NUPTIALITY MODELS AND THE TWO-SEX PROBLEM IN NATIONAL POPULATION FORECASTS *

Nico KEILMAN **

Netherlands Interuniversity Demographic Institute, Voorburg, The Netherlands

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Abstract. This paper describes the two-sex problem in nuptiality models, focusing on applications in national population forecasts. Requirements for a realistic two-sex marriage model are mentioned, together with additional considerations important in the context of official forecasts. Recent nuptiality models violate the requirements to a certain extent and/or do not meet the additional considerations. A new model, used in the 1980-based population forecasts of the Netherlands compiled by the Netherlands' Central Bureau of Statistics, is described. When confronted with the requirements and considerations, the CBS-model, although not ideal, is seen to possess relatively good characteristics.

Résumé. Les modèles de nuptialité et le 'problème des deux sexes' dans les perspectives démographiques nationales

Cet article présente le 'problème des deux sexes' dans les modèles de nuptialité, en mettant l'accent sur leur utilisation dans le cadre de perspectives démographiques nationales. Les conditions requises pour qu'un modèle de nuptialité à deux sexes soit réaliste sont précisées, ainsi que d'autres éléments dont il est important de tenir compte dans le contexte de perspectives démographiques officielles ; dans une certaine mesure, les modèles existants violent les unes et/ou ignorent les autres. Un nouveau modèle – sinon idéal, du moins relativement meilleur à cet égard – est présenté ici : le modèle élaboré par le Bureau Central de Statistiques des Pays-Bas (CBS) et utilisé dans les perspectives démographiques néerlandaises calculées à partir de 1980.

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** Author’s address: N.I.D.I., P.O. Box 955, 2270 AZ Voorburg, The Netherlands.
1. Introduction

Most national population forecasts not only calculate the future total population size but also contain a classification by sex and age. Specification of the future population according to marital status is sometimes given as well.

Why should a population forecaster compute the future numbers of unmarried, married, divorced and widowed persons? Two reasons can be mentioned [Brass (1974, p. 537)].

- It contributes to better extrapolation of demographic phenomena into the future. Both mortality and, particularly, fertility, show considerable variations over the marital states.
- Forecasts of the population by marital status may be of value in their own right. A decomposition by marital status is relevant mainly for the planning of future housing needs, but it is also of significance for the production of consumer durables.

In addition, we can note that any forecasts of the numbers of marriages and marriage dissolutions generated in the process may be useful for the organization of the civil registration system and of the divorce courts, and also for the authorities that deal with widow’s pensions.

However, the specification of an appropriate nuptiality model poses certain methodological problems when sex is used in addition to marital status. The so-called two-sex problem leads to special requirements for the nuptiality model. Hence this part of the complete forecasting model is usually more complex than those parts that deal with other phenomena such as fertility, mortality and migration.

This paper aims at finding solutions for the two-sex problem in nuptiality models in so far as they are used in national population forecasts. The discussions are limited to purely demographic approaches, leaving aside models with equations that explicitly describe relations between demographic (here: nuptiality) variables and non-demographic ones. The reason for this choice lies in our emphasis on national population forecasts which usually model demographic processes in a purely demographic way.

We shall first discuss the nature of the two-sex problem. We summarize the requirements that a realistic two-sex marriage model should fulfill and also mention additional considerations commonly found in
forecasting situations (section 2). Section 3 contains an overview of recent nuptiality models and of the extent to which they meet these requirements. In sections 4 and 5 a new nuptiality model used in the 1980-based national population forecasts of the Netherlands, compiled by the Netherlands’ Central Bureau of Statistics (CBS), is described. Finally (sections 6 and 7), we give an overview of the technical performance of the model when it was used in the CBS population forecasts and also an overall evaluation.

2. Problem description and constraints

2.1. The two-sex problem

Demographic models of observed populations usually do not take account of interactions between the sexes. Most of them deal with the experiences of a single sex: at best the behaviour of males and that of females are treated separately. Attempts to reflect the experience of both males and females simultaneously are confronted with the ‘two-sex problem’. By this we mean that observed sex-specific rates for certain demographic events which involve interaction between the sexes (e.g., nuptiality and fertility) cannot be used consistently together in a demographic model once the model population differs from the observed one. Suppose two sets of age-specific marriage rates are observed, one for males and one for females. Now for those two sets of rates to reflect the behaviour of any population, the total number of marriages derived from the male rates must equal the total number of marriages implied by the female rates. In general, however, the rates taken from one population will not yield consistent numbers of marriages when applied to another population. An analogous situation occurs with divorce, transition to widowhood and fertility if these three phenomena are described for the two sexes separately [Schoen (1981, p. 201)].

There is no generally accepted method available to solve this two-sex problem. Several authors have, however, mentioned requirements (also labeled ‘axioms’) that should be met by a realistic two-sex marriage model.
2.2. Requirements for a realistic two-sex marriage model

Recent overviews of the requirements for realistic two-sex marriage models have been given by Pollard (1977) and Wijewickrema (1980). The most important will be summarized here. Monogamy is presupposed.

(1) **Availability:** the number of marriages for males (females) aged \( x(\gamma) \) cannot exceed the number of eligible males (females) aged \( x(\gamma) \). Thus,

\[
\sum_{y} N_{xy} \leq M_{x} \quad \text{and} \quad \sum_{x} N_{xy} \leq F_{y},
\]

where \( N_{xy} \) is the number of marriages between males aged \( x \) and females aged \( y \) during the period under consideration, and \( M_{x} \) and \( F_{y} \) denote the numbers of eligible males aged \( x \) and eligible females aged \( y \) respectively.

(2) **Monotonicity:** increased availability at any specified age can – ceteris paribus – result only in an increase in marriages. In symbols,

\[
\frac{\partial N_{xy}}{\partial F_{y}} \geq 0 \quad \text{and} \quad \frac{\partial N_{xy}}{\partial M_{x}} \geq 0.
\]

For some age-combinations strict inequalities should occur.

