Appendix from Heathcote et al., “The Macroeconomic Implications of Rising Wage Inequality in the United States” (JPE, vol. 118, no. 4, p. 000)

This appendix is organized as follows. Section A describes the household surveys used in the paper, outlines the sample selection criteria, and compares the time trends for various empirical moments across surveys. Section B discusses in detail the specification, identification, and estimation of our statistical model for individual wage dynamics. Section C illustrates the numerical algorithm designed to compute the equilibrium of the model under both perfect foresight and myopic beliefs. Section D compares the evolution of inequality over the life cycle in the model and in the data. Section E portrays the evolution of some key cross-sectional moments in the perfect foresight model and in the myopic beliefs model.

A. Data Description

Our sources for individual- and household-level data are the Panel Study of Income Dynamics (PSID), the Current Population Survey (CPS), and the Consumer Expenditure Survey (CEX). Since all three data sets are widely used for microeconometric and, more recently, for quantitative macroeconomic research, we shall only briefly describe them here.

PSID.—The PSID is a longitudinal study of a representative sample of U.S. individuals (men, women, and children) and the family units in which they reside. Approximately 5,000 households were interviewed in the first year of the survey, 1968. From 1968 to 1997, the PSID interviewed individuals from families in the sample every year, whether or not they were living in the same dwelling or with the same people. Adults have been followed as they have grown older, and children have been observed as they advance through childhood and into adulthood, forming family units of their own (the “split-offs”). This property makes the PSID an unbalanced panel. Since 1997, the PSID has been biennial. The most recent year available, at the time of our analysis, is 2003. In 2003, the sample includes over 7,000 families. The PSID consists of various independent samples. We focus on the main and most commonly used, the so-called SRC (Survey Research Center) sample, which does not require weights, since it is representative of the U.S. population. Questions referring to income and labor supply are retrospective; for example, those asked in the 1990 survey refer to calendar year 1989.

CPS.—The CPS is a monthly survey of about 50,000 households conducted by the Bureau of the Census for the Bureau of Labor Statistics. The sample is selected to represent the civilian noninstitutional population. Respondents are interviewed to obtain information about the employment status of each member of the household 16 years of age and older. The CPS is the primary source of information on the labor force characteristics of the U.S. population. Survey questions cover employment, unemployment, earnings, hours of work, and other indicators. A variety of demographic characteristics are available, including age, sex, race, marital status, and educational attainment.

In our investigation, we use the Annual Social and Economic Supplement (so-called March Files) in the format arranged by Unicon Research. Computer data files are available only starting from 1968, and the latest year available, at the time of our research, was 2006. In all our calculations, we use weights. As for the PSID, questions referring to income and labor supply are retrospective.

CEX.—The CEX is a survey collecting information on the buying habits of American consumers, including data on their expenditures, income, and consumer unit (household) characteristics. The data are collected by the Bureau of Labor Statistics and used primarily for revising the consumer price index (CPI). The data are collected in independent quarterly interview and weekly diary surveys of approximately 7,500 sample households (5,000 prior to 1999).

We use the data set constructed from the original CEX data by Krueger and Perri (2006) and available on the authors’ Web sites. As is common in most of the previous research, their data use only the interview survey, which covers around 95 percent of total expenditures. Frequently purchased items such as personal care products and housekeeping supplies are reported only in the diary survey. The period covered by their data is 1980–2003. CEX data before 1980 are not comparable to those for the later years. Households that are classified as incomplete income respondents by the CEX and have not completed the full set of five interviews are excluded. We refer to Krueger and Perri (2006) for additional details on the data construction.

Variable definitions.—The calibration of the model and its evaluation are based on a set of cross-sectional first
and second moments constructed from both the PSID and the CPS. The key variables of interest are gross (i.e., before-tax) annual labor earnings, annual hours, hourly wages, and household consumption. We always construct hourly wages as annual earnings divided by annual hours worked. Nominal wages, earnings, and consumption are deflated with the CPI and expressed in 1992 dollars.

In the PSID, gross annual earnings are defined as the sum of several labor income components including wages and salaries, bonuses, commissions, overtime, tips, and so forth. Annual hours are defined as “annual hours worked for money on all jobs including overtime.”

In the CPS, gross annual earnings are defined as income from wages and salaries including pay for overtime, tips, and commissions. Annual hours worked are constructed as the product of weeks worked last year and hours worked last week. Until 1975, weeks worked are reported in intervals (0, 1–13, 14–26, ..., 50–52). To recode weeks worked for 1968–75, Unicon grouped the data in a few years after 1975 by intervals and computed within-interval means. These means from the later years were applied to the earlier years. The variable “hours worked last week at all jobs” is not ideal, but it is the only one continuously available since 1968 and comparable across years. Starting from the 1976 survey, the CPS contains a question on “usual weekly hours worked this year.” Even though levels differ, trends in mean hours, in their variance, and in the wage-hour correlation, which are the focus of our study, are virtually equivalent across the two definitions.

In the CEX, gross annual earnings refer to the amount of wage and salary income before deductions received in the past 12 months. Since we noticed that in the Krueger-Perri file there were some missing values for earnings, we merged earnings data from the CEX Public Release Member files (provided to us by Orazio Attanasio) into the Krueger-Perri file and use the former observations whenever earnings data were missing in the original Krueger-Perri file. Annual hours worked are defined as the product of “number of weeks worked full or part time by member in last 12 months” and “number of hours usually worked per week by member.”

Our benchmark definition for consumption is the same as Krueger and Perri’s, that is, the sum of expenditures on nondurables, services, and small durables (such as household equipment) plus imputed services from owned housing and vehicles. Each expenditure component is deflated by an expenditure-specific, quarter-specific CPI. Household expenditures are equivalized through the census scale. We label this variable ND+. See Krueger and Perri (2006) for further details.

Sample selection.—The objective of our sample selection is to apply exactly the same restrictions to the PSID, CPS, and CEX. We select married households with no missing values for gender, age, and education in which (1) the husband is between 25 and 59 years old, (2) the husband works at least 260 hours per year (a quarter part-time), (3) conditional on working, the hourly wage (annual earnings divided by annual hours) is above half of the minimum wage for both spouses, and (4) income is not from self-employment.

