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Abstract Probabilistic household forecasts to 2041 are presented for Denmark, Finland, and the Netherlands. Future trends in fertility, mortality and international migration are taken from official population forecasts. Time series of shares of the population in six different household positions are modelled as random walks with drift. Brass' relational model preserves the age patterns of the household shares. Probabilistic forecasts for households are computed by combining predictive distributions for the household shares with predictive distributions of the populations, specific for age and sex. If current trends in the three countries continue, we will witness a development towards more and smaller households, often driven by increasing numbers of persons who live alone. We can be quite certain that by 2041, there will be between two and four times as many persons aged 80 and over who live alone when compared with the situation in 2011.

Keywords Probabilistic household forecast · Brass relational method · Random walk with drift · Random share method · One-person households · Cohabiting couples · Married couples · Lone parents · Institutions

Introduction

Household forecasts are useful for the planning of welfare provisions, housing supply, and the demand for consumer durables. For instance, elderly persons who live alone are more vulnerable than those who live with a partner. Therefore, household status is an important determinant of the need for formal and informal support and care for the elderly, in addition to health (e.g. Lakdawalla and Philipson 1999; Muller et al. 1999; Lakdawalla et al. 2003; Grundy and Jital 2007).

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Projections of social welfare spending depend, among other things, on the number of lone parents in the future (Jacobsen and Jensen 2014). Falling fertility rates and more frequent divorce and separation in western countries after World War 2 have led to smaller households (OECD 2011); in turn, this has had a strong impact on housing needs (Van Vliet et al. 1985). Another consequence of falling average household size has been an increasing demand for energy, because of economies of scale in households of different sizes (O'Neill and Chen 2002).

Deterministic household forecasts have been computed since the 1930s. Nowadays, probabilistic household forecasts are increasingly being accepted as a useful means of quantifying uncertainty; see Bijak et al. (2015) and the review in "Earlier work" section of this paper. A recent example of a probabilistic household forecast is given by Christiansen and Keilman (2013). They used an approach based on random household shares to compute such forecasts for Denmark and Finland. The household shares represent the proportions of persons who live in a certain household position (living alone, living with marital partner, living in an institution etc.) at a certain point in time, specific for age and sex. Because the future is uncertain, future values of these shares were treated as random variables. The authors obtained expected values of the multivariate predictive distributions for the shares by means of the multi-state household projection model LIPRO, while time series models of the household shares were used to assess the uncertainty in predicted shares, i.e. the variances and covariances of the distributions.

The purpose of this paper is to simplify the approach used by Christiansen and Keilman, abbreviated as C&K henceforth. Their method requires transition probabilities between household positions as input data. Transition probabilities of this kind are available from the population registers of Denmark and Finland, but few other countries have similar data for their populations as a whole. Panel surveys may be used to compute the required transition probabilities, but one needs a large sample because the probabilities have to be specified for men and women and disaggregated by age. We avoid transition probabilities by formulating multivariate time series models for the shares in each household position. We find that a Random Walk with Drift (RWD) model is a good representation of the data. The typical age patterns of household positions are modelled by means of the Brass Relational method (Brass 1971; Preston et al. 2001). Predictive distributions of the household shares for future years are obtained from the RWD models. In order to compute a probabilistic forecast for the numbers of persons in certain household positions, simulated predictive distributions for the household shares are combined with simulated predictive distributions of the populations, specific for age and sex. Correlations across ages and between men and women are accounted for. Finally, we use simple accounting rules for computing future numbers of *households* of different types, and the uncertainty therein, based on simulated predictive distributions of the number of *persons* by household position.

We apply our model to annual data for Denmark, Finland and the Netherlands. The latter country was added because with only 15 years of data, the series of observations on household shares is relatively short. Hence it is useful to investigate whether the proposed method still gives acceptable results in this situation. The probabilistic household forecasts span the period 2011–2041. The starting point is the population of each country disaggregated by age, sex, and household position in 2011, based on census data from Eurostat. Future trends in fertility, mortality and international migration are taken from official population forecasts.

The contribution this paper makes to existing literature is that it shows how techniques of data dimension reduction, in particular the Brass Relational method to capture the pronounced age patterns of certain household positions, can be used to model and predict patterns of household dynamics. The approach used in this paper is considerably simpler than that of C&K.

Earlier work

C&K reviewed various methods for household projection and forecasting. Here we give a brief summary.

Deterministic household forecasts have a long tradition (e.g. US National Resources Planning Committee 1938; United Nations 1973). *Probabilistic* household forecasts were first introduced around the turn of the century by De Beer and Alders; see Alders (1999, 2001) and De Beer and Alders (1999). Alders and De Beer used stochastic simulation and combined a stochastic population forecast with forecasts of random shares. The shares distribute the population probabilistically over six household positions: individuals could live as a child with parents, live alone, live with a partner, live as a lone parent or in an institution, or belong to another category. For instance, the authors computed the (random) number of lone mothers aged 40 in 2020 as the product of two other random variables, namely the number of women aged 40 in 2020 and the share of 40-year old women who live as a lone mother in 2020. Expected values for population variables and for the shares for specific household positions were obtained from observed time series, but the statistical distributions that were assumed for the shares were based on intuitive reasoning. Perfect correlations across age and sex were assumed for the mortality rates, fertility rates and migration numbers in the stochastic population forecasts, as well as for the random shares. In other words, if a simulated death rate for a certain age was higher than expected, so were the rates for all other ages, and similarly for birth rates (by mother's age) and immigration numbers. In addition the authors assumed perfect auto-correlation for the random shares: when the simulated shares for one particular year were high, they would also be high in the following years.

Scherbov and Ediev (2007) combined a probabilistic population forecast for the population disaggregated by age and sex with random headship rates, and applied their method to the case of Russia. A headship rate reflects the proportion of the population that is the head of a private household, for a given combination of age and sex (United Nations 1973; Jiang and O'Neill 2004). Scherbov and Ediev based a large part of their uncertainty distributions on intuition. Wilson (2013a, b) computed a probabilistic household forecast for Greater Sydney. Household parameters were modelled as a random walk. Standard deviations of the random errors were set based on judgement due to the lack of past errors in estimates of living arrangements and households.

A problem connected to these probabilistic household forecasts is that uncertainty parameters were largely judgemental. Alho and Keilman (2010) improved on this situation by estimating uncertainty parameters from data. Building on the random share method of De Beer and Alders, they applied their approach to Norwegian data. One important drawback of that work was that the uncertainty assessments were based on limited data, and that simplifying assumptions had to be made. As mentioned in the previous section, C&K used long time series data of observed shares for Denmark and Finland, and formal time series methods to quantify the uncertainty connected to household shares in the future. Expected values of the shares were computed using a multi-state model of household dynamics.

Data

The starting point of our forecasts is census data on the population of the three countries, disaggregated by 5-year age group, sex, and seven household positions in 2011. "Household position" is a characteristic of an individual person. It describes the type of position an individual has in his or her household; for instance, dependent child or spouse. Different persons who belong to a certain household can have different household positions. For instance, a household consisting of a cohabiting couple and their child contains two persons with household position "cohabitee" and one with the position "child". Thus, household position should be distinguished from household type, which is a not a characteristic of an individual person, but of a group of persons. The only exception is a household of type "one-person household", which is the same as a person with household position "living in a one-person household".

The census data used in the current paper contain the following household positions (household position code in parentheses):

1. Child living with parent(s) (CHLD)
2. Living in one-person household (SIN0)
3. Living in unmarried cohabitation, with or without children (COH)
4. Living with marital spouse, with or without children (MAR)
5. Living as lone parent (SIN+)
6. Living in a private household, but not in any of the positions described above (OTHR)
7. Living in an institution (INST).

These seven categories follow directly from the classification employed by Eurostat in household statistics based on data from the 2010 census round (Eurostat 2014). Also see Chapter XI of the Census recommendations regarding household data formulated by the United Nations Economic Commission for Europe (UNECE 2006). The categories refer to living arrangement and not marital status. For example, the category MAR does not include all those who are married, but only those who are currently living with a spouse. Persons who live with a partner (household positions 3 and 4) or as a lone parent may have one or more unrelated adults in the same household, provided that these other adults do not form a family of their own. Persons who live in a multiple family household are all coded as OTHR (household position 6). Household members without any parent–child relationship or a relationship between (married or cohabiting) partners also belong to this category.