(3) **Homogeneity:** an increase (decrease) in the number of available members of each sex by the same factor entails an increase (decrease) in the number of marriages by the same factor. If the numbers of unmarried males (females) aged \( x(\gamma) \) are denoted by the vector \( \{ M_{x} \} \) (\( \{ F_{y} \} \)), then

\[
N_{xy}(c \{ M_{x} \}, c \{ F_{y} \}) = c N_{xy}(\{ M_{x} \}, \{ F_{y} \}), \quad c \geq 0.
\]

This requirement states that if the population doubles we expect the number of weddings to double rather than to quadruple or remain unchanged.

(4) **Competition:** the number of marriages \( N_{xy} \) for age combination \((x, y)\) should be a non-increasing [and, over some area of \((x, y)\), strictly decreasing] function of \( M_{x'} \), for \( x' \neq x \), and similarly, in the
case of females, for $F_y$, $y' \neq y$. Thus,

$$\frac{\partial N_{xy}}{\partial M_x} \leq 0, \quad (x' \neq x), \quad \text{and} \quad \frac{\partial N_{xy}}{\partial F_y} \leq 0, \quad (y' \neq y).$$

This axiom stems from the fact that the marriage chances of a male aged 25, for example, are very likely to be decreased by an increase in the supply of eligible males aged 28 or 35. However, the degree of decrease caused by 28-year-old males is not necessarily the same as that caused by 35-year-old ones, as will be stated in the next requirement.

(5) **Substitution** (or relative competition): the negative effect on $N_{xy}$ of an increase in $M_x$ ($x' \neq x$) depends on how close $x'$ is to $x$. More specifically, the decrease in $N_{xy}$ for an increase of $M_x$ is greater than the decrease in $N_{xy}$ for an equivalent increase of $M_{x''}$ if $x'$ is closer to $x$ than $x''$ is. This means that

if $(x'' - x) > (x' - x) > 0$ or if $(x'' - x) < (x' - x) < 0$,

then $\frac{\partial N_{xy}}{\partial M_x} < \frac{\partial N_{xy}}{\partial M_{x''}} < 0$.

Thus an extra supply of 22-year-old single males leads to a stronger decrease in the number of marrying males aged 20 than a similar extra supply of single males aged 25. Similar statements are analogously valid for the sexes interchanged.

(6) **Symmetry with respect to the sexes**: model results should be robust against interchanging the sexes. Symbolically

$$N_{xy}\{\{ M_x \}, \{ F_y \}\} = N_{xy}\{\{ F_y \}, \{ M_x \}\},$$

where the vectors $\{ M_x \}$ and $\{ F_y \}$ contain, in this special case, all relevant variables, hence both stock variables (unmarried persons) and flow variables (marriage rates). This axiom excludes for instance female-dominant models.

The six requirements given above apply to a marriage model. Models describing divorce or transition to widowhood do not have to meet the competition requirement or the substitution requirement (see sections 5 and 6). Moreover, the requirement with respect to symmetry takes a different form in widowhood models.

Not all these requirements are equally important. Availability, for instance, is a very strict requirement whereas competition and substitu-
tion are desirable. A symmetric model could be regarded as more elegant than one which is not symmetric.

Finally there is an essential requirement for both marriage formation and dissolution.

(7) Consistency: in any given period of time the total number of males marrying should equal the total number of females marrying. A stronger version of this axiom should be used when marriages are distinguished according to age-combination of the partners. Then the number of marriages between males aged $x$ and females aged $y$ implied by the male rates must be equal to the number of $(x, y)$ marriages implied by the female rates.

2.3. Considerations in population forecasts

Unlike individual researchers, agencies producing official forecasts often use additional considerations besides those of the preceding section. This is due mainly to the interpretation that is (or should be) given to the notion of a forecast. When we use the term 'population forecast' in this article, we mean a calculation concerning the future size, composition and development of a population using likely assumptions with respect to phenomena influencing the calculation. These assumptions apply to socio-demographic phenomena (for example, future norms and values with respect to the family), to technological phenomena (for example, the development of medical technology), to strictly demographic factors (for example, the mean age at divorce) et cetera.

The strictly demographic factors are represented in the forecasting model by means of parameters. In general, these parameters are related to the risk, for individuals from a certain population category, of experiencing a particular demographic event during a given forecasting period (mortality risks for all persons, marriage risks for unmarried persons over 16 years of age and so on). Producing a population forecast involves the specification of those values for these parameters which represent the most likely (realistic, plausible) development of the demographic phenomenon in question during the forecasting period.

The use of the notion of a population forecast as described above has two consequences that are relevant within the framework of the two-sex problem.
The first consideration concerns the availability of a reliable and sufficiently long series of observed parameter values. Such an historical series is generally necessary (but not sufficient!) to specify the most likely future development for those parameters. An agency producing population forecasts will usually choose a nuptiality model that can be linked to the available demographic statistics.

Second, the demographic parameters should be controllable: any adjustments to the values initially specified that are necessary to meet requirements 1–7 ought to be small. Resulting parameter values should still depict a time series that represents the most likely development. The use of iterative algorithms is consequently not a very attractive strategy, unless one is certain that the final solution is close to the trial solution.¹

3. Some recent nuptiality models

In this section we list a number of recent nuptiality models and the extent to which they meet the requirements and considerations mentioned in the preceding section. Details of the models will be omitted here. Readers not familiar with particular models may consult the references cited. A summarizing review of these models has been given elsewhere [Keilman (1982a,b)].

In table 1 we list nine nuptiality models. Four of them (those for the Netherlands, Norway and Denmark, Sweden, and Great Britain) were used by agencies involved in population forecasting, although the calculations for the Netherlands were largely experimental [CBS (1979)]. It can be seen that all these four models meet the forecasting considerations: data are sufficiently available and no iterations are required. Yet adjustments (‘one-cycle iterations’) are often necessary. The availability and substitution requirements are violated, although, when looking in more detail at these four models, it should be mentioned that failure to meet the availability requirement will occur only in rather extreme cases.

¹ Moreover, when employing an iterative procedure, an analytical check of the model against the requirements of section 2.2 would often be quite difficult.
Table 1
Performance and characteristics of recent nuptiality models: extent to which they meet theoretical requirements and practical forecasting considerations.