The marital status restriction is needed in order to be consistent with the theoretical model. Restriction 1 is imposed to avoid severe sample selection in the hours and wage data due to early retirement. Restriction 2 is imposed since one-quarter of part-time employment is our definition of labor force participation. Restriction 3 is imposed to reduce implausible outliers at the bottom of the wage distribution, which is particularly important since we use the variance of log wages as a measure of dispersion (see Katz and Autor [1999] for a discussion on the importance of trimming earnings data at the bottom). Restriction 4 is imposed since the presence of self-employment income makes it difficult to distinguish between the labor and the capital share, particularly in CPS and CEX data, and to deal with negative labor income.

Table A1 details the sample selection process in the three data sets, step by step. The final sample has 43,123 household/year observations in the PSID, 660,326 household/year observations in the CPS, and 21,556 household/year observations in the CEX.

Top coding.—After we impose our selection criteria, there are only six top-coded observations in the final PSID sample. Since we found that none of the statistics are affected by those few values, we did not make any correction for top-coded values. Roughly 2.1 percent of the earnings values in the final CPS sample are top coded. Top coding of earnings in the CPS changed substantially over the sample period. We follow Katz and Autor (1999) and multiply all top-coded observations by a factor equal to 1.5 up to 1996 and made no correction after 1996, when top-coded observations take on the average value of all top-coded observations, by demographic group instead of the threshold value. We tried with smaller and larger factors, and our findings remain robust. In the final CEX sample there are 362 top-coded observations, that is, around 1.7 percent of the total. Since the top coding changes virtually in the same ways as in the CPS, including the change of approach after 1996, we used the Katz-Autor strategy for the CEX as well.

Comparison across data sets.—Table A2 shows that—over the period in which they overlap (1980–2003)—the
three samples are remarkably similar in their demographic and education structure by gender. Also means of wages, earnings, and hours, by gender, are extremely similar in the three data sets. Finally, average food consumption expenditures in the PSID are very comparable to the CEX estimate.

College graduation data.—The data on college graduation that we use for the calibration of the model refer to the percentage of individuals who have completed college, by gender, age group, and year from 1940 to 2006. The source is table A.2 of the Educational Attainment section on the U.S. Census Bureau Web site, http://www.census.gov/population/www/socdemo.

Comparison between the PSID and the CPS.—Figures A1–A4 compare the time trends in some of the key moments of the joint distribution of hours, wages, and earnings in the CPS and the PSID. The plots show deviations from the means, with means reported in the legend. These four figures demonstrate that, overall, PSID and CPS data line up remarkably well along the vast majority of moments, in terms of both trends and levels. The PSID moments are more volatile because of the much smaller (by over a factor of 15) sample size.

We find that some discrepancies in the trends of a couple of the moments involving women arise toward the end of the sample, when the PSID data are still in “early release” format: the female college premium (fig. A1D) and the correlation between male and female log wages (fig. A4D). The trends for the moments involving men’s data are remarkably aligned across the two data sets. The trend in household log earnings inequality (fig. A4B)—a crucial moment in our study—is somewhat flatter in the PSID. Since the trends in male and female earnings dispersion broadly agree in the two surveys, the smaller increase in household earnings inequality in the PSID should be attributed to the decline in the correlation between male and female wages in the 1990s vis-à-vis the small rise of this correlation in the CPS over the same period. The trend of the variance of household log earnings in the CEX lies somewhere in between the PSID and the CPS. For example, over the last two decades of available data (1984–2003), the CEX data show a rise of 0.08 log points vis-à-vis an increase of 0.12 in the CPS and 0.05 in the PSID. See Heathcote, Perri, and Violante (2010) for further discussion on the comparability of trends in household earnings inequality across household surveys.

B. Identification and Estimation of the Wage Process

1. Statistical Model

In the paper we posit the following statistical model of the log wage residuals for individual \( i \) of age \( j \) at time \( t \). For all \( j, t \),

\[
y_{i,j,t} = \eta_{i,j,t} + v_{i,j,t} + \tilde{v}_{i,j,t},
\]

where \( \tilde{v}_{i,j,t} \sim N(0, \lambda^i) \) is a transitory (i.e., uncorrelated over time) component capturing measurement error in hourly wages, \( v_{i,j,t} \sim N(0, \xi_i) \) is a transitory component representing genuine individual productivity shock, and \( \eta_{i,j,t} \) is the persistent component of labor productivity. In turn, this persistent component is modeled as follows. For all \( j, t > 1 \),

\[
\eta_{i,j,t} = \rho \eta_{i,j-1,t-1} + \omega_{i,j,t},
\]

where \( \omega_{i,j,t} \sim N(0, \lambda^i) \). For all \( t \), at age \( j = 1 \), \( \eta_{i,j,t} \) is drawn from the time-invariant initial distribution with variance \( \lambda^i \). We assume that \( \omega_{i,j,t}, \tilde{v}_{i,j,t}, v_{i,j,t}, \) and \( \eta_{i,j,1} \) are orthogonal to each other and independent and identically distributed across individuals in the population.

The choice of this statistical model was guided by three considerations. First, the autocovariance function for wages (across ages) shows a sharp drop between lag 0 and lag 1. This pattern suggests the presence of a purely transitory component, which likely incorporates classical measurement error in wages. Second, there are strong life cycle effects in the unconditional variance of wages: in our sample, there is almost a twofold increase in the variance between age 25 and age 59. This suggests the existence of a persistent component in individual productivity. This component is modeled as an AR(1) process. Third, the nonstationarity of the wage process is captured by indexing the distributions for productivity innovations by year rather than by cohort, following the bulk of the literature, which argues that cohort effects are small compared to time effects in accounting for the rise in wage inequality in the United States (e.g., Heathcote, Storesletten, and Violante 2005).

For all \( j \), at \( t = 1 \) the distribution of labor productivity is assumed to be in its steady state with variances \( \{\lambda^i, \lambda_i, \lambda^t_i, \lambda^x \} \). This assumption is made to maintain consistency with the model’s solution and simulations. Note
that some of the variances \( \{ \lambda_1, \lambda_2^* \} \) are time varying whereas others \( \{ \lambda^*, \lambda^* \} \) are not. We restrict the variance of measurement error \( \lambda^* \) to be constant for identification purposes, and as explained in the main text, we use an external estimate to identify its size.