No age restrictions have been imposed on persons who have a certain household position. In particular, children (CHLD) and lone parents (SIN+) can be of any age. In practice, observed or predicted numbers of persons aged 85, say, with positions CHLD or SIN+, will not be interpreted as such, but should be assigned to a different position, for instance to the group of other. Moreover, we have ignored persons aged younger than 15 in the following positions: SIN0, COH, MAR, and SIN+.

By defining the seven household positions given here we follow the approach of C&K. Clearly, alternative classifications are possible, for instance by household size. This will be relevant in applications connected to the demand for housing. We assume that the current classification has a somewhat broader application. Also note that there is considerable overlap between groupings in terms of household size and in terms of the seven categories above, since around 40 % of the private households (and close to one-fifth of the population) in the three countries consist of one-person households (Eurostat 2014).

Time series of annual data on the number of people in these seven household positions were obtained from the population registers of Denmark, Finland and the Netherlands. The data relate to 1 January of the years 1981–2007 for Denmark, of the years 1988–2009 for Finland, and of the years 1996–2010 for the Netherlands. The data comprise the seven household positions listed above, men and women, and ages 0–4, 5–9, …, 85–89, and 90+.

Modelling household shares

Household positions

The purpose of the modelling exercise is to obtain predictive distributions for the household shares in each country, disaggregated by sex and 5-year age group. We write $V(j,x,s,t,c)$ for the number of people in household position $j = 1, 2,\ldots,7$ who are at age $x = 0, 1,\ldots$ and are of sex $s = 1$ or 2, at time $t = 0,1, 2,\ldots$ in country $c = 1, 2, 3$. The sum $\Sigma_j V(j,x,s,t,c)$ gives the population $W(x,s,t,c)$ of age x and sex s at time t for country c, irrespective of household position. Household position j has share $\alpha(j,x,s,t,c) = V(j,x,s,t,c)/W(x,s,t,c) = \alpha_j(x,s,t,c)$. The household positions are numbered as follows: CHLD ($j = 1$), SIN0 ($j = 2$), COH ($j = 3$), MAR ($j = 4$), SIN+ ($j = 5$), OTHR ($j = 6$), INST ($j = 7$).

A share for a particular household position as defined here is different from other variables frequently used in demographic research about household members, such as headship rate and household membership rate. A headship rate starts from the notion of head of household; cf. the definition in "Earlier work" section. The shares in the current analysis circumvent this notion. A household membership rate is the proportion of persons (of a given age and sex) who are members of a specified type of household, as a proportion of all persons (of that age-sex combination; Linke 1988, 116; Haupt et al. 2003). The household shares in the current analysis focus solely on persons, irrespective of the type of household in which they live.

For modelling random evolution of the shares, a logit transformation was applied. Building on earlier work (Alho and Keilman 2010; Christiansen and Keilman 2013; Wilson 2013a, b), we have opted for a hierarchy of household positions using a variant of continuing fractions. This led to six types of fractions to be modelled (all specific for age, sex, time, and country). The following fractions were defined, given age, sex, time, and country:

1. The share of CHLD;
2. The relative share of COH and MAR out of the total share of one minus the share of CHLD;
3. The relative share of MAR out of the share of COH and MAR;
4. The relative share of SIN0 and INST out of the total share of SIN0, SIN+, OTHR, and INST;
5. The relative share of SIN0 out of the share of SIN0 and INST;
6. The relative share of SIN+ out of the total share of SIN+and OTHR.

Because of the hierarchy, predicted shares in the logit scale at a higher level are independent of predicted shares at a lower level. The particular sequence 2–6 above is based upon the idea that important shares (numerically and behaviourally) ought to be modelled first, and the less important shares last. Hence persons who live together with a partner (points 2 and 3 above), or alone (points 4 and 5) are given priority. The positions of SIN0 and INST are often difficult to distinguish for elderly persons, due to unclear registration rules for persons who live de facto in an institution (C&K). Therefore initially they are treated as one group (point 4).

Children have been singled out from the beginning, because their shares are assumed to be constant over time. The age pattern for this household position shows very little variation: for ages under 15, the shares are almost 100 % (some children live in multi-family households and hence have household position OTHR, while a few live in institutions). For ages 15–19 and 20–24 the shares fall rapidly, and they are close to zero for ages beyond 25. Hence any systematic changes over time in the age patterns are difficult to identify. There is evidence that the home-leaving behaviour of young adults changes over time. For instance, Stoeldraijer (2014) found a consistent upward trend in home-leaving age in the Netherlands since the start of the economic crisis in 2008. Important explanations for changes in the home-leaving age are the economic situation of young adults, the country's economic situation in general, and the housing market (Matsudaira 2016). These conditions fluctuate in the long run. Indeed, when analysed for a longer period, i.e. 1995–2011, the mean age at leaving the parental home for Dutch young adults shows no systematic trend; cf. Figure 3.2.1 in Stoeldraijer (2014). Also note that the home-leaving age concerns a flow variable, whereas our assumption of a constant share for CHLD concerns a stock variable, namely the children/young adults who live with their parents, as a share of all children/young adults of the relevant age. By its nature, the mean age at leaving the parental home will be more volatile than the mean age of those who live with their parent(s). The latter mean age shows only minor fluctuations in the Dutch data for the years 1996–2010.

Finally, we have selected the household position OTHR as a remainder, which is in agreement with the nature of this position as we have defined it.

Note that the hierarchy of household positions specified above is different from the one used by C&K. Unlike C&K we start by singling out the share of children for reasons stated above. Next, we model the combined share of married and cohabiting persons, because household positions for persons who live as a couple are numerically important at many ages (roughly for ages 25–70; see Figs. 3, 4, 5 to be discussed below).

Temporarily suppressing indices for age, sex, time, and country, the logit transforms of the fractions 2–6 above are

$$\begin{aligned}
\xi_2 &= \text{logit}((\alpha_3 + \alpha_4)/(1 - \alpha_1)) \\
\xi_3 &= \text{logit}(\alpha_4/(\alpha_3 + \alpha_4)) \\
\xi_4 &= \text{logit}((\alpha_2 + \alpha_7)/(\alpha_2 + \alpha_5 + \alpha_6 + \alpha_7)) \\
\xi_5 &= \text{logit}((\alpha_2)/(\alpha_2 + \alpha_7)) \\
\xi_6 &= \text{logit}(\alpha_5/(\alpha_5 + \alpha_6))
\end{aligned}$$

In this way, five series were constructed, for each combination of country, sex, and age group.

A random walk with drift model

Much attention has been given to the age pattern of the shares of each household position. There are many possible modelling strategies for such age patterns. Keilman (2016) gives a brief review, and concludes that a Brass type of relational model works well, at least compared to the more popular Lee-Carter model. The reason is the particular form of the latter model. When the trend in the shares is downward for some ages and upward for other ages, one of the parameters of the Lee-Carter model (namely the age profile b_x), may have both positive and negative values. When in addition the general time trend in the shares is systematically upwards or downwards, the result is a strong distortion in the age pattern of predicted shares; see Lee and Miller (2001) for a discussion of this issue in the context of mortality.

Originally intended to model age-specific survival from birth to age x, the Brass relational model (Brass 1971) can be written as

$$Y(x) = a + b.Y^S(x) + e(x),$$

where $Y(x)$ is the probability of survival from birth to age x in logit transformed form, while $Y^S(x)$ is a standard age pattern of survival, also in logit form. a and b are coefficients to be estimated from the data, and $e(x)$ is an error term with zero expectation and constant variance. Changing the parameter a shifts the age pattern up or down, while *b* changes its slope. See, for example, Preston et al. (2001) for a thorough discussion.

The model is linear in its parameters. Hence, as a first step, we used the method of ordinary least squares (OLS) to estimate the Brass model applied to the age pattern of logit transformed fractions $\xi_k(x,t)$. The standard age pattern $\xi_k^S(x)$ was defined as the average value of $\xi_k(x,t)$, where for each *k* the average was taken over all years *t*, for a given combination of age, sex and country. For each *k* we obtained estimates of parameters *a* and *b* that varied over time, between sexes and between countries. In terms of the coefficient of determination (R^2), the fit was excellent in almost all cases, with R^2 values larger than 0.9 and many of them larger than 0.97.