<table>
<thead>
<tr>
<th>Requirements</th>
<th>Forecasting considerations</th>
</tr>
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<tbody>
<tr>
<td>Availability</td>
<td>Mono-</td>
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<td></td>
<td>tony</td>
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<tr>
<td></td>
<td>ability</td>
</tr>
</tbody>
</table>

**Models constructed by forecasting agencies**

(1) The Netherlands [CBS (1979)]
   - no
   - yes
   - yes
   - no
   - no
   - yes
   - yes
   - yes
   - yes
   - no

(2) Norway [Brunborg et al. (1981)]
   - Denmark [Danmarks Statistik (1970), Spohr (1972)]
   - yes
   - no
   - yes
   - no
   - yes
   - yes
   - yes
   - yes
   - no
   - no

(3) Sweden [Statistiska Central Byrå (1970), Widén (1969)]
   - no
   - no
   - yes
   - no
   - no
   - no
   - yes
   - yes
   - yes
   - yes
   - no

(4) Great Britain [Daykin and Leete (1979)]
   - no
   - yes
   - yes
   - yes
   - no
   - yes
   - yes
   - yes
   - yes
   - no

**Models constructed by individual researchers**

(5) Panmictic circles [Henry (1968, 1972)]
   - yes
   - no
   - yes
   - no
   - no
   - yes
   - yes
   - yes
   - yes
   - no

(6) Shah and Giesbrecht (1969)
   - yes
   - d
   - yes
   - ?
   - ?
   - yes
   - yes
   - yes
   - yes
   - (yes)

(7) Iterative adjustments [McFarland (1975)]
   - yes
   - yes
   - yes
   - ?
   - ?
   - yes
   - yes
   - yes
   - yes
   - (yes)

(8) Harmonic means [Schoen (1981)]
   - yes
   - yes
   - yes
   - no
   - no
   - yes
   - yes
   - no
   - no
   - no

(9) Generalized harmonic means [Pollard (1975)]
   - yes
   - yes
   - yes
   - yes
   - yes
   - yes
   - yes
   - yes
   - no
   - no

---

a Consistency is achieved for marriages and marriage dissolutions for males and females irrespective of age.
b Consistency is achieved for marriages and (if applicable) marriage dissolutions by age-combination of the partners.
c For divorce at the level of age-combinations, for marriage and transition to widowhood at the level of total numbers irrespective of age.
d Under certain conditions (see text).
e Problematic (see text).
f Partially.
Models 5–9 were constructed by individual researchers. Typically, they describe only marriage. Hardly any attention is paid to divorce or transition to widowhood. But one may argue that these could easily be modelled by borrowing the main principles from the respective marriage models.

The Shah and Giesbrecht model meets the monotonicity requirement only under certain circumstances. It contains parameters that control the interval within which marriage rates can be adjusted in order to meet the consistency requirement. But it can be shown that with certain values of these control parameters the model does not fulfill the monotonicity requirement. Furthermore, this model as well as the McFarland model employs an iterative procedure. Hence to make statements on competition and substitution numerical experimentation is necessary.

Schoen (1981, p. 209) claims that his ‘harmonic means’ model fulfills the competition requirement. He bases his conclusion upon a numerical experiment, in which the marriages by age-combination in a two-sex nuptiality–mortality life table derived from Swedish observations for 1973 are compared with the marriages computed for the hypothetical situation in which the ‘observed’ marriages of one particular age combination \((x = 30–34, y = 30–34)\) were taken as twice the number actually observed. This kind of sensitivity analysis clearly relates to a notion of ‘competition’ different from the usual one. In fact, in his model \(\frac{\partial N_{xy}}{\partial M_{x}} = 0\) for all \(x' \neq x\), and the like for females. This was already noted by Pollard (1977, p. 298).

Pollard’s ‘generalized harmonic means’ model fulfills all the requirements of section 2.2. However, it poses problems with respect to available data. Marriages for a certain age combination \((x, y)\) are computed as the product of a marriage rate for these ages and a risk population. In calculating this population at risk the model uses parameters reflecting the relative attractiveness of males aged \(x\) to females aged \(y\) and vice versa. For each age combination \((x, y)\) a male and a female attractivity parameter must be employed. Their numerical values cannot easily be established though [Wijewickrema (1980, p. 36)] and Pollard made no suggestions whatsoever on this topic.

From the preceding paragraphs we conclude that finding a realistic nuptiality model is not an easy matter. Competition and substitution in particular cause problems, except in Pollard’s model. Moreover, many models are rather complicated and hence not very transparent. For
instance all the models, except those of Sweden and Great Britain, use a matrix of marriages specified by the age-combination of partners. For a forecast that gives the future population distributed by age, sex and marital status only, a decomposition by the combined ages is not strictly necessary. Such a detailed forecast may give better results, but that can only be established after compiling competitive forecasts. We shall take up this point in section 7.

The importance of a realistic nuptiality model should not be overstated. Simulations show that unless there are severe shortages for one sex, all models will produce rather similar results [Schoen (1981, p. 213), Keyfitz (1971, p. 102)]. However, the performance of the various models might differ substantially in relatively unbalanced situations [McFarland (1975, pp. 77–81)]. The rapid decrease in the annual number of live births observed in the Netherlands during the late sixties and early seventies might cause such imbalances during the nineties of the present century. Males usually marry females two or three years younger than themselves, and those born in the sixties or seventies will be confronted with fewer such females than those born in earlier years. Under such circumstances, attention to the choice of model may be important.

4. The CBS-marriage model

4.1. Equations

For each projection year the calculations consist of three steps. The first step computes an initial estimate of the number of marrying persons (first marriages as well as remarriages) by age and sex. The number of marrying males aged \( x \) is given by

\[
U_x = U_S + U_D + U_W
\]

\[
= n_x(M_x^S + \frac{1}{2}I_x^S) + n_x(M_x^D + \frac{1}{2}I_x^D) + n_x(M_x^W + \frac{1}{2}I_x^W).
\]  

(1)

Here \( N \) denotes the number of marrying persons, \( M \) is the number of eligible males present at the beginning of the calendar year and \( I \) is the net number of immigrants during the year. The superscripts \( S \), \( D \) and \( W \) denote the marital states single (never married), divorced and widowed respectively. The marriage rates are written as \( n \).
Giving initial values \( n^{S(1)} \), \( n^{D(1)} \) and \( n^{W(1)} \) to the marriage rates results in an initial value for the number of marrying males aged \( x \). Let \( N_x^{(1)} \) denote this initial number. Similar reasoning gives \( N_y^{(1)} \), the initial number for marrying females aged \( y \).