2. Identification: An Example

We now describe the identification procedure for the case in which \( t = 1, 2, 4 \) and \( j = 1, 2, 3 \). This is a useful example to illustrate our case in which, after a certain date, the PSID survey becomes biennial and data for some intermediate years (\( t = 3 \) in the example) are missing. Let \( \mathbf{T} \) denote the \((1 \times 10)\) parameter vector \( \{ \lambda_1, \lambda_2, \lambda_3^*, \lambda_4, \lambda_5^*, \lambda_6^*, \lambda_7^*, \lambda_8^*, \lambda_9^*, \lambda_10^* \} \). The key challenge is to identify parameters at date \( t = 3 \).

Define the theoretical moment

\[
m_{t+n}^{ij}(\mathbf{T}) = E(y_{ij,t+n|t+n}),
\]

The expectation operator is defined over all individuals \( i \) of age \( j \) at time \( t \) present both at \( t \) and at \( t+n \). In our simple example, we have a total of 12 such moments that we can construct from available data.

The covariance between periods \( t = 1 \) and \( t = 2 \) for the entry cohort of age \( j = 1 \) at \( t = 1 \) is

\[
m_{1,2}^1 = E[(\eta_{1,1} + v_{1,1})(\eta_{1,2} + v_{1,2})] = \rho \lambda^*,
\]

and the same covariance between periods \( t = 2 \) and \( t = 4 \) is

\[
m_{1,4}^1 = E[(\eta_{1,1} + v_{1,1})(\eta_{1,4} + v_{1,4})] = \rho^2 \lambda^*.
\]

This pair of moments identifies \( (\rho, \lambda^*) \).

At \( t = 1 \), the variance for the entry cohort

\[
m_{1,1}^1 = E[(\eta_{1,1} + v_{1,1})^2] = \lambda^* + \lambda_1^*
\]

identifies \( \lambda_1^* \) given knowledge of \( \lambda^* \).

From the variance of the age group \( j = 2 \) at time \( t = 1 \),

\[
m_{1,1}^2 = E[(\eta_{2,1} + v_{2,1})^2] = \rho^2 \lambda^* + \lambda_2^* + \lambda_1^*,
\]

we can identify \( \lambda_2^* \), given knowledge of the initial variance \( \lambda^* \) and of \( \lambda_1^* \).

At \( t = 2 \), the two variances for age groups \( j = 1, 2 \),

\[
m_{2,2}^2 = E[(\eta_{1,2} + v_{1,2})^2] = \lambda^* + \lambda_2^*,
\]

\[
m_{2,2}^2 = E[(\eta_{2,2} + v_{2,2})^2] = \rho^2 \lambda^* + \lambda_2^* + \lambda_2^*,
\]

identify \( \lambda_2^* \) and \( \lambda_2^* \).

At \( t = 4 \), we can construct the three variances

\[
m_{4,4}^1 = E[(\eta_{4,4} + v_{4,4})^2] = \lambda^* + \lambda_4^*,
\]

\[
m_{4,4}^2 = E[(\eta_{2,4} + v_{2,4})^2] = \rho^2 \lambda^* + \lambda_4^* + \lambda_4^*,
\]

\[
m_{4,4}^3 = E[(\eta_{3,4} + v_{3,4})^2] = \rho^4 \lambda^* + \rho^3 \lambda_4^* + \lambda_4^* + \lambda_4^*.
\]

As usual, the variance of the entrant cohort identifies \( \lambda_1^* \), given knowledge of the initial variance \( \lambda^* \). Comparing the variance of new cohorts with the variance of age 2 cohorts identifies \( \lambda_2^* \), the variance of the current persistent shock. Finally, the variance of the age \( j = 3 \) cohort contains the variance of the persistent shock that hit at the previous date, and this allows identification of \( \lambda_3^* \).

Two remarks are in order. First, we can identify \( \lambda_1^* \) in spite of a lack of data for \( t = 3 \) because the \( \omega \) shock hitting individuals at time \( t = 3 \) persists into \( t = 4 \), a date for which observations are available. Thus, comparing wage dispersion between a new cohort and an old cohort at \( t = 4 \) allows us to identify \( \lambda_1^* \) since there are no cohort effects. Second, in general, one cannot separately identify persistent and transitory shocks in the last year of the sample. Here we can, thanks, once again, to the assumption of no cohort effects in the initial variance \( \lambda^* \).

The only parameter left to identify is \( \lambda_2^* \). Transitory shocks at \( t = 3 \) do not show up in moments at any other \( t \), and thus we need to impose a restriction to complete our identification. There are several possible choices. We opt for assuming that the cross-sectional variance of wages in the population in the missing years is a weighted average
of the variance in the year before and in the year after. In our specific example, if we let \( \tilde{m}_{j,t} \) be the cross-sectional variance of log wages at time \( t \), then we assume that \( \tilde{m}_{j,3} = (\tilde{m}_{2,2} + \tilde{m}_{4,4})/2 \). Given our knowledge of all the parameters \( \{ \rho, \lambda^L, \lambda^U \} \), one can reconstruct the cross-sectional variance component due to the cumulation of the persistent shocks up to \( t = 3 \). The difference between the total variance and the part due to persistent shocks identifies residually the transitory component \( \lambda^T \).

3. Estimation

**Parameter vector.**—We have available survey data for 1967–96, 1998, 2000, and 2002. Even though, theoretically, the variance of the persistent shocks \( \lambda^T \) is identified in the missing years, in practice the fact that the lack of data occurs toward the end of the sample substantially reduces the amount of information available to estimate such parameters. Moreover, as explained, identification in the missing years hinges on the no–cohort effects assumption. Therefore, we choose to take a cautious approach and estimate \( \lambda^T \) only for those years when data are available. In simulating the model, we assume that the variance of the persistent shocks for the missing years is a weighted average of the two adjacent years.