In most cases we noticed a gradual increase or decrease over time in the estimates of *a* and *b*. To illustrate, the upper panel of Fig. 1 shows the annual estimates of the Brass model parameters for Danish women, for $k = 4$ (SIN0 plus INST as a share of the total share of SIN0, SIN+, OTHR, and INST) and $k = 6$ (SIN+ as a share of

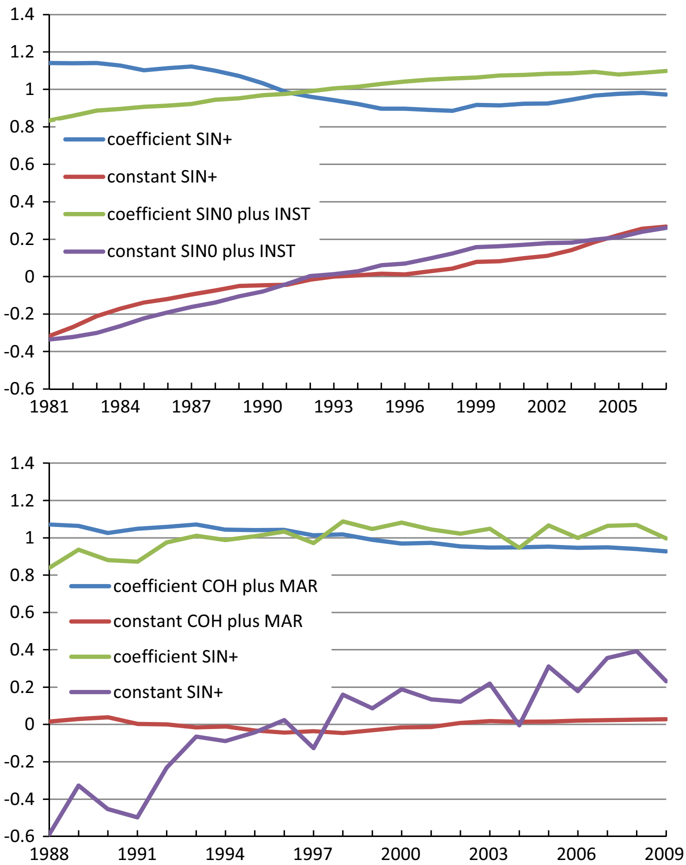


Fig. 1 Estimates of Brass model parameters (constant and coefficient) when model is fitted to data from each year separately. *Upper panel*: data for women in Denmark. *Lower panel*: data for men in Finland. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

SIN+ plus OTHR). The latter two cases were selected because they fitted best to Danish data (as judged by a value of R^2 equal to 0.996) or had the worst fit ($R^2 = 0.912$) in terms of a model in which the parameters are assumed to follow a straight line; see model (1) below. Since the standard age pattern is defined as the average pattern for the period, the constant a is close to zero around the middle of the period, while the coefficient b is close to one. In both cases, the constant term increases regularly; for lone mothers, however, the rotation of the age pattern is somewhat irregular. For the case of SIN0 and INST combined the slope of the age profile increases systematically. The lower panel shows estimates for Finnish men, for the cases $k = 2$ (COH plus MAR; $R^2 = 0.976$) and $k = 6$ (SIN+; $R^2 = 0.994$).

The gradual developments in parameter estimates suggested that a and b could be written as linear functions of time, i.e.

$$\xi_k(x,t) = (A_k + a_k.t) + (B_k + b_k.t).\xi_k^S(x) + e_k(x,t). \tag{1}$$

In order to avoid spurious correlation, we detrended this model by taking first differences, and found

$$\Delta\xi_k(x,t) = a_k + b_k.\xi_k^S(x) + d_k(x,t), \tag{2}$$

where $\Delta\xi_k(x,t) = \xi_k(x,t) - \xi_k(x,t-1)$ and $d_k(x,t) = \Delta e_k(x,t)$ is an error term.

Model (2) defines $\xi_k(x,t)$ as a random walk with drift (RWD). The drift $a_k + b_k.\xi_k^S(x)$ consists of a part a_k that is common for all ages, whereas $b_k.\xi_k^S(x)$ is an age-specific part. The term $\xi_k^S(x)$ preserves the age pattern in the random walk increments for each type of fraction k. The innovation variance is $\sigma_k^2 = \mathrm{Var}(d_k(x,t))$. Figure 2 gives a schematic representation of the model.

Note that the age pattern of $\Delta\xi_k(x)$ relates to fixed *calendar years*. By its nature, the Brass relational method does not include any effects for *birth cohorts*. This issue is discussed in "Conclusions and discussion" section.

In a second step, parameters a_k and b_k of the model in expression (2) were estimated by OLS (across ages x) assuming an innovation variance independent of age and time. For each type of fraction $\xi_k(x,t)$ ($k = 2, 3, 4, 5, 6$), country-specific estimates were very similar, and differences were not significant in most cases. Thus in a third step, the model was re-estimated for all three countries simultaneously. In addition, we did not distinguish by sex: the results from the second step showed small differences between men and women, except for k equal to 4, which reflects the chance of living either alone or in an institution. For women, the estimate of a_4

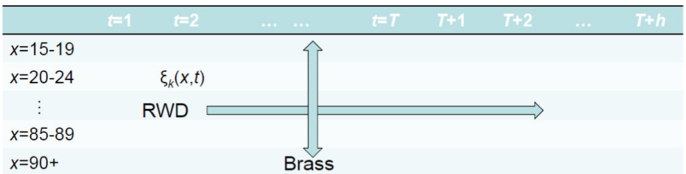


Fig. 2 Schematic representation of the random walk with drift (RWD) model in expression (2). The variable x denotes age group, t denotes time. Predictions start at time T

turned out to be significantly lower (but still positive) than that for men. A possible explanation is that women's chances of living alone have increased relatively slowly, because their survival chances have increased more slowly than those of men. Table 1 gives the parameter estimates. Note that for $k = 6$ (lone parents), neither estimates is significantly different from zero, and hence the process is likely to follow a random walk without drift.

To check for possible autocorrelation in the data, we formulated an extension of model (2) with $d_k(x,t)$ error terms written as a first-order autoregressive process, that is,

$$\Delta\xi_k(x,t) = a_k + b_k.\xi_k^S(x) + d_k(x,t) + \rho.d_k(x,t-1) + u_k(x,t).$$

The parameters of this model were estimated by means of the Prais-Winsten method (Greene 2003, p. 273). A careful country-by-country and sex-by-sex comparison for the five cases $k = 2$–6 revealed that the estimates of a_k and b_k were very close to those of model (2) above. In one case, namely $k = 2$ (married and cohabiting persons combined) we found systematic first-order autocorrelation, with estimates of ρ between 0.35 and 0.55, depending on sex and on country. In all other cases the autocorrelation pattern was much less systematic, with both positive and negative estimates of the autocorrelation coefficient. Strong autocorrelation was found for lone parents ($k = 6$), with estimates equal to -0.76 and $+0.69$ for men in Finland and women in Denmark respectively. In all other cases the estimates indicated much weaker autocorrelation. Of the 30 estimates, 22 were positive, while eight were negative.

Views on the issue of autocorrelation differ: some consider it to be model misspecification; others treat it as a problem with the data (Greene 2003, p. 253). We decided to ignore possible autocorrelation, and to use the simpler model in expression (2) instead, for the following reasons. The autocorrelation pattern was not very systematic and hence difficult to interpret. Only two out of 30 cases showed relatively strong autocorrelation. Finally, with rather short time series (27 years of data for Denmark, fewer for the other two countries) we wanted to keep the number of model parameters to a minimum. The consequence is that estimators for the

Table 1 Parameter estimates for model (2)

k	a_k		b_k		Cov(a_k,b_k)
	Estimate	t value	Estimate	t value	
2	−0.0005697	−0.7	−0.0076073	−7.6	−5.75e−7
3	−0.0432034	−11.8	0.0083405	5.2	−4.88e−6
4 (men)	0.0385686	27.6	−0.0033708	−1.8	−1.48e−6
4 (women)	0.0211024	18.1	0.0040122	4.4	−3.60e−7
5	0.0652686	14.2	−0.0109857	−6.3	−7.16e−6
6	0.0313597	1.1	0.0121778	0.3	0.0010577

Student t values based on robust standard errors parameters in Table 1 still are unbiased, but that standard errors and t-values are incorrect. On average (across the 60 standard errors for three 3 countries, two sexes, two model parameters, and five cases k = 2–6), the standard errors of model (2) turned out to be 5 % lower than those of the model with autocorrelated errors. In 16 of the 30 cases, the standard errors of model (2) were larger than those of the extended model.