The first step yields a total number of marrying males \( N_x^{(1)} \) not necessarily equal to the total number of marrying females \( N_y^{(1)} \). In the second step consistency is achieved. The initially inconsistent numbers are averaged to find the total number of marriages \( N^{(2)} \), using a harmonic mean.

\[
N^{(2)} = \frac{2 \sum_x N_x^{(1)} \cdot \sum_y N_y^{(1)}}{\left( \sum_x N_x^{(1)} + \sum_y N_y^{(1)} \right)}.
\]

The third and final step calculates fresh estimates of the number of marriages for each type at each age. This is done by proportionally rating the initial numbers up or down. For males the adjustment factor, \( \lambda^m \), is

\[
\lambda^m = N^{(2)} \left/ \sum_x N_x^{(1)} \right/ 2 \sum_y N_y^{(1)} \left/ \left( \sum_x N_x^{(1)} + \sum_y N_y^{(1)} \right) \right. \right.
\]

(2)

Analogously, the female adjustment factor, \( \lambda^f \), is given by

\[
\lambda^f = 2 \sum_x N_x^{(1)} \left/ \left( \sum_x N_x^{(1)} + \sum_y N_y^{(1)} \right) \right. \right.
\]

(3)

These adjustment factors allow the calculation of the final numbers of marriages, \( N_x^{(2)} \) and \( N_y^{(2)} \), for each marital status.

4.2. The marriage model compared with the requirements

To what extent does the CBS-model meet the requirements of section 2.2?

We would like to make two remarks before putting the model to the test. Firstly, in the axioms with respect to monotonicity and competition, only the sign of the partial derivatives is important, not their numerical value. Hence instead of \( M_x \), \( F_y \) and the like we could just as
well read \( N_x^{(1)} \), \( N_y^{(1)} \) and the like since, for example, \( \partial N_x^{(1)}/\partial M_x > 0 \) [see eq. (1)].

A second remark concerns the variable \( N_{xy} \). In the CBS-forecasts this variable is not computed: only the age-specific numbers of marrying men \( (N_x = \Sigma_y N_{xy}) \) and marrying women \( (N_y = \Sigma_x N_{xy}) \) are available. Calculation of the two-dimensional variable \( N_{xy} \) is not necessary in the framework of a projection engine that computes future population size and compositions by sex, age and marital status. Therefore, when comparing the CBS-model with the requirements stated in section 2.2, we shall have to consider only two marriage functions, viz. \( N_x^{(2)} \) and \( N_y^{(2)} \). Since the model is symmetric in the sexes, only one sex has to be treated.

With respect to availability, it can be shown that in practice this requirement will always be fulfilled. Difficulties might arise, but only if the initial marriage rate were to exceed 0.5 at any age. Consider never-married males aged \( x \) for instance. Their adjusted marriage rate can be written as \( n_x^{S(1)} \cdot \lambda^m \), being the initial rate multiplied by the adjustment factor. Hence the availability condition will be violated if \( n_x^{S(1)} \cdot \lambda^m \) exceeds one. From section 4.1 it will be clear that this is equivalent to

\[
\left( n_x^{S(1)} \cdot \frac{2 \Sigma_y N_y^{(1)}}{\Sigma_y N_y^{(1)}} \right) / \left( \Sigma_x N_x^{(1)} + \Sigma_y N_y^{(1)} \right) > 1, \quad \text{or}
\]

\[
n_x^{S(1)} > \frac{1}{2} + \frac{1}{2} \left( \Sigma_x N_x^{(1)} / \Sigma_y N_y^{(1)} \right).
\]

This means that the availability condition, when applied to the CBS-model, will not cause any problems when the initial rate, \( n_x^{S(1)} \), does not exceed 0.5. In recent years it has taken on a maximum value of about 30\%, for ages around 25 years [CBS (1978, p. 41)]. Even with future values of \( n_x^{S(1)} \) greater than 0.5, the availability requirement will often be fulfilled. In section 6.1 we shall see that in times of great imbalance between the sexes, we can still expect the value of \( \lambda^m \) to be between 0.8 and 1.2. This corresponds to \( \frac{2}{3} < \Sigma_x N_x / \Sigma_y N_y < \frac{5}{3} \). Thus, for all \( n_x^{S(1)} \) smaller than \( \left( \frac{1}{2} + \frac{1}{2} \cdot \frac{2}{3} \right) = \frac{5}{6} \), the CBS-model meets the availability criterion. For females and for remarriage the line of thought is identical to that for single males.
Monotonicity, homogeneity and competition create no difficulties. For example, considering the latter requirement we write

\[
\frac{\partial N_x^{(2)}}{\partial N_x^{(1)}} = -2 \frac{N_x^{(1)} \cdot \sum_y N_y^{(1)}}{\left( \sum_x N_x^{(1)} + \sum_y N_y^{(1)} \right)}
\]

\[
= -2 \left( N_x^{(1)} \cdot \sum_y N_y^{(1)} \right) \left( \sum_x N_x^{(1)} + \sum_y N_y^{(1)} \right)^2 < 0 \quad (x' \neq x).
\]

From this it also follows that the CBS-model does not fulfill the substitution axiom. A further differentiation with respect to \( M_{x'}^S \) gives

\[
\frac{\partial N_x^{(2)}}{\partial M_{x'}^S} = -2 n^{S(1)} x' \cdot \sum_y N_y^{(1)} \left( \sum_x N_x^{(1)} + \sum_y N_y^{(1)} \right)^2.
\]

In other words, for \( x < x' < x'' \), the negative effect on \( N_x^{(2)} \) of an increase in \( M_{x'}^S \) exceeds the negative effect of an increase in \( M_{x''}^S \) if, and only if, \( n^{S(1)} x' \) is greater than \( n^{S(1)} x'' \). For some age-combinations \( x' \) and \( x'' \) this condition will hold, for instance for \( 25 \leq x < x' < x'' \) since the maximum of the curve representing the Netherlands' age-specific marriage rates for single males is around 25 years. For \( x < x' < x'' \leq 25 \), however, \( n^{S(1)} x' \) is less than \( n^{S(1)} x'' \). Hence for these ages we find no proper substitution effects in the CBS-model. For females and for remarriage, analogous considerations lead to similar conclusions.