Moreover, as we have explained above, separating the variances of persistent and transitory shocks in the last year of the sample hinges also on the, arguably restrictive, assumption of no cohort effects. Therefore, we choose not to estimate these two variances for 2002, but rather we use the 2002 survey only to improve our estimation of the structural variances up to 2000 (by constructing covariances between 2002 and the previous years). To sum up, we estimate \( \rho, \lambda^L, \lambda^U \), and \( \{ \lambda^L_{1967}, \ldots, \lambda^L_{1998}, \lambda^L_{2000}; \lambda^U_{1967}, \ldots, \lambda^U_{1998}, \lambda^U_{2000} \} \) for a total of \( L = 66 \) parameters. Denote by \( \mathbf{Y} \) the \((L \times 1)\) parameter vector.

**Empirical moments.**—Every year \( t \), we group individuals in the sample into 10-year adjacent age cells indexed by \( j \), the first cell being age group 29 containing all workers between 25 and 34 years old, up until the last cell for age group 54 with individuals between 50 and 59. Our sample length and age grouping imply \( T = 33 \) and \( J = 26 \). Let \( m_{j,t}(\mathbf{Y}) \) be the theoretical covariance between wages in the two age group/year cells determining the triple \((j, t, n)\), exactly as in (A1). For every pair \((j, t)\), let \( \hat{n}(j, t) \) be the maximum number of moments involving individuals of age \( j \) at time \( t \) that can be constructed from the sample (taking into account the fact that some years are missing).

The moment conditions used in the estimation are of the form

\[
E(t_{i,j,t,n})[\tilde{y}_{i,j,t} \cdot \tilde{y}_{i,j+n,t+n} - m_{i,t+n}(\mathbf{Y})] = 0,
\]

where \( t_{i,j,t,n} \) is an indicator function that equals one if individual \( i \) has observations in both periods/age groups determined by \((j, t, n)\) and zero otherwise. The empirical counterpart of these moment conditions becomes

\[
\hat{m}_{i,t+n} - m_{i,t+n}(\mathbf{Y}) = 0,
\]

where

\[
\hat{m}_{i,t+n} = \frac{1}{I_{i,t,n}} \sum_{j=1}^{I_{i,t,n}} y_{i,j,t} \cdot \tilde{y}_{i,j+n,t+n}
\]

is the empirical covariance between wages for individuals of age \( j \) at time \( t \) and wages of the same individuals \( n \) periods later. Note that \( I_{i,t,n} = \sum_{j=1}^{I_{i,t,n}} t_{i,j,t,n} \), since not all individuals contribute to each moment.

**Estimator.**—The estimator we use is a minimum distance estimator that solves the following minimization problem:

\[
\min_{\mathbf{Y}} [\hat{\mathbf{m}} - \mathbf{m}(\mathbf{Y})] \mathbf{W}^{-1} [\hat{\mathbf{m}} - \mathbf{m}(\mathbf{Y})], \tag{A2}
\]

where \( \hat{\mathbf{m}} \) and \( \mathbf{m}(\mathbf{Y}) \) are the vectors of the stacked empirical and theoretical covariances with dimension \( N = \sum_{i=1}^{I_{i,t,n}} \sum_{j=1}^{I_{i,t,n}} \hat{n}(j, t) \), and \( \mathbf{W} \) is an \((N \times N)\) weighting matrix. In our estimation, \( N = 9,634 \).

To implement the estimator, we need a choice for \( \mathbf{W} \). The bulk of the literature follows Altonji and Segal (1996), who found that in common applications there is a substantial small-sample bias in the estimates of \( \mathbf{T} \); hence using the identity matrix for \( \mathbf{W} \) is a superior strategy to using the optimal weighting matrix characterized by Chamberlain (1984). With this choice, the solution of equation (A2) reduces to a nonlinear least-squares problem.

Standard errors are computed by block bootstrap, using 500 replications. Bootstrap samples are drawn at the household level, with each sample containing the same number of households as the original sample. Resulting
standard errors thus account for arbitrary serial dependence, heteroskedasticity, and additional estimation error induced by the use of residuals from the first-stage regressions.

The estimates are plotted in figure 3 in the paper and are also reproduced in table A3.

C. Numerical Algorithm

First we describe how we pick the sequence for the scaling variable $Z_t$. Then we review the details of the timing assumptions. Next we describe how we solve for decision rules and how we compute steady states, and we calibrate the parameter set endogenously. Finally, we describe how we handle the transition phase in which the wage structure evolves.

1. $Z$ Sequence

The assumption that the economy is open and faces a constant pretax world interest rate $r$ implies a constant capital to aggregate effective labor ratio since

$$r = (\alpha Z_t K_t^{-1} H_t^{-1}) = \frac{K_t}{H_t} = Z_t^{\alpha/(\alpha-1)} \left[ \frac{1}{\alpha} (r + \delta)^{\alpha/(\alpha-1)} \right]. \quad (A3)$$

When the expression for $K_t/H_t$ is substituted into the equilibrium expressions for prices $p$, defined in equation (11), it is clear that prices for different types of labor are functions of the technology parameters $\{Z_t, \lambda^t, \lambda^v\}$ and of the aggregate quantities of the different types of labor supplied, $H_t = [H_t^{m,h}, H_t^{v,h}, H_t^{w,h}, H_t^{f,h}]$. We denote these functions by $p(Z_t, \lambda^t, \lambda^v, H_t)$.

Let $H(\lambda^t, \lambda^v, H_t)$, as defined in (1), be the function defining aggregate effective labor supply. The path for $Z_t$ is assumed such that, given the initial steady-state quantities of labor input, $H_t$, average individual after-tax earnings for agents working age is equal to one at each date. This implies

$$\frac{1}{2} \int_{z \geq 0} (1 - t^n)(1 - \alpha) Z_t^{\alpha/(\alpha-1)} \left[ \frac{1}{\alpha} (r + \delta)^{\alpha/(\alpha-1)} \right] H(\lambda_t^v, \lambda_t^h, H_t) = 1. \quad (A4)$$

2. Timing

Prior to 1965 we assume that the economy is in an initial steady state in which parameters $\lambda_t$ and prices $p_t$ are constant. In 1965 new information is revealed and agents revise expectations: instead of thinking that $\lambda_t$ and $p_t$ will persist forever, they now foresee the exact time-varying future paths for $\{\lambda_t\}_{t=1965}^{2000}$ and $\{p_t\}_{t=1965}^{2000}$.