Predictions

Starting from a known value $\xi_k(x,T)$, a future value h years ahead ($h = 1, 2,\ldots$) is

$$\xi_k(x,T+h) = \xi_k(x,T) + h.\left(a_k + b_k.\xi_k^S(x)\right) + d_k(x,T+1) + \cdots + d_k(x,T+h).$$

Hence an h-step ahead forecast is

$$E[\xi_k(x,T+h)] = \xi_k(x,T) + h.\left(\hat{a}_k + \hat{b}_k + .\xi_k^S(x)\right), \tag{3}$$

where E[.] denotes expectation, and a_k and b_k have been replaced by their estimated values. The forecast error $F_k(x,T + h)$ equals $\xi_k(x,T + h) - \mathrm{E}[\xi_k(x,T + h)]$. Given our assumptions, its variance is

$$\begin{aligned}\mathrm{Var}[F_k(x,T+h)] &= \mathrm{Var}\left[\sum_{i=1}^{h} d_k(x,T+i) - h.\left(\hat{a}_k + \hat{b}_k.\xi_k^S(x)\right)\right] \\ &= h.\sigma_k^2 + h^2.\mathrm{Var}[\hat{a}_k] + h^2.\left(\xi_k^S(x)\right)^2.\mathrm{Var}[\hat{b}_k] - 2.h.\xi_k^S(x).\mathrm{Cov}\left[\hat{a}_k, \hat{b}_k\right],\end{aligned} \tag{4}$$

where $\mathrm{Cov}\left[\hat{a}_k, \hat{b}_k\right]$ denotes the covariance between $\hat{a}_k$ and $\hat{b}_k$.

The estimated models (2) were used to extrapolate the logit-transformed fractions $\xi_k(x,t)$ to 2041. Figures 3, 4 and 5 give selected results for observed and predicted shares $\alpha_j(x)$, where the predictions were obtained by back transformation of the logit-transformed fractions $\xi_k(x,T + h)$; see "Appendix". Figures 3, 4 and 5 show a continuation of historical trends, in line with the assumptions. The trends are very similar in the three countries. Cohabitation will become more prevalent, in particular among young adults. For persons aged 60–80, the most dominant position will still be living with a marriage partner. Among the oldest old (aged 80+) we can expect a slight increase in the chances of living with a partner, but a somewhat stronger increase in the chances of living alone. Much of these two trends is caused by a strong fall in the shares of elderly who live in an institution (not shown here), resulting in the "deinstitutionalization" of elder care that started in the 1970s in Denmark, and a stronger focus on other forms of long-term care such as sheltered housing and home services (Daatland 1997).

These findings for the elderly are in line with the conclusions of Spersrud (2012) based on simulations of household trends in Finland during the years 2009–2039. She used the multi-state projection model LIPRO and simulated future numbers of men and women aged 80 and over who live in an institution, with a partner (spouse or cohabitee), or alone. Assuming a constant number of persons in an institution, she found that the chances of a person aged 80+ to live alone or with a marital spouse

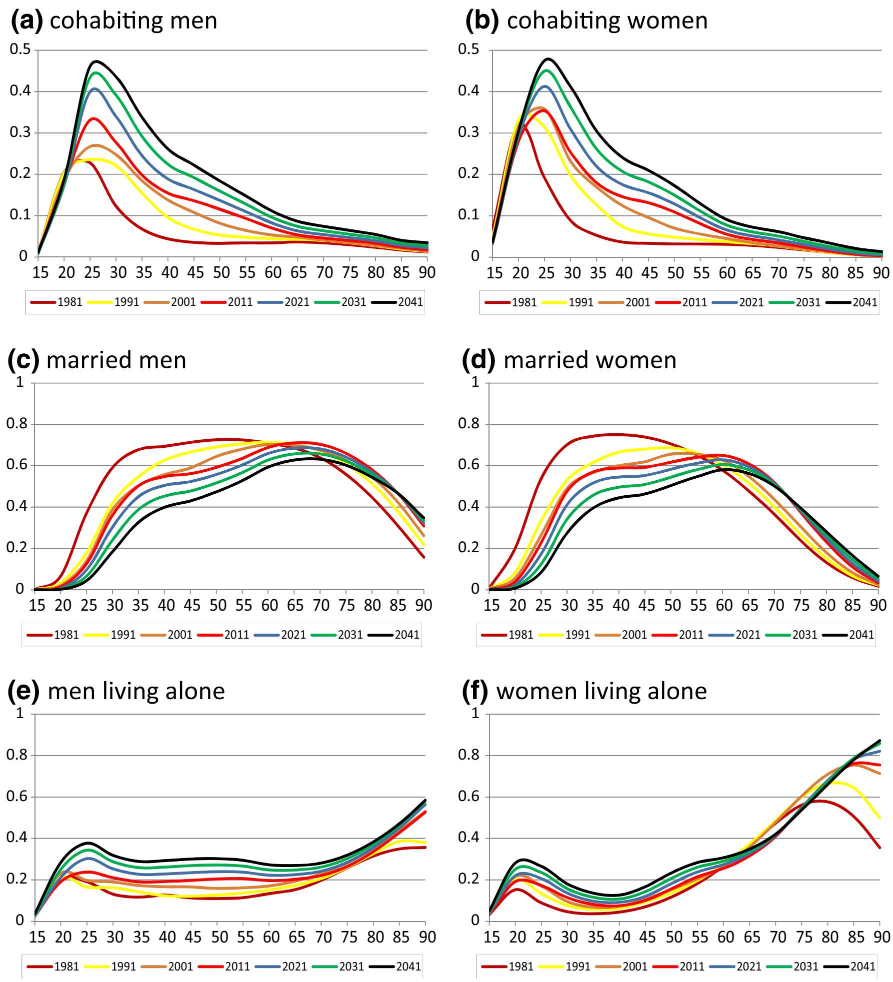


Fig. 3 Observed (1981–2011) and predicted (2021–2041) shares of persons in selected household positions, by age, Denmark. *Data sources*: 1981–2001 register data; 2011 census data, Eurostat 2014, 2021–2041 model extrapolations. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

would increase by 3–4 percentage points from 2009 to 2039. The assumption of constant institutional capacity together with a strong increase in the number of persons aged 80+ (by a factor 2.8 over the period) results in falling shares of elderly persons who live in an institution. On the other hand, if for the entire period, the capacity of institutions was assumed to be twice as large as that in 2009, the share of elderly who lived alone in 2039 would be the same as in 2009, while the share of persons with a spouse would increase by 2 percentage points. In general, Spersrud (2012) found that the numbers of elderly who live alone are strongly affected by the assumed number of places in institutions. The fact that both entry rates into an

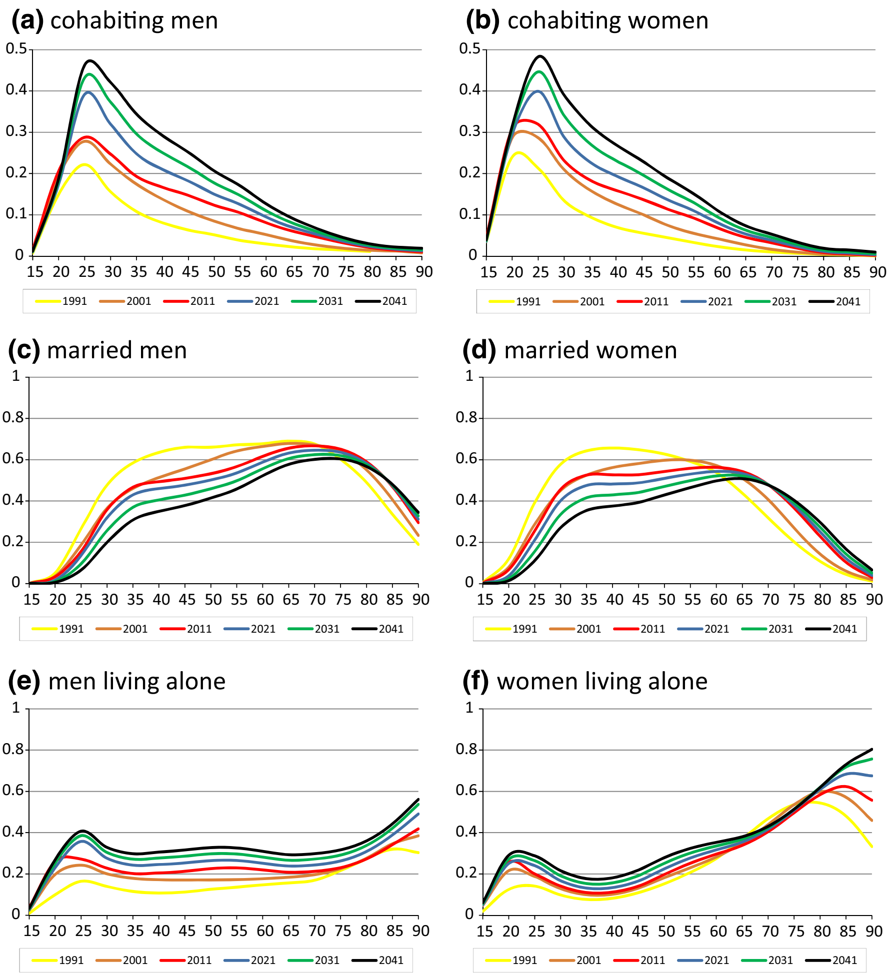


Fig. 4 Observed (1991–2011) and predicted (2021–2041) shares of persons in selected household positions, by age, Finland. *Data sources*: 1991–2001 register data; 2011 census data, Eurostat 2014; 2021–2041 model extrapolations. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

institution and exit rates back to a private household in Finland are higher for elderly who live alone than for those who live with a partner is one explanation.

