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Using Deterministic and Probabilistic Population Forecasts

The relevance of probabilistic population forecasts

Population forecasts inform us about the size of a population and the numbers of men and women that will be in various age groups in the future. These forecasts are important for planning purposes, for instance, to analyse future educational facilities, public pension expenditures, housing needs, etc. Other social and economic variables also play a part, e.g. participation rates for college and university students, retirement behaviour, and household size. But given a particular time frame for a forecast, the size and the age pyramid of a population are generally easier to predict than the other social and economic variables a planner needs, hence population forecasts are routinely made by statistical agencies in most countries of the world.

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Figure 1 shows an example of such a forecast, which was recently computed by Statistics Norway (Statistics Norway 2006). Norway's population is expected to increase from 4.6 million in 2005 to 6.1 million in 2060. However, we cannot exclude stronger or weaker growth. Therefore, we see two additional forecast variants, one resulting in 7.4 million persons in 2060 and the other arriving at a figure of 4.9 million. These three forecast variants are based on different assumptions for fertility, mortality and international migration. In the main variant featuring moderate population growth, the forecasters assume for 2060 a fertility level of 1.8 children per woman on average, a life expectancy of 86 years for men and 90 years for women, and net immigration of 16 000 persons. The other two variants result when one assumes higher or lower values for fertility, life expectancy, and immigration.[1]

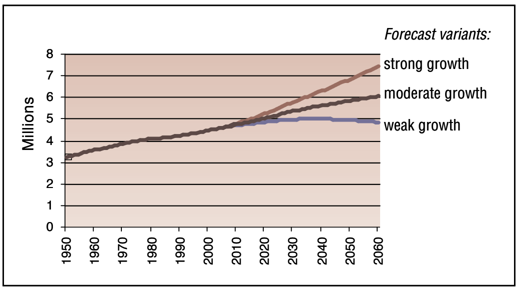

Figure 1. Population size, Norway. Registered 1950–2005. Forecast 2006–2060.

1: Statistics Norway has also published nine other combinations. One example is the so-called 'Strong ageing' variant, which results from combining low fertility, high life expectancy and low immigration.

Forecast variants

The practice of computing more than one forecast variant is standard among statistical agencies (Keilman and Cruijsen 1992). It goes back at least to 1947, when Whelpton and colleagues published their forecast for the United States. Statistical agencies follow this practice because they want to account for the fact that the future is inherently uncertain, and that different forecast assumptions will lead to different forecast outcomes. However, one major problem is that the conventional approach is entirely *deterministic*, i.e. statistical distributions are not included in the forecasting model. Hence, no probabilities are attached to the variants and this poses a problem for the user of the forecast, who has to select one of the variants as input for his analysis.[2] Therefore we advocate the use of *probabilistic* population forecasts which state the likelihood of the various outcomes. Probabilistic forecasts give future population size and age pyramids not as one number (or perhaps a few, depending on the number of variants), but as a whole range of probability distributions. The future is inherently uncertain, yet some demographic developments are more probable than other developments. The probability distributions tell us *how much* more probable. Thus, the user of a probabilistic forecast is informed about the likely magnitude of the errors, and how these errors vary across age groups or between the sexes. When a decision maker is able explicitly to deal with forecast uncertainty, this will lead to better decision making. As soon as he knows the expected costs involved in decisions based on forecast results that turn out to be wrong at a later stage, an optimal strategy can be chosen. Unfortunately, nearly all official forecasts are deterministic, not probabilistic – Statistics Netherlands is the only known exception (Alders and De Beer 1998). However, demographers and statisticians have developed methods to calculate probabilistic forecasts. By way of illustration, I shall present a probabilistic population forecast for Norway, discuss its advantages for the user compared to a deterministic forecast, and show how a probabilistic forecast can be used in practice.

A probabilistic forecast for Norway

The probabilistic forecast for Norway is part of a recently completed research project, called "Uncertain Population of Europe" (UPE). The aim of the project was to compute the probability distributions of future demographic variables such as population size, age groups etc. for 18 countries in Europe, including Norway. Details can be found at http://www.stat.fi/tup/euupe/, in Alho *et al.* (2006) and Alders *et al.* (2007). I will provide some selected results for Norway.