The first and last years for which we estimate using our PSID sample are 1967 and 2000 (see Sec. B). The path for $\lambda_t$ is time varying for $1967 \leq t \leq 2000$ in such a way that the wage structure in the model evolves precisely as in the data over this period. Prices are time varying between 1965 and 1967, even though all technology parameters in $\lambda_t$ are constant, because agents adjust their education and labor supply decisions in anticipation of future changes in the wage structure. This affects the relative supplies of different types of labor and, thus, relative prices.

We assume that by 2021, prices for the four types of labor have converged to their final steady-state values, denoted $p_{t=2021}$. These prices are such that the model replicates the observed college premium and gender gap for 2002, the last year of our PSID sample. Adjustment to the final steady state is slow because it takes time for the educational composition of the workforce to adjust to the final steady-state values, and while this adjustment is taking place, the relative supplies of different types of labor are changing.

We need to make assumptions for the path for $\lambda_t$ during the transition period. For $t > 2000$ we assume that the wage risk parameters $(\lambda^t, \lambda^v)$ are constant and equal to the estimated values for 2000, denoted $(\lambda^t_{2000}, \lambda^v_{2000})$. We assume that the path for $\lambda^h_t$ over the period $2000 < t < 2021$ is such that the relative price $p_{t=2000}^{m,h}/p_{t=2001}^{m,h}$ is constant at the value that replicates the observed male college premium in 2002. The path for $\lambda^h_t$ is such that the model gender premium is equal at each date to that observed in 2002. Note that these assumptions imply that both $(\lambda^h_t, \lambda^v_t)$ and $p_t$ are time varying between 2000 and 2021.

Recall that some parameters are calibrated internally, as described in Section IV. In addition to all these parameter
values, agents need to know the sequences for equilibrium prices \{p_\tau\} in order to solve their problems. In practice, we proceed as follows. We first solve for initial and final steady states to set the internally calibrated time-invariant parameter values and the steady-state values for the technology parameters and to solve for the associated steady-state prices. Given these parameters and prices, we then solve for the transition in order to fill in the sequence \{Z_\tau, \lambda^m_\tau, \lambda^f_\tau\}_{\tau=1965}^{1980} and \{p_\tau\}_{\tau=1980}^{2020}.

3. Decision Rules

The household decision problems are standard finite-horizon dynamic programming problems. We start in the last period of life, \(J\), and work backward by age. Solving for decision rules in the retirement stage of the life cycle is relatively simple since there is no labor market risk and the only decision for the household is how to divide income between consumption and savings. Solving for decisions in the working stage of the life cycle is more challenging computationally because the state space is large: for each household type and for each age, we need to keep track of household wealth and of the persistent and transitory stochastic components of the wage for both the husband and the wife.

We assume that the transitory shocks and the innovations to the persistent component can each take two values, but we allow the cumulated value of the persistent component to be continuous. At each age, we approximate decision rules for consumption using piecewise trilinear functions defined over wealth and the male and female persistent components (one function for each possible combination of age and mix of education and transitory shocks within the household). We make our grid finer at low levels of wealth and allow the number of grid points for persistent shocks to increase with age, given that our estimates indicate a high value for the autoregressive coefficient \(\rho\). We use the “endogenous grid” method for Euler equation iteration, as described by Carroll (2006).

The key idea is that at each point in the state space, one considers a grid over current shocks and next-period wealth and then uses the intertemporal first-order condition to compute implied current wealth. This can be accomplished very quickly because it avoids having to solve a nonlinear equation (the Euler equation) numerically. The method is also well suited to dealing with borrowing constraints: setting the value for next-period assets at the constraint determines the “endogenous” value for current assets below which the constraint must bind. As explained by Barillas and Fernández-Villaverde (2006), it is straightforward to extend this method to the case in which labor supply is endogenous, as in our economy.

For prime-working-age households, the actual number of points on our grid for individual states is 253,920; we solve for optimal consumption and labor supply choices at each of these. This number corresponds to four possible education pairs, 30 values for household wealth, 23 values each for the persistent component for the husband and the wife, and two values each for the transitory component. Of course, this gives us decisions for only one cohort at one particular age: we ultimately need to compute decisions for 75 ages for each of 93 different cohorts. The total number of points at which we compute decisions is thus approximately 800 million. To simulate the economy, we simulate 20,000 households for each education composition and for each cohort and then create cross-sectional moments by weighting appropriately by education (given enrollment rates and matching probabilities) and by cohort (given survival probabilities).

4. Steady States and Internal Calibration

It is useful to postpone determining \((\bar{\kappa}_m, \bar{\kappa}_f, v^m, v^f)\) and to simply assume that there exist values for these parameters that deliver the target graduation rates by gender in the two steady states, \((q^m_*, q^f_*, q^m_1, q^f_1)\). This way, in what follows, we can avoid solving for the education decisions.

We guess values for parameters \((Z_\tau, Z^m_\tau, Z^f_\tau, \lambda^m_\tau, \lambda^f_\tau, \beta, a, \psi, b)\) and equilibrium prices \((p^{m,h}_*, p^{m,h}_1)\). Given the production technology and the calibration strategy, these guesses are sufficient to construct the remaining steady-state prices as follows.

First, since the selection issue for men is assumed to be minor, given guesses for \((p^{m,h}_*, p^{m,h}_1)\) and the observed college premia in 1967 and 2002, we immediately have \((p^{m,l}_*, p^{m,l}_1)\). For example, if \(\Pi_*\) is the ratio between the average wage of male college graduates relative to male high school graduates at the start of the sample, we set

\[
p^{m,l}_* = \frac{p^{m,h}_*}{\Pi_*}.
\]
Second, given the guesses for \((X_i^C, X_i^L)\), from (11), we can recover steady-state prices for female labor. For example, in the first steady state,

\[
p_{m,h}^i = p_{m,i}^i = \frac{1}{p_{m,i}^j} = \frac{1 - X_i^C}{X_i^L}. \tag{A6}
\]

In the initial steady state, the solution to the household’s problem delivers a set of decision rules as well as associated value functions \(V_i^j\), and expected start of working life values \(V_i^0\). Then we move to the matching stage. Given the enrollment rates \((q_i^m, q_i^l)\) and the target degree of assortative matching \(q_i\), we can compute matching probabilities \((\pi_i^m, \pi_i^l)\) using the equation defining the correlation between education levels within the household (7) and the consistency conditions of the form (5). The same logic applies to the final steady state.