The symmetry condition is completely fulfilled. A final remark pertains to consistency. The consistency requirement presupposes that the net number of 'foreign marriages' equals zero. The 1980-based population forecasts concern the de jure population, i.e., those persons who have habitual residence in the Netherlands. The consistency requirement assumes that the number of males resident in the Netherlands who marry females with residence abroad equals the number of females residing in the Netherlands who marry males living abroad. In practice, the net number of these 'foreign marriages' has been about 500 to 1,000 per year in recent years, roughly only 1% of the total
number of marriages. 'Foreign marriages' were, therefore, disregarded in these forecasts.

The consequence of the fact that in the CBS-model the substitution axiom is not met, can be evaluated as follows. Consider for instance the year 2000, in which, according to the low variant of the forecasts (described in section 6 below), the CBS-model computed 1,912 marrying males aged 20 \( (N_{x}^{(2)} = 1,912 \text{ for } x = 20) \). Expression (4) permits us to calculate the deviation in \( N_{x}^{(2)} \) if for instance the number of single males aged 22 were to increase by, say, 10,000 persons – i.e., to be some 12% more than the originally computed number of 84,000 males [CBS (1982, p. 72)]. In the low variant in the year 2000, a marriage rate of \( n_{x}(1) = 0.06317 \ (x' = 22) \) was used and the model initially calculated \( \Sigma N_{x}^{(1)} = 89,835 \) and \( \Sigma N_{y}^{(1)} = 82,992 \). Hence expression (4) results in 7.0 fewer marriages for males aged 20 due to such an additional supply of 10,000 single males aged 22. However, an additional 10,000 single males aged 25 (an increase of some 14%) would result in 15.5 fewer marriages for males aged 20 since \( n_{x}(1) = 0.13978 \) for \( x'' = 25 \). Thus, an extra supply of 25-year-old single males has a greater negative influence on marrying males aged 20 than an extra supply of 22-year-old ones. The influence is so small, however, that this unrealistic ('reversed') substitution effect cannot be regarded as important in practice. Other calendar years and other variants lead to comparable numerical values and hence to the same conclusions.

5. The CBS-divorce and widowhood model

Like marriage, divorce and transition to widowhood must show consistency. The total number of male divorces should be equal to the total number of female divorces in a given year. For widowhood, the consistency requirement has two parts. On the one hand, the number of new widowers should equal the number of married females who die in a certain year. On the other hand, the number of married males who die should be equal to the number of new widows.

The requirements concerning availability, monotonicity, homogeneity and symmetry (with an adapted interpretation for widowhood) should also be met.

The computations for divorce and widowhood are similar to those for marriage. First, initial rates are specified by age and sex. Then
initial numbers are calculated for persons who divorce, experience widowhood or die in the married state. Addition over all ages for each sex shows to what extent adjustment is necessary to achieve consistency. For divorce, the adjusted number is the harmonic mean of the initial numbers for the two sexes. For transition to widowhood, only the initial numbers of new widowers and widows are adjusted. Their counterparts, the numbers of dying married females and males respectively, remain unaltered.

In practice, this model fulfills all the requirements for a realistic divorce and widowhood model, i.e., availability, monotonicity, homogeneity, symmetry and consistency.

6. Results

We can now discuss the results concerning nuptiality in the 1980-based population forecasts, using the CBS-nuptiality model as a part of the complete forecasting model. We shall limit ourselves mainly to 'methodological' results, more specifically the degree to which the initial calculations of the numbers of persons changing their marital status in a given year had to be adjusted to obtain the final numbers. More substantive demographic results are given in the relevant CBS-publications [CBS (1982, 1984)].

The 1980-based population forecasts of the CBS contain three variants – High, Medium and Low. These variants were obtained by combining different assumptions for fertility, first marriage and external migration with a fixed set of assumptions for other variables (mortality, widowhood, divorce, remarriage) [CBS (1982, p. 17)]. Summary indicators of the (initial) assumptions are given in table 2, and a very brief description of those assumptions most relevant for this article is given below. More detailed information on this subject is to be found elsewhere [CBS (1982, ch. 2); CBS (1984, ch. 3)].

As far as marriage is concerned, a decrease in the proportions ever marrying is assumed. Whereas the proportions are 90% (males) and 95% (females) for persons born around 1940, they are assumed to be 65–75% (males) and 75–85% (females) for persons born around 1970 (after adjustments due to the marriage model). For the coming decades it is expected that the mean age at first marriage will be some two years higher than nowadays, and that remarriage of divorced and widowed persons will continue to decrease.
Table 2
Summary indicators of (initial) assumptions, 1980-based population forecasts of the Netherlands.