The results show that the odds are four to one (80 per cent chance) that the population of Norway, now 4.7 million, will number between 4.79 and 5.16 million individuals in the year 2020, and 4.84–6.76 million in 2050; see Figure 2. The interval for 2050 illustrates that long-term uncertainty is quite large; see also Figure 3. Continued growth to 2050 is probable and a decrease in population size is unlikely, but we cannot exclude such

2: Instead of uncertainty variants, the alternative interpretation of variants is that of scenarios which depict alternative futures. In this case, too, the user does not know the probability of these variants.

a trend. The probability for a population size in 2050 below the current 4.7 million is an estimated 6 per cent. Similar probability results were computed for men and women of all ages.

How do these probabilistic forecast results compare with those obtained by a conventional deterministic forecast? Statistics Norway's moderate growth forecast predicts a population size in 2050 of 5.84 million. This is slightly higher than UPE's median forecast of 5.68 million, but well inside the 80 per cent prediction interval; see Figure 2.

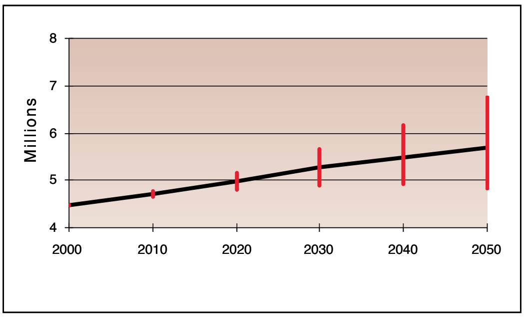

Figure 2. Total population, Norway. Median forecast (black) and 80% prediction intervals (red)

Problems related to deterministic population forecast

a) A limited number of variants leave room for politically motivated choices by the users

A probabilistic forecast forces the user to consider a whole range of results, with probabilities attached to them; see, for instance, Figure 3. The probability for one single number is zero. In contrast, a deterministic forecast includes only a limited number of outcomes, typically three or four, and no probabilities. A forecast user, when confronted with the choice between these few variants, is likely to take his decision based on subjective or political grounds, depending on vested interests. The construction firm that plans the building of a new school is more likely to use a high forecast for the future number of pupils than the school board that has to bear the costs.

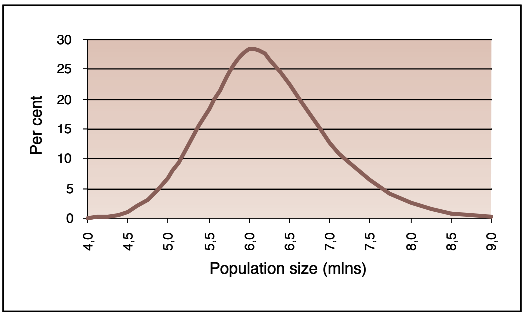

Figure 3. Predictive distribution total population size, Norway 2050

b) Two variants that are extreme for one variable are not necessarily extreme for another variable

The legal pensionable age in Norway is 67. In 2050, the population aged 67 and over will number between 1 090 000 and 1 388 000, depending on low or high population growth, and according to the deterministic forecast computed by Statistics Norway (2006). However, the Old Age Dependency Ratio (OADR), i.e. the number of 67+ as a ratio of the number aged 20–66, equals 0.377 for low population growth, and 0.368 for high population growth. The gap between these two is much smaller than one would expect based on the absolute numbers. The reason for this inconsistency is that the population of working age in this forecast is perfectly correlated with the number of elderly. In the high growth variant, for every year in which life expectancy is high, immigration is also high, and *vice versa* for the low growth variant. A probabilistic forecast does not necessarily assume perfect correlation between these two age groups. Starting from the high and low numbers for the over 67 mentioned above, the probabilistic forecast for Norway predicts a 60 per cent probability for the population aged 67+ to total 1 090 000 to 1 388 000 individuals in 2050, corresponding to a 60 per cent interval for the OADR in 2050 stretches from 0.343 to 0.446. Thus the relative width of the interval for those over 67 as well as for the OADR in 2050 in the probabilistic forecast are of comparable value, as opposed to the narrow relative distance between the same variables in the deterministic forecast. In general, two variants in a traditional forecast that are extreme for one variable are not necessarily extreme for another variable (Lee 1998).