Note that the reduction of the shares for Danish and Finnish married men and women around age 30 stopped by 1991, but it continues in the projection to 2041. Other factors being the same, this poor model fit should show up as relatively wide prediction intervals and uncertain predictions for those men, caused by a poor fit of model (2). Indeed, the simulations show that uncertainty in the shares of married men and women in the two countries (for instance, as measured by the standard deviation or by the width of the prediction interval) increases from a very low level

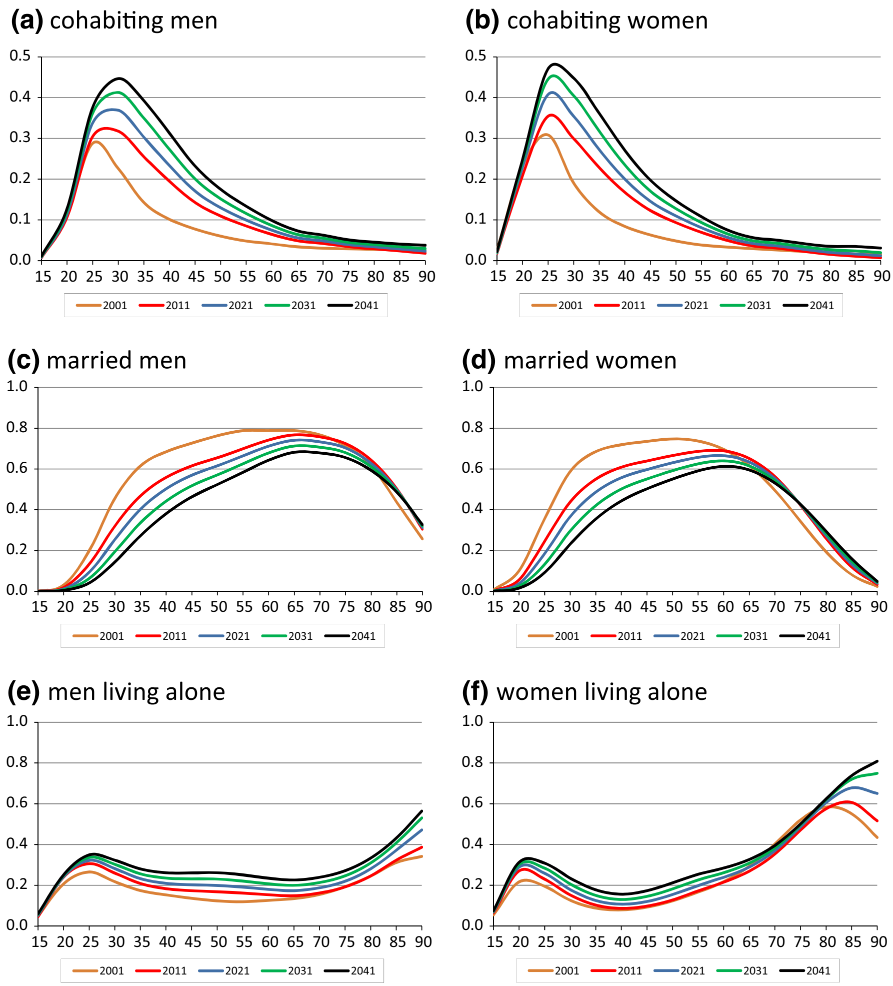


Fig. 5 Observed (2001–2011) and predicted (2021–2041) shares of persons in selected household positions, by age, Netherlands. *Data sources*: 2001 register data; 2011 census data, Eurostat 2014, 2021–2041 model extrapolations. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

at age 15–19 to a maximum around ages 35–39. For later ages it falls again (numbers not given here).

Variances and correlations

The logit-transformed fractions $\xi_k(x,T + h) = \xi_k(x,s,T + h,c)$ are assumed to have a multivariate normal distribution, with expected values given by expression (3). Expression (4) specifies their variances. We have no reason to assume that the uncertainty in the forecasts differs between countries. Therefore, we computed the average across the three countries of the standard age pattern $\overline{\xi_k^S(x,s)} = \sum_c \xi_k^S(x,s,c)/3$ and replaced ξ_k^S in expression (4) by this average.

Covariances/correlations remain to be specified. The fractions $\xi_k(x,s,t,c)$ are correlated across ages x, across sexes s, and between countries c. Since each fraction is modelled as a Random Walk with Drift process, it has zero autocorrelation. Inter-country correlations may be ignored as long as we present results for the populations of the three countries separately. Correlations across ages and between men and women were estimated from the residuals of model (2).

For $k = 2$, 3, 4, 5, and 6, we found correlations between sexes equal to 0.626, 0.598, 0.624, 0.891, and 0.065, respectively. Given the low estimate for lone parents ($k = 6$), we have assumed independence between men and women for this group. Reasons for becoming and remaining a lone parent are often very different for men and women. Differences in the estimates for the other groups ($k = 2$–5) are hard to interpret. Therefore we took the median of the four numbers above, which is 0.623.

Following earlier work (Alho and Keilman 2010, C&K) we assumed an AR1 process for the errors in the age dimension. There was little systematic difference in the estimated correlations across ages. Inspecting correlations for different types of fractions ($k = 2,\ldots,6$), we found extremely high estimated age correlation for the share of COH plus MAR ($k = 2$; median value across ages equal to 0.982). An intuitive explanation is that the age pattern for living with a partner is very regular. In the simulations described below we have assumed that ages are perfectly correlated for this group. In other words, when the simulated value of ξ_2 is high/low for a particular age group, it is also high/low for all other age groups. For the other types ($k = 3$–6) there was no systematic pattern. We have used the median correlation across ages and types, which is 0.756.

Household forecasts

Method

Below we present selected forecast results for the three countries for the years 2021, 2031, and 2041. The starting point was the household structure in 2011 based on census information; see Eurostat (2014). Figure 6 shows the age pyramids of the three countries together with the household structures of these populations.

Not surprisingly, most children and adolescents younger than 20 years of age live with one or both parents; see the green bars at the bottom of the pyramids. Among adults, married couples constitute the vast majority, although cohabitation is frequent among adults under the age of 30. Age and sex patterns of those who live alone are strikingly similar across the three countries. The graphs show a consistent pattern of more young men than young women living alone. One explanation is that when a young couple (cohabiting or married) with children breaks up, in many cases the man leaves the household and lives alone for some time, while the woman becomes a lone mother. The sex ratio among one-person households is reversed for the elderly. This is due to three factors: men are often a few years older than women