<table>
<thead>
<tr>
<th></th>
<th>Cohort 1950</th>
<th>Cohort 1960</th>
<th>Cohort 1970</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>M</td>
<td>H</td>
</tr>
<tr>
<td><strong>Fertility</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average number of children</td>
<td>1.85</td>
<td>1.86</td>
<td>1.87</td>
</tr>
<tr>
<td>Median age at childbearing (yrs.)</td>
<td>26.6</td>
<td>26.6</td>
<td>26.6</td>
</tr>
<tr>
<td><strong>First marriage</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion ultimately marrying (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>85.0</td>
<td>85.5</td>
<td>85.7</td>
</tr>
<tr>
<td>F</td>
<td>91.5</td>
<td>91.6</td>
<td>91.8</td>
</tr>
<tr>
<td>Median age at first marriage (yrs.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>23.7</td>
<td>23.7</td>
<td>23.8</td>
</tr>
<tr>
<td>F</td>
<td>21.7</td>
<td>21.7</td>
<td>21.7</td>
</tr>
<tr>
<td><strong>Year 1970</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(observed)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Year 1980</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(observed)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Year 1990</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Divorce</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standardized general divorce rate (%)</td>
<td>M</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0.67</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>--------------------------------</td>
<td>----</td>
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<td>-----</td>
</tr>
<tr>
<td><strong>Median age at divorce (yrs.)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>36.6</td>
<td>35.5</td>
<td>35.0</td>
</tr>
<tr>
<td>F</td>
<td>35.2</td>
<td>33.3</td>
<td>33.0</td>
</tr>
<tr>
<td><strong>Remarriage of divorcees</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standardized general remarriage rate (%)</td>
<td>M</td>
<td>13.19</td>
<td>7.50</td>
</tr>
<tr>
<td>F</td>
<td>8.37</td>
<td>5.20</td>
<td>4.00</td>
</tr>
<tr>
<td>Median age at remarriage (yrs.)</td>
<td>M</td>
<td>36.0</td>
<td>36.6</td>
</tr>
<tr>
<td>F</td>
<td>33.0</td>
<td>33.1</td>
<td>33.0</td>
</tr>
<tr>
<td><strong>Remarriage of widowed persons</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standardized general remarriage rate (%)</td>
<td>M</td>
<td>2.67</td>
<td>1.50</td>
</tr>
<tr>
<td>F</td>
<td>0.35</td>
<td>0.21</td>
<td>0.15</td>
</tr>
<tr>
<td>Median age at remarriage (yrs.)</td>
<td>M</td>
<td>56.8</td>
<td>57.5</td>
</tr>
<tr>
<td>F</td>
<td>56.1</td>
<td>55.5</td>
<td>55.0</td>
</tr>
<tr>
<td><strong>Mortality</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life expectancy at birth (yrs.)</td>
<td>M</td>
<td>70.7</td>
<td>72.4</td>
</tr>
<tr>
<td>F</td>
<td>76.5</td>
<td>79.1</td>
<td>79.5</td>
</tr>
<tr>
<td><strong>External migration</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net-immigration (×1000)</td>
<td>M + F</td>
<td>33.5</td>
<td>53.1</td>
</tr>
</tbody>
</table>

a No variants distinguished.
b Low variant.
c Medium variant.
d High variant.
According to the hypotheses made, the proportion of married persons experiencing \textit{divorce} will continue its rise observed during the seventies, from about 20\% (its present value) to about 25\% at the end of the century. However, the rise is assumed to level off in the eighties.

The decline of mortality risks observed during the seventies is assumed to proceed in all age classes. This means that \textit{widowhood} risks will decrease too.

\textbf{6.1. Adjustment factors for marriage}

In section 4.1 the adjustment factor for marriage was defined as the proportion by which the initial rates (and also the initial numbers of persons marrying) have to be adjusted in order to achieve consistency. From expressions (2) and (3) it follows that the sum of the male adjustment factor, $\lambda^m$, and the female adjustment factor, $\lambda^f$, equals two. Hence $\lambda^f$ can be obtained from $\lambda^m$ and vice versa. Therefore we need to consider only one factor.

Fig. 1 shows the development of the adjustment factor for males. In the period 1980–2000 it shows the same behaviour in all variants: after some initial fluctuations between 1980 and 1985 a period with a cyclical behaviour starts. In the years between 1985 and 2030 it is less than one. This means that in the initial situation there is an oversupply of marrying males; this excess increases to 4.5–6.0\% (in 1998 when $\lambda^m$ is 94.0–95.5\%) and decreases to 2.5–3.5\%.

What could cause this development? The share of remarriages in all marriages is taken to be 15–20\% for the coming decades: the majority of marrying persons will never have been married before [CBS (1982, p. 45)]. More than 80\% of these first marriages occur in the age group 20–34 years. The ratio of the number of never-married females to that of never-married males, both aged 20–34, will, therefore, explain the development of the adjustment factor to a great extent. This is particularly true for the years from about 1995 onwards. In this period the marriage rates were assumed to be (almost) constant in time [CBS (1982, p. 18)]. This means that variations in the numbers of marriages (and hence in the adjustment factor) will be almost completely determined by variations in the never-married population.

When the initial first-marriage assumption of the CBS-forecasts was specified, a lasting and often increasing excess in the supply of never-married males in the years to come was, in fact, taken into account.
Fig. 1. Adjustment factors: Marriages of men.

Source: CBS.
This was expressed by the assumption that for the generations born in the period 1930–1970 the difference between the male proportion ever marrying and the corresponding female proportion will increase. For instance, this proportion for females born around 1940 is some 5% more than the proportion for males. However, for the cohorts born around 1970 the initial difference increases to about 8% (see table 2). Nonetheless this excess in male supply was underestimated, since the male adjustment factor is continuously below one.

The relation suggested above between the sex ratio of never-married persons aged 20–34 and the adjustment factor is confirmed by the first two columns of table 3. A comparison between this table and fig. 1 shows that a period in which the sex ratio decreases (1990/1995–2000, but particularly 2015–2030) corresponds roughly with a period in which the adjustment factor decreases. For the years between 1980 and 1990/1995 the relation between the adjustment factor and the sex ratio is much weaker. This is caused by the diverging trends in the marriage rates for this period [CBS (1982, pp. 56, 57)].

For every forecast year the sex ratio in the low variant is larger than that in the high variant. Hence, in the high variant a relatively large excess male supply exists. With respect to this excess in the male

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Sex ratio (number of women per 1000 men) by population category and forecast variant. a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calender year</td>
<td>Never-married persons 20–34 years</td>
</tr>
<tr>
<td></td>
<td>Low variant</td>
</tr>
<tr>
<td>1980</td>
<td>608</td>
</tr>
<tr>
<td>1985</td>
<td>674</td>
</tr>
<tr>
<td>1990</td>
<td>713</td>
</tr>
<tr>
<td>1995</td>
<td>723</td>
</tr>
<tr>
<td>2000</td>
<td>717</td>
</tr>
<tr>
<td>2005</td>
<td>724</td>
</tr>
<tr>
<td>2010</td>
<td>737</td>
</tr>
<tr>
<td>2015</td>
<td>746</td>
</tr>
<tr>
<td>2020</td>
<td>745</td>
</tr>
<tr>
<td>2025</td>
<td>737</td>
</tr>
<tr>
<td>2030</td>
<td>731</td>
</tr>
</tbody>
</table>

a Source: Calculated from CBS (1982), tables 2–4.
supply, no difference is initially present between the three forecast variants. Therefore, a tension between the sexes is seen to have resulted, a tension which is greater in the high variant than in the low variant. This agrees with fig. 1: almost everywhere the low variant $\lambda^m$ is closer to 100% than is the high variant $\lambda^m$.