c) When interpreted as uncertainty intervals, coverage probabilities are small in the short run and large in the long run

As noted above, Statistics Norway has formulated a low growth and a high growth variant. These result in 4.9 and 6.8 million inhabitants in 2050, respectively. The UPE results tell us that the interval between 4.9 and 6.8 million in 2050 has a coverage probability of 78 per cent. But for 2010, when the low-high interval in Statistics Norway's forecast ranges from 4.72 to 4.78 million, the coverage probability is a mere 47 per cent. Coverage probabilities for the low-high interval that increase rapidly with increasing forecast horizon are a common problem for deterministic forecasts. The reason is that these forecasts implicitly assume perfect correlation over time. In the high variant, fertility (or life expectancy or migration) is assumed to be high in one year, and it is 100 per cent certain that it will also be high one year later, and the same applies to the low variant. This is not a realistic assumption. A probabilistic forecast does not show this defect.

Using loss functions to assess the results of a probabilistic forecast: An illustrative example

The Norwegian system for public old-age pensions is not sustainable in the long run. Therefore, the Norwegian government proposed a pension reform in the spring of 2007 (see Report No. 5 to the Storting 2006–2007). An important element of the new system is the so-called life expectancy adjustment: when mortality is low and people live longer, annual pension benefits will be lower than when mortality is high, all other things being equal. In the proposed pension system, the annual pension benefits for a retired person are equal to the total earned pension rights at the time of retirement divided by the remaining period of life expectancy in the population. Individuals may account for a possible increase in life expectancy (and thus lower annual pension benefits) by retiring later or by saving more. To fix ideas, assume that a person aged 55 plans to retire at the age of 62, at which time he expects to have earned certain pension rights. The planning consists in determining how much additional saving will be required up to age 62, given a desired level of annual pension benefits. Remaining life expectancy at age 62 has to be predicted years into the future. Write that forecast as Fe62. When this person reaches age 62 and the actual period of life expectancy Ae62 turns out to be higher than the forecast, the actual annual benefits will be lower than predicted. This will imply a loss for the individual, which will be larger the stronger the underprediction is. When life expectancy is over-predicted (Fe62>Ae62), the individual has saved 'too much'. The person's *loss function* quantifies his loss as a function of the forecast error. When a probabilistic forecast is available at the time the decision is taken, Fe62 is a stochastic variable, as is the loss function. Thus, one may compute the expected loss, and the individual will select a life expectancy value which minimizes his expected loss. For simplicity, assume that the individual's loss function is in accordance with the following linear form

$$\text{loss} = c(\text{Fe62} - \text{Ae62}) \text{ for Fe62} > \text{Ae62}$$
$$= \lambda c(\text{Ae62} - \text{Fe62}) \text{ for Fe62} \leq \text{Ae62}.$$

c translates the forecast error Fe62-Ae62 into costs, while λ reflects the degree of symmetry in the loss function. λ>1 implies that an underprediction is more severe than an equally large overprediction (i.e. the individual perceives having saved 'too much' between ages 55 and 62 as less severe than receiving too low benefits after age 62). For this particular form of the loss function, the optimal choice of the forecast variable is that value for which the predictive distribution of Fe62 equals λ/(λ+1) (e.g. Alho and Spencer 2005). Thus for a risk-neutral person who has symmetric loss function, λ=1 and the median is the optimal choice. When loss is non-symmetric, the optimal choice depends on λ and on the form of the predictive distribution of Fe62. Figure 4 plots the optimal value of the remaining life expectancy for changing λ, assuming that Fe62 is normally distributed with expectation equal to 20 years and standard deviations (denoted as s), equal to two and four years. For s = 2 years, a person whose λ equals 2.25 selects a remaining life expectancy of 21 years – one year higher than the median. When the uncertainty in the predictions becomes larger, the individual becomes more cautious. For instance, for s = 4 years, this person would select 22 years – the shift compared with the median value becomes twice as large.

