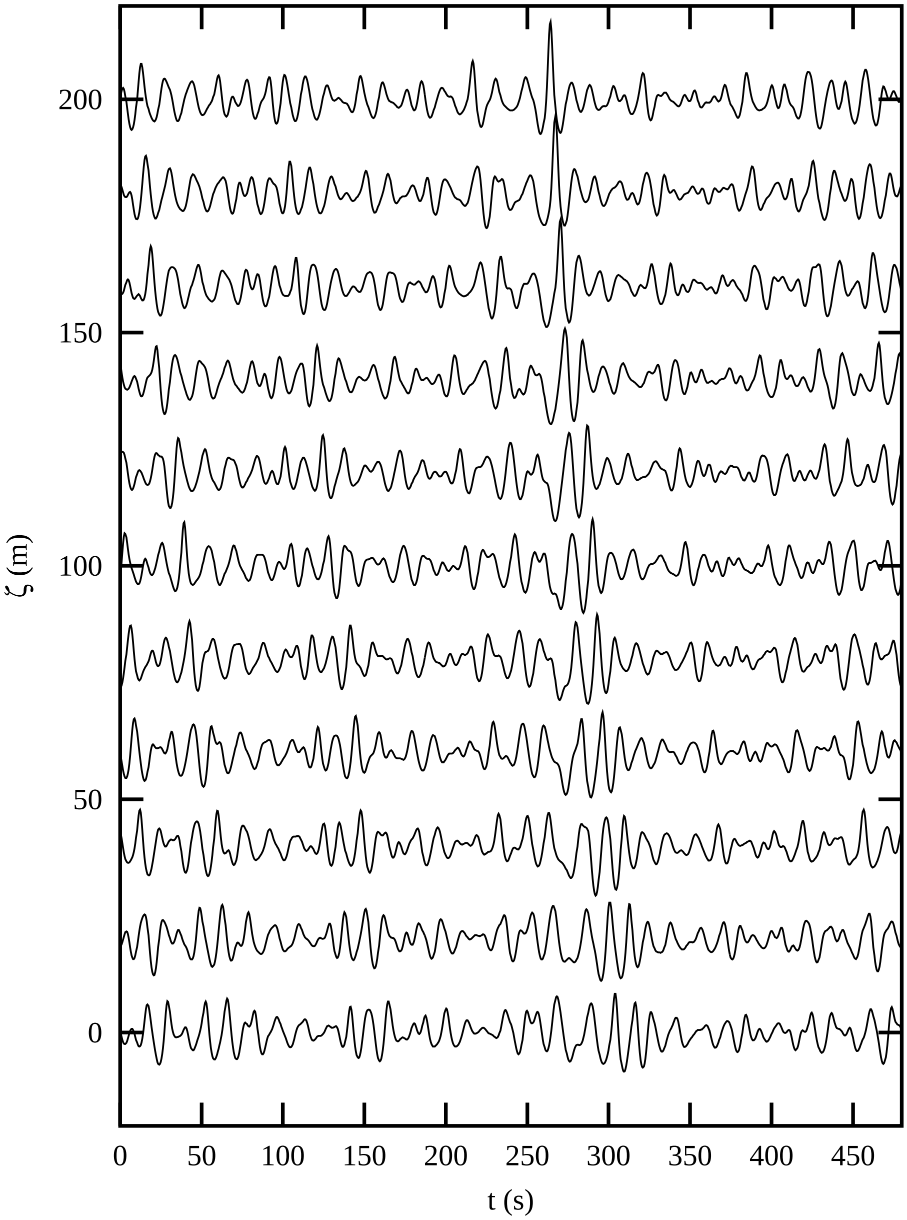

Fig. 5. Time histories at 50 meters intervals downstream (top to bottom).

As far as the "New Year" freak wave that hit "Draupner" is concerned, the most important conclusion is probably that it did not occur suddenly and unexpectedly. There is rather reason to believe that large waves approached the platform for at least a minute or so, and would likely have been an impressive view for a daring observer.

Acknowledgments
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