Finally a conclusion regarding the distance between the adjustment factors in the high and the low variants. Starting in the mid-nineties, this distance is almost constant in time (1.0–1.5%). Moreover, it is much smaller than the amplitude of the waves. These observations lead to the conclusion that the influence of the marriage assumptions upon the adjustment factors (with an additional amplification due to different fertility assumptions after the years around 2000) is much smaller than the influence of the structure of the current population.

6.2. Adjustment factors for divorce

Fig. 2 represents the development of the male adjustment factor for divorce. The relation existing between the male factor and the female factor is the same as in the case of marriage. Fig. 2 shows a more or less regularly rising set of curves, from almost 101% in 1980 to 101–102% in 2030. A comparison with fig. 1 results in the following observations:

- the adjustment factors for men here exceed 100%,
- the adjustments are relatively small, and
- cyclical movements are almost complete absent here, or, possibly, possess a relatively long wavelength.

The behaviour of the divorce adjustment factor can roughly be explained by possible imbalances between male and female married persons. This is illustrated by the third and fourth column of table 3, which show the sex ratio for married persons. Since the variation in age at divorce is greater than that at marriage all ages were considered here.

In the high variant, the small increase in the adjustment factor corresponds with a small increase in the number of married females per 1000 married males. In the low variant the adjustment factor shows hardly any development at all; at the same time, the sex ratio of the married population is almost constant.

The fact that the divorce adjustment factors are considerably lower than the marriage ones can mainly be explained by the following circumstance. By definition, divorce is related only to married persons,
Fig. 2. Adjustment factors: Divorces of men.

Source: CBS.
and the extent to which married males experience this event differs only slightly from the extent to which married females do so [see, for instance, CBS (1982, p. 27)]. Marriage, however, is not only related to never-married persons, but also to divorced and widowed ones. For each of these three groups relatively large differences between the sexes can be observed in the degree to which marriages are contracted [see, for instance, table 2 and particularly CBS (1982, pp. 24, 27 and 29)]. This makes it easier to extrapolate male and female divorce rates than male and female marriage rates in a consistent manner.

6.3. Adjustment factors for transition to widowhood

From section 5 it follows that the widowhood adjustment factors do not show the same relation as those for marriage and divorce. We present, therefore, both $\lambda^m$ and $\lambda^f$.

Fig. 3 illustrates the behaviour of the widowhood adjustment factors. In general, the tension for this phenomenon -- $(1 - \lambda^m)$, $(1 - \lambda^f)$ -- is greater than that for marriage (and thus substantially greater than that for divorce). It should be realized, however, that with marriage the initial inconsistency was divided equally between males and females. With transition to widowhood only the initial widowhood rates for one sex were adjusted, whereas the mortality rates for the other sex remained unchanged. A better comparison, therefore, would be between $|1 - \lambda^m|/2$ and $|1 - \lambda^f|/2$ for widowhood on the one hand and $\lambda^m$ and $\lambda^f$ for marriage (and divorce) on the other. In that case the tension for new widows is of the same order as that for marriage, while the tension for new widowers is about half as large.

The widowhood adjustment factors are almost without exception lower than 100%, which means that both male and female widowhood rates were initially too high. In the period 1980–2030, however, a decreasing trend can be observed in the tension, especially for males.

Again the behaviour of the adjustment factors is partly explained by the sex ratio of the population at risk. Over 80% of new widowers are aged between 50 and 85 years: the sex ratio of this group is shown, therefore, in the last two columns of table 3. The development of this variable is globally similar to that of the male adjustment factor for widowhood whereas the female factor is roughly inversely proportional to it. The relation between the adjustment factor and the sex ratio is weaker here, however, than in the cases of marriage and divorce. This is
Fig. 3. Adjustment factors: Transition to widowhood.

Source: CBS.
partly caused by the death patterns of married persons at higher ages, which were not taken into account here.

7. Concluding remarks

We have shown that the nuptiality model of the 1980-based population forecasts of the Netherlands' Central Bureau of Statistics gives a practical solution for the two-sex problem within the framework of nuptiality. It fulfills almost all the requirements mentioned in the literature for a realistic marriage model, i.e., availability, monotonicity, homogeneity, competition and symmetry between the sexes. Only the substitution requirement is not met. In addition to meeting these requirements, it also responds to some considerations that are often used by official forecasters: availability of historical observations for the model's parameters, controllability of those parameters, and avoidance – if possible – of iterative procedures. Nevertheless some remarks, suggested partly by the previous sections, can be made with respect to the CBS-model.

First of all, the present model might be extended with a device that computes the number of marriages by age-combination of the partners, \( N_{xy} \), given the number of males marrying, \( N_x \), and females marrying \( N_y \). The same applies to divorce and transition to widowhood. This device would be useful for two reasons:

- it would permit better comparison between the resulting CBS-marriage model and some other marriage models, namely those which compute marriages by age-combination (Henry's 'panmictic circles' model, the Shah and Giesbrecht model, McFarland's 'iterative adjustment' model, Schoen's 'harmonic means' model and finally Pollard's 'generalized harmonic means' model, see table 1);
- it would open up the possibility of computing an updated stock of married couples distinguished by age-combination of the partners. Of course, information on the initial stock (derived from a census for instance) and an assumption with respect to the decomposition of married external migrants by age-combination are necessary for this.

Such a provision consists of an algorithm that determines the cells of a table \( (N_{xy}^{(2)}) \) with given marginals (the \( N_x^{(2)} \) and \( N_y^{(2)} \)). Several options
for this can be studied, for instance the least-squares formulation and the maximum-entropy formulation. The former approach seems to be particularly worthwhile studying since an analytical solution exists [Hortensius (1979)]. In contrast, the maximum-entropy problem needs an iterative solution procedure (the method of bi-proportional adjustments) unless an unrealistic assumption of independence between male and female age-specific marriage behaviour is made [Reynolds (1977, p. 144)].