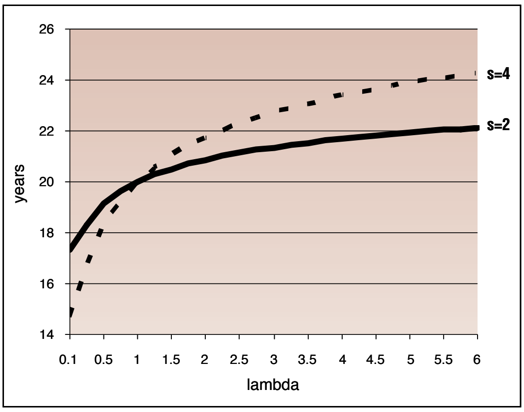

Figure 4. Optimal choice for e62

Loss functions in general are difficult to establish, in particular when non-monetary variables are of central concern. However, an important first step towards a full analysis is to check the degree of symmetry in the loss function. Is an underprediction more harmful or less harmful than an overprediction of the same magnitude? In the context of public old-age pensions, predictions of life expectancy that are too low imply a deficit in the pension fund. All else being equal, this could result in a cut in the benefits or in other welfare programs, or a rise in taxes. This compares unfavourably with a life expectancy prediction that turns out to be too high, *ex post facto*. Hence, the managers of the public pension fund will most likely select life expectancy values that tend to be too high, rather than too low.

Final remarks

Although probabilistic forecasts are well suited to reflect forecast uncertainty, there are certainly issues connected with such forecasts. One important one is that the uncertainty parameters for probabilistic forecasts themselves are uncertain. Frequently, they result from extrapolations of observed uncertainty statistics, either model-based extrapolations or ones that are more intuitive. Thus, a possible strategy is to be cautious and not underestimate the uncertainty of the forecast (Alho *et al.* 2006). One practical issue is that the users have to know how to handle forecast results in the form of probability distributions, rather than one number. In the short term, forecast uncertainty is not important, at least not in general for most forecast results at the country level. In the long run, however, users should be aware of the costs attached to employing a forecast result that subsequently turns out to be too high or too low, see above. They should ask themselves whether an immediate decision based on the uncertain forecast is necessary, or whether they can wait for a while until a new forecast possibly shows less uncertainty. If an immediate decision is required, they should try to determine the most essential features of the loss function, and base their decisions on that.

In his British Academy Annual Lecture on 1 December 2004, the Bank of England's Governor Mervyn King stressed that in a wide range of collective decisions, it is vital to think in terms of probabilities (King 2004). We must accept the need to analyse the uncertainty that inevitably surrounds these decisions. In order to frame a public discussion in terms of risks, the public needs to receive accurate and objective information about the risks. Transparency and honesty about risks should be an essential part of both the decision-making process and the explanation of decisions. If population projections are to inform policy decisions, then uncertainty of these projections must be assessed. In some areas, greater uncertainty might lead to postponement of actions. In other policy arenas, e.g. education planning, greater uncertainty might indicate that the best polices would be those most easily changed as the future unfolds. For example, a school planner facing uncertain projections of enrolment growth might decide to rent additional space for schools rather than building or buying space. Explicitly estimating the degree of uncertainty in population projections encourages consideration of alternative population futures and the full range of implications suggested by these alternatives (Lee and Tuljapurkar 2007).

However, the public has great difficulty in understanding probabilities, and handling them. Whether occupied with weather, or inflation, or population trends in the future, forecasters should develop appropriate techniques for communicating uncertainty to the users of their services. The type of charts presented in Figure 2 for future population size of Norway are commonly used by Norges Bank in its monetary reports, and meteorologists use them for their weather reports. Population forecasters should also consider using such charts.

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