At this point, we can simulate the economy to compute cross-sectional moments. We do two simulations, one for each steady state, and compute the set of statistics that correspond to our target calibration moments and equilibrium conditions. Since technology parameters and equilibrium prices vary across steady states, so do household decisions and cross-sectional moments. We want to calibrate the model economy to replicate certain features of the U.S. economy (e.g., mean hours worked) on average across the sample period. We implement this by computing average empirical target statistics across the sample period and searching for parameter values such that these are reproduced in the model when averaging across the two steady-state simulations.

To verify that the guesses for prices \((p_{m,h}^i, p_{m,l}^i)\) are in fact consistent with equilibrium requires knowledge of each argument of the equilibrium pricing functions, since we need to verify that \(p_{m,l}^i = p(Z_i, X_i^L, X_i^L, H_i)\). The vector of aggregate effective hours worked by each type of labor, \(H_i\), can be computed within the simulation. The technology parameters \(Z_i\) and \(X_i^L\) are part of the guess. However, we still need to compute the implied value for \(X_i^L\). Since, without selection, the observed skill premium \(\Pi_i\), is equal to the price ratio \(p_{m,l}^i/p_{m,l}^j\), we can compute \(X_i^L\) using the ratio of the expressions for the marginal products of male skilled and unskilled labor:

\[
\Pi_i = \frac{p_{m,h}^i}{p_{m,l}^i} = \frac{X_i^L}{1 - X_i^L} \implies X_i^L = \frac{\Pi_i}{\Pi_i + c_i}. \tag{A7}
\]

where

\[
c_i = \left[\frac{X_i^L H_i^m + (1 - X_i^L) H_i^l}{X_i^L H_i^l + (1 - X_i^L) H_i^m}\right]^{1/\theta}. \tag{A8}
\]

To recap, we guess a vector \((Z_i, Z_l, X_i^L, X_i^C, \beta, a, \psi, b, p_{m,h}^i, p_{m,l}^i)\), solve the model, and check whether or not the corresponding eight target calibration moments and two equilibrium conditions for prices are satisfied. The targets (see Sec. IV) (i) are average no behavioral response after-tax earnings equal to one in each steady state, (ii) replicate the gender premium in each steady state, (iii) replicate the average wealth to average income ratio, (iv) replicate realistic redistribution from the pension system (see below). If any of these conditions are not satisfied at the initial guess, we use multidimensional Newton-Raphson methods to update the guess. Then we resolve decision rules and simulate again, iterating in this fashion to convergence.

One parameter (and corresponding calibration target) requires more discussion: the value for the lump-sum transfer \(b\) received by all retirees. Recall that the goal is to set \(b\) so that the dispersion of discounted lifetime earnings plus pension income in the final steady state of our economy is the same as in an alternative economy featuring the actual U.S. Old-Age Insurance system.

To compute U.S. Social Security System benefits for a model household, we first compute average monthly earnings throughout working life (AIME). The AIME value is the input for a formula that calculates Social Security benefits as follows: 90 percent of AIME up to a first threshold (bend point) equal to 38 percent of average individual earnings, plus 32 percent of AIME from this bend point to a higher bend point equal to 159 percent of average earnings, plus 15 percent of the remaining AIME exceeding this last bend point. These are the actual bend points of the U.S. Social Security System in 2007.

Once we have calculated the monthly Social Security benefits of the husband and wife within the couple, we compute household benefits \(b_{HS}^i\) as the maximum between (a) the sum of the two benefits and (b) 1.5 times the highest of the two benefits. This rule is called the spousal benefit rule in the U.S. pension system. We assume that pension benefits in the U.S. system are taxed at half the labor income tax rate, which is a reasonable approximation. We repeat this procedure for every household in the artificial panel and then compute the within-cohort variance.
of the log of lifetime household earnings plus Social Security. Next, we perform a similar calculation given our alternative hypothetical pension system characterized by a lump-sum pension, \( b \). The desired value for \( b \) is the value that equates the dispersion in discounted lifetime income across the two systems.

The last step in the steady-state stage of the solution method is to compute the education cost distribution parameters \((\bar{R}^g, \bar{R}^m, \xi^g, \xi^m)\). We do this by first using equation (4) to compute expected values of education by household type in both steady states. We then solve a simple set of four nonlinear equations of the form (3), one for each gender and for each steady state, to compute the four utility cost parameters that are consistent with the empirical enrollment rates. This procedure allows us to perfectly replicate the target graduation rates by gender in 1967 and 2002.

5. Transitional Dynamics

Once all parameter values are known, it remains to solve for prices from 1965 (when information about future changes in the wage structure is revealed) to 2020 (the last year of transition).

We first guess sequences \( \{p_{t,m}^{1929}\}_{t=1965}^{1967}, \{p_{t,m}^{1966}\}_{t=1965}^{1967}, \) and \( \{\lambda_t^g\}_{t=1965}^{1967} \). Given these guesses, we can construct prices for each type of labor at each date as follows: (i) for \( t < 1965 \), prices are given by \( p_v \); (ii) for \( 1965 \leq t < 1967 \), prices for male labor are given by the guess \( (p_{t,m}^{1965}, p_{t,m}^{1966}) \) and prices for female labor can be determined given \( \lambda_t^f = \lambda_t^g \) using the expression for the gender premium (A6); (iii) for \( 1967 \leq t < 2020 \), \( p_{t,m}^{1967} \), can be readily computed given the guess \( p_{t,m}^{1967} \) and the empirical college premium by applying (A6), whereas prices for female labor are implied by the guess for \( \lambda_{t,m}^f \) and equation (A7); (iv) for \( t \geq 2021 \), prices are given by \( p_{t,m} \).

Given all the prices, we solve each cohort’s problem, beginning with the cohort that enters the labor force in year \( t = 1965 - j = 1929 \) and ending with the cohort that enters the labor force in year 2021. We then compute cohort-specific expected values \( \lambda \) for each household type.