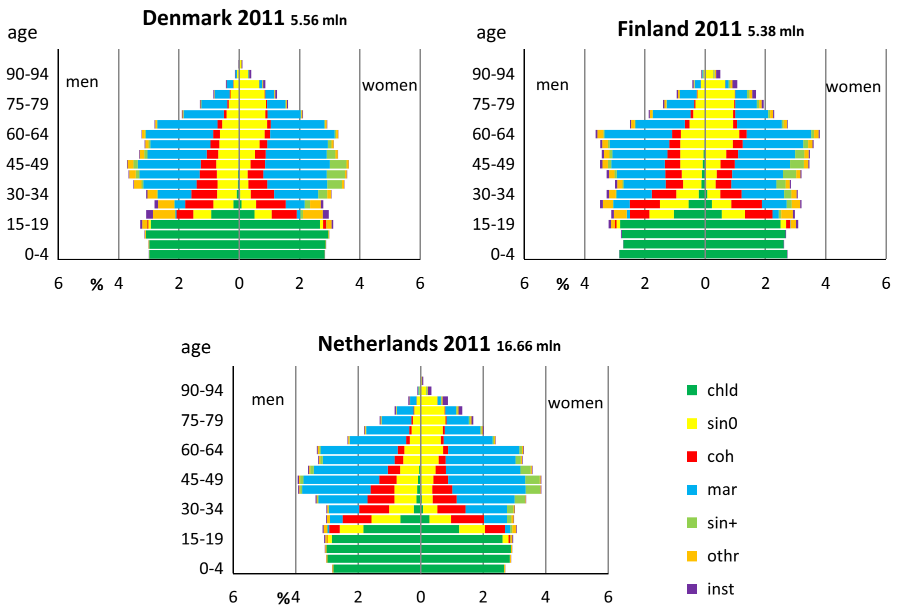


Fig. 6 Household structure of the population in three countries. Explanation of legend: "chld": child living with parent(s); "sin0": person living alone; "coh": person living with cohabitee; "mar": person living with marital spouse; "sin+": lone parent; "othr": other private household position; "inst": person living in institution. *Source*: Eurostat (2014). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

when they form a couple; mortality among (married, cohabiting) men is higher than among women; and, after union dissolution, women are less likely to repartner than men (Peters and Liefbroer 1997; United Nations 2010; US Census Bureau 2014). All this leads to more elderly women than men who live alone.

Results of the household forecast are based on 1000 stochastic simulations for the household shares, combined with 1000 simulations for the populations.

Both the shares and the populations are for men and women separately, and specific for 5-year age groups and for household positions. For example, the number of lone mothers ($j = 5$) aged x at 1 January 2021 in country c equals $\widehat{\alpha}(5, x, 2, 2021, c).\widehat{W}(x, 2, 2021, c)$. The assumption here is that the share $\widehat{\alpha}$ and the population number $\widehat{W}$ are independent. Reasons why this assumption is justified are discussed by Alho and Keilman (2010). Note that numbers of cohabiting men and women (COH) are computed independently from each other. Hence there is no provision for maintaining the obvious link between these two. The same is true for household position MAR. This issue is discussed further in "Conclusions and discussion" section.

For the Netherlands we used the results (1000 simulations) of the official probabilistic population forecast published by Statistics Netherlands (Van Duin and Stoeldraijer 2014). For Denmark and Finland, the stochastic population forecasts are updates of the results from the Uncertain Population of Europe (UPE) project. The aim of that project was to compute stochastic population forecasts for 18 European countries, including the countries of the current paper. For more information about the methodology and assumptions see Alho et al. (2006), Alders et al. (2007), Alho et al. (2008) and the website http://www.stat.fi/tup/euupe/.

We used Juha Alho's Program for Error Propagation (PEP) to calculate the stochastic population forecasts for Denmark and Finland. This program takes as input the base population and predicted mortality rates and fertility rates (for women) as well as net migration, all by one-year age groups for all the forecast years. In addition one must specify uncertainty parameters for these rates and the rates' co-variances across time, age, and between the sexes. Fertility, mortality, and net migration are assumed to be independent of each other. The program then draws sample values from a normal distribution, and transforms these into correlated errors. Adding the errors to the specified rates creates a sample path for the vital rates. This sample path together with the base population is then used to calculate a sample path for the future population, using a cohort component model. The process is repeated as many times as required.

UPE results were updated by changing the commencement years to 2011, and by using expected values for age-specific death rates, birth rates, and net migration numbers from recent population forecasts of the two countries. The remaining assumptions, that is, the variances and co-variances for the mortality rates, fertility rates, and net migration, were left unchanged. The assumption here is that the volatility of fertility, mortality, and migration for the period 2011–2041 in the two countries is the same as that assumed in the UPE-project for corresponding lead times of 10, 20, and 30 years.

To compute numbers of *households* based on *persons* in various household positions, we made the following assumptions.

- Each one-person household, lone father household, or lone mother household corresponds with one person with household position SIN0, SIN+ (men) or SIN+ (women), respectively.
- The numbers of cohabiting and married couples equal half the numbers of persons with household positions COH and MAR, respectively.
- The number of other households equals the number of persons with household position OTHR divided by a fixed ratio, which equals the number of households of type 'other' relative to the number of persons with household position OTHR. For all future years, the ratios were assumed to be 2.05, 3.99, and 5.86, for Denmark, Finland, and the Netherlands, respectively. The latter figures are based on information from the census of 2011 in each country (Eurostat 2014).

Selected results

Table 2 shows that predicted developments in important household types in the three countries are as one could expect, given our assumptions. Numbers of one-person households and of cohabiting couples will increase to 2041, whereas there will be fewer married couples. These developments reflect our assumptions of a continuation of historical trends in household shares. Except for in Finland, numbers

Table 2 Private households and population size—observed 2011 (census numbers) and predicted 2021–2041 (averages across 1000 stochastic simulations) in millions, coefficients of variation (CV, %), lower and upper bounds of 80 % prediction intervals (millions)

	One person households	Cohabiting couples	Married couples	Lone fathers	Lone mothers	All private households (incl. other private households)	Population size
Denmark							
2011	0.95	0.30	1.03	0.03	0.15	2.54	5.56
2021							
Average	1.19	0.40	1.08	0.03	0.12	2.85	5.83
CV	9.8	17.1	7.4	48.4	29.0	2.5	1.0
Interval	[1.04,1.34]	[0.31,0.48]	[0.97,1.18]	[0.009,0.04]	[0.07,0.16]	[2.76,2.93]	[5.75,5.90]
2031							
Average	1.39	0.48	1.02	0.02	0.11	3.04	6.10
CV	13.0	21.3	11.8	52.2	35.2	3.9	2.8
Interval	[1.16,1.62]	[0.35,0.60]	[0.86,1.17]	[0.01,0.04]	[0.06,0.15]	[2.89,3.20]	[5.88,6.31]
2041							
Average	1.54	0.53	0.96	0.02	0.10	3.19	6.32
CV	15.7	24.8	14.7	59.4	39.4	5.5	5.3
Interval	[1.23,1.87]	[0.36,0.70]	[0.78,1.14]	[0.01,0.04]	[0.05,0.16]	[2.97,3.42]	[5.92,6.76]
Finland							
2011	1.04	0.30	0.93	0.03	0.14	2.52	5.38
2021							
Average	1.18	0.36	0.93	0.04	0.13	2.69	5.64
CV	9.8	18.2	8.7	68.7	43.3	3.0	1.5
Interval	[1.04,1.33]	[0.28,0.44]	[0.82,1.03]	[0.01,0.07]	[0.06,0.20]	[2.58,2.80]	[5.53,5.75]
2031							
Average	1.27	0.42	0.87	0.03	0.12	2.76	5.82
CV	13.6	22.7	13.7	69.4	47.8	5.0	4.0

Table 2 continued

	One person households	Cohabiting couples	Married couples	Lone fathers	Lone mothers	All private households (incl. other private households)	Population size
Interval	[1.05,1.51]	[0.30,0.55]	[0.71,1.03]	[0.01,0.07]	[0.04,0.20]	[2.57,2.94]	[5.52,6.11]
2041							
Average	1.25	0.48	0.82	0.03	0.11	2.72	5.92
CV	18.3	25.5	17.0	76.5	50.6	7.4	6.8
Interval	[0.97,1.55]	[0.32,0.64]	[0.64,1.00]	[0.00,0.06]	[0.04,0.19]	[2.47,2.97]	[5.44,6.43]
Netherlands							
2011	2.71	0.92	3.27	0.09	0.41	7.48	16.7
2021							
Average	3.11	0.85	3.51	0.09	0.35	7.96	17.3
CV	10.0	19.0	6.1	54.7	34.1	2.6	1.0
Interval	[2.72,3.52]	[0.65,1.05]	[3.22,3.78]	[0.03,0.16]	[0.19,0.50]	[7.69,8.23]	[17.1,17.5]
2031							
Average	3.52	0.93	3.42	0.10	0.35	8.38	17.8
CV	13.0	25.1	9.5	58.3	42.0	3.9	2.7
Interval	[2.93,4.11]	[0.64,1.23]	[3.00,3.83]	[0.03,0.18]	[0.15,0.54]	[7.96,8.81]	[17.2,18.4]
2041							
Average	3.97	0.97	3.29	0.11	0.38	8.78	17.8
CV	14.9	29.9	11.6	64.8	46.4	5.3	4.3
Interval	[3.24,4.76]	[0.63,1.35]	[2.80,3.76]	[0.03,0.20]	[0.15,0.61]	[8.18,9.41]	[16.8,18.8]

of private households grow faster than population numbers. As a consequence, the average size of private households will fall. This development is explained by a strong growth of one-person households, by some 40 % or more for the period 2011–2041. Finland is an exception: one-person households grow by no more than 20 % during the period. This increase is counteracted by a decline in married couples by 12 %. As a result, the increase in the total number of households (8 %) is less than that in population size (10 %), and average household size will increase slightly from 2.1 in 2011 to 2.2 in 2041.