Before computing $N_{xy}$, the solution of any method should be put to the test of the requirements of section 2.

Secondly, Pollard’s ‘generalized harmonic means’ model deserves further exploration in view of its potential use for population forecasts. It computes the number of marrying couples with age-combination $(x, y)$ as follows:

$$N_{xy} = \left( v_{xy} \cdot M_x \cdot F_y \right) \left/ \left( \sum_x g_{xy} \cdot M_x + \sum_y h_{xy} \cdot F_y \right) \right.$$  

where $v_{xy}$ is a scaling factor which can be interpreted as a two-dimensional marriage rate, and the weights \{$g_{xy}$\} and \{$h_{xy}$\} reflect the mutual attractivity between the sexes. To infer the weights’ numerical value, use could be made of the preference index of independent partner choice described by Brackel (1970). In this index the usual percentages of marrying persons ($N_{xy}/\sum_y N_{xy}$ for males and $N_{xy}/\sum_x N_{xy}$ for females) are corrected with the accompanying sex ratios. Applied to marriages in the Netherlands between 1948 and 1965, it appears to give very regular patterns of partner choice for the different ages of males (or females) who make their choices. For a given age-combination, this index – denoted by $V_{xy}$ – is defined as

$$V^m_{xy} = \left( N_{xy} \cdot M_x \right) \left/ \left( F_y \cdot \sum_y N_{xy} \right) \right. \quad \text{for males, and}$$

$$V^f_{xy} = \left( N_{xy} \cdot F_y \right) \left/ \left( M_x \cdot \sum_x N_{xy} \right) \right. \quad \text{for females.}$$

These male and female indexes cannot be interpreted directly as the
required weights since $\sum_x V_{xy}^m \neq 1$ and $\sum_y V_{xy}^f \neq 1$. We define, therefore,

$$V_{xy}^m = V_{xy}^m / \sum_x V_{xy}^m \quad \text{for males, and}$$

$$V_{xy}^f = V_{xy}^f / \sum_y V_{xy}^f \quad \text{for females.}$$

Historical observations on $N_{xy}$, $M_{x}$ and $F_{y}$ open the possibility of computing time series for $V_{xy}^m$ and $V_{xy}^f$. Moreover, the substitutions $g_{xy} = V_{xy}^f$ and $h_{xy} = V_{xy}^f$ enable us to compute the scaling factors, $v_{xy}$. Having analyzed the historical series of the three sets of model parameters (the $v_{xy}$, the $g_{xy}$ and the $h_{xy}$), extrapolations of their numerical values could be made in order to forecast the number of marriages with the aid of Pollard's model.

Meanwhile the conclusion can be drawn that the CBS-model is a further step on the road leading to an ideal solution for the two-sex problem within the framework of official national forecasts. Compared with other models described by individual researchers, the CBS-model

- is more realistic (apart from Pollard's model, which has other drawbacks),
- responds better to the considerations used in official forecasts, and
- has a relatively simple structure.

Other models, used in official forecasts, perform less adequately when the requirements for a realistic model are considered.

Finally we should note that this article has dealt exclusively with formal marital states: never-married, married, divorced and widowed. Their relevance for describing real-life developments can, however, be questioned. Due to an increasing trend in cohabitation outside wedlock, the behaviour of unmarried persons (either never-married or formerly-married) will become more and more like the behaviour of formally married persons. This leads to the suggestion that the value of population forecasts containing formal marital status is likely to decrease in the future. In fact, for the reasons mentioned, the population forecasts of Sweden produced after 1970 no longer contain marital status.

This does not mean though that studies that aim at solving the two-sex problem related to marital status are without value. The
methodological results and insights can also be used in models with a slightly different definition of marital status. It is not just the formal status that should be considered, but rather (or perhaps also) the classification cohabiting/non-cohabiting. The methodology of two-sex models can probably also be applied in studies that try to cope with the emergence and spread of the less formal living arrangements observed in developed countries in the last decade.

References


Brass, W., 1974, Perspectives in population prediction, illustrated by the statistics of England and Wales, Journal of the Royal Statistical Society A 137, no. 4, 532-583.


Centraal Bureau voor de Statistiek (CBS), see entries under Netherlands, Centraal Bureau voor de Statistiek.

Daykin, C. and R. Leete, 1979, Projections of the population by marital condition, Statistical News 44, 44.17-44.24.

Denmark, Danmarks Statistik, 1970, Befolkningsprognoser 1970 (Population forecasts 1970), Statistiske Undersøgelser. no. 27 (Danmarks Statistik, Copenhagen).


Hortensius, D.G., 1970, De beste schatting voor het "binnenwerk" van een tabel (The best estimation of the cells of a table), Unpublished paper of the CBS, no. 9397-70-SA.

Keilman, N.W., 1982a, Nuptiality models and the two-sex problem in national population forecasts – with an emphasis on the Netherlands, NIDI-working paper, no. 34.

Keilman, N.W., 1982b, Het nuptialiteitsmodel in prognose 1980 (The nuptiality model in the 1980-based population forecasts), Maandstatistiek van de bevolking 30, no. 9, 16-36.


Netherlands, Centraal Bureau voor de Statistiek (CBS), 1979, De toekomstige ontwikkeling van de burgerlijke staat van mannen (Future trends in the marital status of men) (Staatsuitgeverij, The Hague).

N. Keilman / Nuptiality models and the two-sex problem

Netherlands, Centraal Bureau voor de Statistiek (CBS), 1984, Prognose van de bevolking van Nederland na 1980, deel II: Modelbouw en formulering van hypothesen (Forecasts of the population of the Netherlands after 1980, Part II: Model building and formulation of assumptions) (Staatsuitgeverij, The Hague).


Schoen, R., 1981, The harmonic mean as the basis of a realistic two-sex marriage model, Demography 18, no. 2, 201–216.


Spohr, H., 1972, A matrix model for the distribution of a population according to age, sex and marital status, Statistisk Tidsskrift III, 10(5), 367–376.


Widén, L., 1969, Methodology in population projection, Demographic Institute, Report no. 9 (Demographic Institute, University of Gothenburg, Gothenburg).