To compute cross-sectional moments and aggregate effective hours for each type of labor, we need the education composition of the workforce at each date. Recall that in each year we set the gender-specific means of the education cost distributions so as to exactly replicate the empirical graduation rates. Given rates \( q_t^g \) and the target degree of assortative matching \( \rho^g \), we compute matching probabilities \( \pi^g_t \) and thus the education composition for the year \( t \) cohort. Given these probabilities and the values \( \lambda \), we can calculate expected education values \( M^g_t \). Finally, we use the equilibrium schooling condition (3) to check whether the guessed enrollment rates are correct. Enrollment rates allow us to derive the household composition for each cohort.

Once we have decision rules and household composition for all cohorts, we can simulate the economy and compute time series for the model-implied gender premium and compare this to its empirical counterpart. This is the basis for updating the sequence \( \lambda_t^g \).

To establish whether the guesses for prices are consistent with equilibrium, we need to check whether the guessed prices are equal to those implied by applying the functions \( p(Z, \bar{R}^g, \xi, H) \). To check this, we need the time series \( \{Z_t\} \) and \( \{\lambda_t^g\} \) in addition to aggregate effective hours for each type of labor, \( \{H_t\} \). We generate series for \( \{H_t\} \) by simulation and use these series, along with the (guessed) sequences for \( \{\lambda_t^g\}, \{p_{t,m}^{1967}\}, \) and \( \{p_{t,m}^{1967}\}, \) to compute a time series \( \lambda_t^g \) using the time \( t \) equivalent of equations (A7). We then use equation (A4) to construct a time series \( \{Z_t\} \) such that in the hypothetical counterfactual that \( H_t = H_t \), for all \( t \), average individual earnings would be time invariant. We are then in a position to compute the model-implied equilibrium price sequences.

After comparing the guessed price sequences to the model-implied price sequences, we update our guesses. We then solve again the problem for all cohorts, resimulate, and check again for market clearing in all labor markets and for the appropriate gender wage gap, iterating until convergence.

Finally, we compute how the means of the education cost distribution must evolve over time to replicate observed college completion. Given the probabilities \( \pi^g_t \) and the values \( \lambda^g_t \), we can calculate expected education values \( M^g_t \). We then use the equilibrium schooling condition (3) to reverse-engineer the mean education costs that generate \( q_t^g \) given \( M_t^g \).

6. The Myopic Version of the Model

The numerical approach for the myopic version of the model differs slightly from the perfect foresight case described above. The basic structure is the same: we guess sequences of prices and then solve for decisions cohort by cohort. To solve for decisions of a cohort that reaches the maximum age \( J \) in year \( t + J \), we proceed recursively, starting from the end of the life cycle. In year \( t + J \), this cohort’s consumption is a function of year \( t + J \) prices. However, in year \( t + J - 1 \), this cohort assumes that year \( t + J - 1 \) prices will prevail in both years \( t + J - 1 \) and \( t + J \). Thus, to solve for this cohort’s consumption, as a function of the state variables, at age \( J - 1 \), we first generate an
anticipated age $J$ consumption function given year $t + J - 1$ prices and then use the intertemporal Euler equation with this anticipated age $J$ consumption function as an input to solve for the true age $J - 1$, date $t + J - 1$ decision rule. In this fashion, we move step by step backward through the life cycle to compute decision rules at earlier and earlier ages.

D. Model-Data Comparison over the Life Cycle

Although the focus of the exercise is on changes in cross-sectional inequality over time, it is useful to check the performance of the model along the life cycle dimension. Here we report the life cycle dynamics in the mean and variance of household earnings and consumption for the cohort that is 25–29 years old in 1980—the initial year of the consumption sample—and we compare it to the 1980 cohort in the model. See figure A5. The model somewhat overestimates the rise in mean household earnings, mostly after age 40, but it replicates the other life cycle facts remarkably well.

E. Comparison between the Economy with Perfect Foresight and the Economy with Myopic Beliefs

Figures A6–A8 plot the key cross-sectional moments analyzed in Section V.A of the paper. The figures contain the time series of the various moments in the economy with perfect foresight and in the economy with myopic beliefs. As we emphasized in the main text, the evolution of these moments in the two economies is remarkably similar, as evident from figures A7 and A8.

The main reason for this close correspondence is that in both economies the college enrollment rate by gender is the same since we replicate the empirical series by design. In order to match these series, the underlying cost distributions must differ across models, as documented in figure A6. In particular, in the mid-1970s, when the skill premium was temporarily low, the myopic model calls for a substantially lower utility cost of education than what was estimated in the perfect foresight economy, since it has to reproduce the same fraction of college graduates with much worse expectations about the returns to college education. The education cost for women is estimated to be lower than for men also early in the period, when few women went to college, because the large gender wage and hours gaps kept monetary returns to college education low for women. Finally, note that, in both steady states, the average cost is the same under the two expectation models: in the steady state the future is like the present, which explains why both lines start and end together.

References


Fig. A1.—Comparison between CPS and PSID samples of married households
Fig. A2.—Comparison between CPS and PSID samples of married households
Fig. A3.—Comparison between CPS and PSID samples of married households
Fig. A4.—Comparison between CPS and PSID samples of married households
Fig. A5.—Model-data comparison: evolution of household earnings and equivalized consumption (mean and variance of the logs) over the life cycle of the cohort that is 25–29 years old in 1980.

Fig. A6.—Average education cost for perfect foresight and myopic beliefs
Fig. A7.—Cross-sectional hours distribution: comparison between perfect foresight and myopic beliefs

Fig. A8.—Further comparison between perfect foresight and myopic beliefs
### Table A1. Sample Selection in PSID, CPS, and CEX

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial sample (married households)</td>
<td>...</td>
<td>68,860</td>
<td>...</td>
</tr>
<tr>
<td>Age of husband between 25 and 59</td>
<td>10,274</td>
<td>58,586</td>
<td>354,256</td>
</tr>
<tr>
<td>Hours worked of husband ≥ 260</td>
<td>1,927</td>
<td>56,659</td>
<td>87,466</td>
</tr>
<tr>
<td>Wage husband &gt; half minimum wage</td>
<td>1,215</td>
<td>55,444</td>
<td>87,466</td>
</tr>
<tr>
<td>Wage wife &gt; half minimum wage</td>
<td>1,723</td>
<td>53,721</td>
<td>32,021</td>
</tr>
<tr>
<td>Income husband not from self-employment</td>
<td>8,784</td>
<td>44,937</td>
<td>28,330</td>
</tr>
<tr>
<td>Income wife not from self-employment</td>
<td>1,814</td>
<td>43,123</td>
<td>12,216</td>
</tr>
</tbody>
</table>