Now we turn to uncertainty in the predictions. Table 2 reports the coefficient of variation (CV) for each prediction, defined as the standard deviation across 1000 simulations divided by the average value. Thus the CV is a *relative* measure of uncertainty. First note that uncertainty increases with increasing forecast lead time, as one could expect. Second, relative uncertainty is small for numerous households. Predictions of married couple households and of one-person households are more certain than those of cohabiting couples, and much more certain than predictions of lone-parent households. A different way of expressing uncertainty is by means of the lower and upper bounds of a prediction interval. The prediction intervals in Table 2 reflect prediction uncertainty in the *absolute* sense. According to the model, chances are 80 % that the number of private households in Denmark in 2041 will be between 2.97 and 3.42 million, up from 2.54 million in 2011. A number higher than 3.42 million is not impossible, but chances for that to occur are only 10 %. Similarly, fewer than 2.97 million households cannot be excluded either—the model predicts that this chance, too, is 10 %. The prediction intervals become wider when we look further into the future—this is another expression of increasing uncertainty, as is the case with the CV. Note that in all three countries the prediction interval for one-person households in 2041 is *wider* than that for all private households. In other words, numbers of one-person households are more difficult to predict than all households. The explanation is that random variations in the numbers of households of various types cancel out when these are added together. For instance, shares for COH and MAR are negatively correlated: when the simulated share for MAR is relatively high, the share of COH is often lower than expected, and vice versa.

Earlier we noted an increase in the forecasted average household size in Finland, from 2.1 in 2011 to 2.2 in 2041. The 80 % prediction interval is [2.0, 2.4], while 28 % of all simulations resulted in an average size of 2.1 or less. This means that an increase is probable, but not very certain. Indeed, while average household size in industrialized countries has fallen for a number of decades, recent information suggests that this development may have come to an end in some cases. See Christiansen (2015, p. 4) for Sweden, United States Census Bureau (2016) for the USA, and .id (2012) for Australia. Cooper et al. (2015) find that the stagnation in the State of Queensland in Australia was observed in the big cities, and suggest it might be linked to the decline in housing affordability.

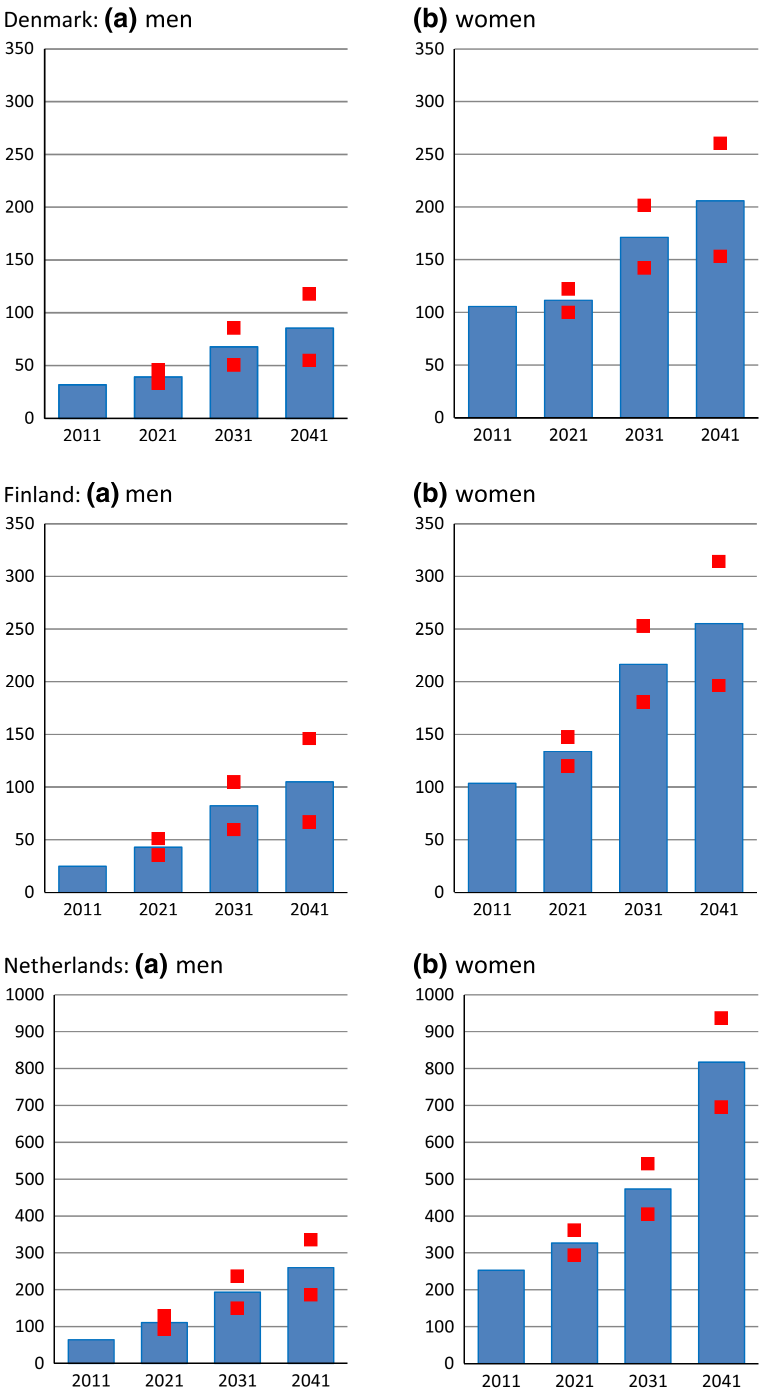
How do the results for Denmark and Finland in Table 2 compare with the findings of C&K? Their results are very different from ours. First, whereas Table 2 shows a slight decrease in the number of households in Finland towards the end of the period, C&K find uniformly growing numbers, caused in particular by more one-person households and more married couples. For Denmark, they find that the number of households grows slowly, caused by relatively moderate increases in numbers of one-person households and of cohabiting couples. (The trend in married couple households is similar to that in Table 2.) The diverging findings were to be expected, because the models for household shares differ strongly between the two approaches. More interestingly, uncertainty around predicted numbers in Table 2 is much larger than that found by C&K. The reasons are not entirely clear, but one explanation is that the latter two authors assumed an estimation variance of the drift estimate in the RWD model equal to the innovation variance divided by the number of observations of the time series minus one. In the current paper we used estimation variance for the drift based on robust standard errors from OLS-regression; cf. expression (4) and Table 1. Although many of the estimates are strongly significant, the error term variances are relatively high (R^2 values are between 0 and 8 %).

Van Duin et al. (2013) report a probabilistic household forecast for the Netherlands for the period 2013–2060, based on the approach developed by De Beer and Alders (1999) and Alders (1999, 2001). The authors predict more or less stable numbers of households that consist of a married or a cohabiting couple (4.3 million in 2041; cf. 3.3 + 1.0 = 4.3 million in Table 2) and slight increases for one-person households (3.7 million in 2041; cf. 4.0 million in Table 2) and lone parents (536,000 in 2041; 482,000 in Table 2). The total number of private households is expected to grow to 8.5 million (8.8 million in Table 2), with a 67 % prediction interval stretching from 8.0 million to 9.1 million. In our case the 67 % interval (in millions) is [8.32, 9.23] (not reported in Table 2). An updated forecast with results for the period 2016–2060 shows essentially the same trends (slightly fewer one-person households, and slightly more lone parents); yet our numbers are within the 67 % prediction intervals of the 2016 update (Statistics Netherlands 2016). Thus official household trends for the Netherlands are very similar to ours, in spite of a methodology that is very different. More detailed results will likely show larger differences.

The comparison between the results in this paper and those in other studies leads to very different conclusions. On the one hand, C&K find smaller prediction uncertainty for Denmark and Finland than that reported in Table 2. On the other hand, independent stochastic household forecasts for the Netherlands are very similar, at least in terms of the numbers that we could compare. Because of this, and the fact that the current model is much less complicated than that of C&K, we tend to give somewhat more credibility to the results in this paper than those of C&K. At the same time we have to be cautious and additional testing is needed before one can draw a more definite conclusion.

The oldest old

Figure 7 shows what we expect for the "oldest old" (persons aged 80 and over) who live alone, if current trends continue. This age group is of considerable interest for policy makers. Although the health condition of the oldest old may improve in the years to come, many of these will be in need of formal and informal care. Our simulations predict that in 2041, there will be between two and four times as many


Denmark: (a) men
(b) women
Finland: (a) men
(b) women
Netherlands: (a) men
(b) women
0
50
100
150
200
250
300
350
400
500
600
700
800
900
1000
2011
2021
2031
2041

◀**Fig. 7** Men (panels (**a**)) and women (panels (**b**)) aged 80+ who live alone. Observed 2011 (census numbers), and predicted 2021–2041 (averages across 1000 stochastic simulations), in thousands. *Red dots* represent upper and lower bounds of 80 % prediction intervals, in thousands. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

men and women aged 80 and over who live alone, compared with the numbers in the censuses of 2011. The growth in these numbers is relatively certain, cf. the increasing lower bound of the 80 % prediction interval. The predictions indicate that whereas in 2011 there were between 330 and 420 elderly women who lived alone for every 100 men living alone, by 2041 this sex ratio is likely to be somewhat more balanced, but still very skewed, at between 240 and 310.

Persons in institutions

The populations of the three countries in this study are ageing, similar to those of other countries. If the *share* of elderly persons who live in an institution is kept constant, the result will be a strong increase in the *number* of persons in such institutions. The historical downward trend in shares for household position INST in the three countries is extrapolated by our RWD model. Whether the result will be more or fewer persons in institutions in the future is an empirical matter. Table 3 shows that the capacity of institutions for the elderly may have to be increased for Finland, where the number of persons who live in an institution is predicted to grow by a factor of four during the thirty-year period. In Denmark the downward trend in the shares results in a decrease in the institutionalized by one third; for the Netherlands the numbers are about constant between 2011 and 2031, but increase to 2041. The wide prediction intervals and the large CV values indicate that it is difficult to predict this variable accurately.

Table 3 Persons who live in an institution—observed 2011 (census numbers) and predicted 2021–2041, average across 1000 stochastic simulations (thousands), coefficient of variation (CV, %), lower and upper bounds of 80 % prediction interval (thousands)

	2011	2021	2031	2041
Denmark				
Average	81.5	69.0	64.2	55.2
CV		21.2	33.2	48.4
Interval		[51.5, 87.7]	[40.6, 93.8]	[28.3, 87.3]
Finland				
Average	110.7	171.5	287.5	436.1
CV		21.2	29.7	32.7
Interval		[128.2, 223.0]	[184.5, 403.1]	[259.6, 629.0]
Netherlands				
Average	219.3	224.9	260.8	326.8
CV		21.3	32.4	42.0
Interval		[165.8, 287.2]	[165.8, 376.2]	[180.8, 492.8]

Conclusions and discussion

We showed how techniques of data dimension reduction can be used to model and predict patterns of household dynamics. The aim was to simplify the method for probabilistic household forecasting used earlier by Christiansen and Keilman (2013). We have computed probabilistic household forecasts for Denmark, Finland, and the Netherlands, spanning the period 2011–2041. The starting point was the population of each country disaggregated by age, sex, and household position as reported in the census round of 2011. Future trends in fertility, mortality and international migration were taken from official population forecasts. Time series of shares of the population in six different household positions were modelled as random walks with drift. Brass' relational model preserved the age patterns of the household shares. Probabilistic forecasts for households were computed by combining predictive distributions for the household shares with predictive distributions of the populations, specific for age and sex.

The results show a continuation of current trends towards more and smaller households, often driven by increasing numbers of persons who live alone. Numbers of households increase faster than population size, which leads to falling average household sizes. A very consistent finding is that more numerous households are easier to predict than households that are less numerous, at least when uncertainty is considered in a relative sense. One can expect a strong growth in the numbers of persons aged 80 and over who live alone in the three countries. A doubling of these numbers towards 2041 is extremely likely. The sex ratio will become slightly less skewed: today there are between three and four elderly women in the three countries who live alone for every elderly man who does so; that ratio is likely to decrease somewhat, to between two and three by 2041.

The contribution we hope to make to the literature is to show how one may simplify the data-hungry approach of Christiansen and Keilman (2013) by avoiding transition probabilities between household positions. At the same time, a few unresolved issues should be mentioned.

First, in "Modelling household shares" section we defined a random walk with drift (RWD) process for the fractions $\xi_k(x,t)$. The drift equals $a_k + b_k.\xi_k^S(x)$, where $\xi_k^S(x)$ is a standard age pattern. This standard is defined period-wise to account for year-to-year changes in the fraction $\xi_k(x,t)$. The term $\xi_k^S(x)$ preserves the age pattern in the random walk increments. *Cohort* effects in the age profiles are not accounted for. For example, one could assume that an increasing share of women who cohabit at age 25 in 1995 will mean larger shares of cohabiting women aged 45, 20 years later. To implement such cohort effects in the Brass relational model would require a standard profile for birth cohorts, in addition to one for periods.

A second issue is that of coherence between men and women. In the observed data for Denmark, Finland, and the Netherland there is a strong correspondence between the numbers of men and women in household types COH and MAR. The numbers are not exactly equal, caused by partnership formation and marriage across international borders, same-sex couples, and errors in the registration, but the numbers are close. This coherence is lost when we predict shares for cohabiting and married men and women separately. When using one random walk model for men and one for women, a practical ad-hoc solution to the problem of coherence between men and women is to adjust predicted numbers of men and women in household positions COH and MAR.

Finally, the household hierarchy that we used can be justified on intuitive grounds, but the consequences of choosing one particular hierarchy are not known. We modelled household shares for persons who live with a partner first. Next, we modelled shares for persons who live alone or in an institution. Lone parents and other persons came last. Christiansen and Keilman (2013) used a somewhat different hierarchy. To analyse the consequences of a specific hierarchy for the probabilistic forecasts would require extensive simulations. This is left as a topic for future research.

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Appendix: Back transformation from ξ to α

In "Modelling household shares" section the shares α_j are transformed into fractions ξ_k. In this "Appendix" we outline the back transformation from ξ_k to α_j. We suppress indices for age, sex, time, and country. The starting point is the set of expressions that transform the shares α_j into fractions ξ_k.

$$\begin{aligned}
\xi_2 &= \text{logit}((\alpha_3 + \alpha_4)/(1 - \alpha_1)) \\
\xi_3 &= \text{logit}(\alpha_4/(\alpha_3 + \alpha_4)) \\
\xi_4 &= \text{logit}((\alpha_2 + \alpha_7)/(\alpha_2 + \alpha_5 + \alpha_6 + \alpha_7)) \\
\xi_5 &= \text{logit}((\alpha_2)/(\alpha_2 + \alpha_7)) \\
\xi_6 &= \text{logit}(\alpha_5/(\alpha_5 + \alpha_6))
\end{aligned}$$

There are many equivalent expressions for the α_j written as functions of the ξ_k. One of these is the following set

$$\begin{aligned}
\alpha_2 &= (1 - \alpha_1)\exp(\xi_4)\exp(\xi_5)/\{(1 + \exp(\xi_2))(1 + \exp(\xi_4))(1 + \exp(\xi_5))\} \\
\alpha_3 &= (1 - \alpha_1)\exp(\xi_2)/\{(1 + \exp(\xi_2))(1 + \exp(\xi_3)\} \\
\alpha_4 &= \alpha_3 \exp(\xi_3) \\
\alpha_6 &= (1 - \alpha_1 - \alpha_3 - \alpha_4)/\{(1 + \exp(\xi_4))(1 + \exp(\xi_6)\} \\
\alpha_5 &= \alpha_6 \exp(\xi_6) \\
\alpha_7 &= \alpha_6 \exp(\xi_4)(1 + \exp(\xi_6))/(1 + \exp(\xi_5))
\end{aligned}$$

By assumption, α_1 is independent of ξ_k ($k = 2, 3,\ldots 6$).

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