### Table A2. Comparison across PSID, CPS, and CEX Samples

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average age of men</td>
<td>39.15</td>
<td>40.94</td>
<td>41.26</td>
</tr>
<tr>
<td>Average age of women</td>
<td>37.0</td>
<td>38.62</td>
<td>39.2</td>
</tr>
<tr>
<td>Fraction of male college graduates</td>
<td>.31</td>
<td>.31</td>
<td>.31</td>
</tr>
<tr>
<td>Fraction of female college graduates</td>
<td>.24</td>
<td>.24</td>
<td>.24</td>
</tr>
<tr>
<td>Average earnings of men</td>
<td>39,674</td>
<td>40,182</td>
<td>38,441</td>
</tr>
<tr>
<td>Average earnings of women</td>
<td>15,097</td>
<td>14,199</td>
<td>15,570</td>
</tr>
<tr>
<td>Average hours worked by men</td>
<td>2,223</td>
<td>2,252</td>
<td>2,225</td>
</tr>
<tr>
<td>Average hours worked by women</td>
<td>1,258</td>
<td>1,227</td>
<td>1,286</td>
</tr>
<tr>
<td>Average hourly wage of men (1992 $)</td>
<td>18.09</td>
<td>18.44</td>
<td>17.49</td>
</tr>
<tr>
<td>Average hourly wage of women (1992 $)</td>
<td>9.55</td>
<td>9.33</td>
<td>9.83</td>
</tr>
<tr>
<td>Average household earnings (1992 $)</td>
<td>54,772</td>
<td>54,381</td>
<td>54,011</td>
</tr>
<tr>
<td>Average food consumption (1992 $)</td>
<td>4,626</td>
<td>...</td>
<td>4,082</td>
</tr>
</tbody>
</table>
Table A3: Parameter Estimates of Wage Process

<table>
<thead>
<tr>
<th>Persistent Component</th>
<th>Transitory Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>.9733 (.0066)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>.1242 (.0067)</td>
</tr>
<tr>
<td>$\lambda_{1967}$</td>
<td>.0076 (.0024)</td>
</tr>
<tr>
<td>$\lambda_{1968}$</td>
<td>.0151 (.0077)</td>
</tr>
<tr>
<td>$\lambda_{1969}$</td>
<td>.0079 (.0039)</td>
</tr>
<tr>
<td>$\lambda_{1970}$</td>
<td>.0087 (.0044)</td>
</tr>
<tr>
<td>$\lambda_{1971}$</td>
<td>.0074 (.0043)</td>
</tr>
<tr>
<td>$\lambda_{1972}$</td>
<td>.0219 (.0067)</td>
</tr>
<tr>
<td>$\lambda_{1973}$</td>
<td>.0065 (.0038)</td>
</tr>
<tr>
<td>$\lambda_{1974}$</td>
<td>.0030 (.0022)</td>
</tr>
<tr>
<td>$\lambda_{1975}$</td>
<td>.0094 (.0050)</td>
</tr>
<tr>
<td>$\lambda_{1976}$</td>
<td>.0067 (.0042)</td>
</tr>
<tr>
<td>$\lambda_{1977}$</td>
<td>.0083 (.0038)</td>
</tr>
<tr>
<td>$\lambda_{1978}$</td>
<td>.0132 (.0047)</td>
</tr>
<tr>
<td>$\lambda_{1979}$</td>
<td>.0075 (.0039)</td>
</tr>
<tr>
<td>$\lambda_{1980}$</td>
<td>.0171 (.0052)</td>
</tr>
<tr>
<td>$\lambda_{1981}$</td>
<td>.0118 (.0052)</td>
</tr>
<tr>
<td>$\lambda_{1982}$</td>
<td>.0179 (.0046)</td>
</tr>
<tr>
<td>$\lambda_{1983}$</td>
<td>.0180 (.0066)</td>
</tr>
<tr>
<td>$\lambda_{1984}$</td>
<td>.0208 (.0061)</td>
</tr>
<tr>
<td>$\lambda_{1985}$</td>
<td>.0158 (.0058)</td>
</tr>
<tr>
<td>$\lambda_{1986}$</td>
<td>.0249 (.0053)</td>
</tr>
<tr>
<td>$\lambda_{1987}$</td>
<td>.0045 (.0039)</td>
</tr>
<tr>
<td>$\lambda_{1988}$</td>
<td>.0226 (.0048)</td>
</tr>
<tr>
<td>$\lambda_{1989}$</td>
<td>.0144 (.0055)</td>
</tr>
<tr>
<td>$\lambda_{1990}$</td>
<td>.0054 (.0047)</td>
</tr>
<tr>
<td>$\lambda_{1991}$</td>
<td>.0182 (.0058)</td>
</tr>
<tr>
<td>$\lambda_{1992}$</td>
<td>.0078 (.0054)</td>
</tr>
<tr>
<td>$\lambda_{1993}$</td>
<td>.0303 (.0072)</td>
</tr>
<tr>
<td>$\lambda_{1994}$</td>
<td>.0087 (.0055)</td>
</tr>
<tr>
<td>$\lambda_{1995}$</td>
<td>.0114 (.0063)</td>
</tr>
<tr>
<td>$\lambda_{1996}$</td>
<td>.0163 (.0069)</td>
</tr>
<tr>
<td>$\lambda_{1997}$</td>
<td>.0190 (.0049)</td>
</tr>
<tr>
<td>$\lambda_{1998}$</td>
<td>.0219 (.0068)</td>
</tr>
<tr>
<td>$\lambda_{1999}$</td>
<td>.0216 (.0049)</td>
</tr>
<tr>
<td>$\lambda_{2000}$</td>
<td>.0212 (.0079)</td>
</tr>
</tbody>
</table>

Note.—Minimum distance estimates of the parameters of the wage process in eq. (9). Standard errors (in parentheses) are obtained by blocks-bootstrap based on 500 replications. See Sec. B for details.