Private Affluence and Public Poverty: A Theory of Intergenerational Predation through Debt.*

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Abstract

We propose a dynamic politico-economic theory of public good provision and debt accumulation whose driver is the intergenerational conflict between successive generations of voters endowed with a low altruism toward their offspring . The Markov equilibrium with repeated voting is equivalent to a social planning problem where the decision maker has quasi-geometric discounting, with a low short-term discount rate. This translates into a demand for fiscal discipline, i.e., low taxes and low debt accumulation. In spite of low altruism, the political equilibrium may converge to an equilibrium with moderate debt and positive public good provision. Debt accumulation is kept in check by the desire of privately affluent young voters to avert a future scenario of public poverty. However, fiscal discipline is weakened if agents can leave negative bequests to their heirs. In this case, the economy may attain immiseration.

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1 Introduction

According to the Pigouvian theory (see, e.g., Barro 1979), government debt should be used to smooth tax distortions in response to fiscal shocks. In reality, however, debt policy has often been used to redistribute resources in favor of current voters. For instance, in recent decades the governments of Italy and Greece have expanded their debt and expenditure during years of economic boom, imposing a heavy burden on the generations entering today's depressed labor markets. In these and other instances, debt becomes a predatory instrument that snatches resources from the future generations.

The drive to accumulate debt, combined with limits on what revenue can be collected in taxes, could drag societies into what Galbraith (1958) coined *private affluence and public poverty*. However, the evidence suggests that there are equilibrating forces that tend to forestall Galbraith's scenario. A number of governments (e.g., Germany, Scandinavian countries) follow prudent debt policies to ensure the future sustainability of the welfare state. Bohn (1998) shows that debt is mean reverting, namely, the debt-GDP ratio tends to fall when it reaches excessive levels. For instance, many countries that accumulated high debt levels during WWII reduced this debt subsequently.

Motivated by these observations, this paper proposes a dynamic politico-economic theory of debt and fiscal policy emphasizing the intergenerational conflict. We address a number of related questions. First, can the fear of future public poverty limit debt accumulation? Second, to what extent does intergenerational altruism curb predatory fiscal policies? Third, how do institutional constraints such as inheritance laws affect debt dynamics in the short and long term? Finally, what is the link between private and public wealth dynamics?

To answer these questions, we study the equilibrium of a small open economy with overlapping generations where different cohorts are linked by altruistic ties. The theory builds on the model of Song, Storesletten and Zilibotti (2012) – henceforth, SSZ12. Agents consume a private good and a public good provided by the government. Public good provision is financed by levying distortionary taxes and by issuing debt that can be sold in an international market at an exogenous interest rate.[1] Fiscal policy is determined sequentially by elected governments without commitment. We deviate from SSZ12 in two important respects. First, while SSZ12

[1] In SSZ12, we consider a world economy comprising a continuum of small open economies, where the world interest rate is determined endogenously. In this paper, we simplify the analysis and focus on a single small open economy facing an exogenous interst rate.

analyzes a standard OLG model, here we introduce and emphasize altruistic links that turn the model into a dynastic one. Yet, altruism is low, and living generations would like to redistribute resources in their own favor. Second, the dynastic model allows us to compare alternative environments. In the benchmark model, we rule out negative bequests. As an extension, we analyze a counterfactual where we impose no restriction on private wealth accumulation within dynasties, other than an intertemporal budget constraint.

To start with, we show that a politico-economic equilibrium *à la* Lindbeck and Weibull (1987) with repeated voting is equivalent to the allocation chosen by a benevolent social planner who attaches positive weights on the discounted utility of both young and old agents. The planner's welfare evaluation incorporates the altruistic links of individual agents. The planner's decision problem can be represented as an intertemporal maximization with quasi-geometric discounting (and, hence, time-inconsistent preferences): the planner's discount factor is larger in the first period than in all subsequent periods.

Next, we provide a recursive representation of the politico-economic equilibrium. We focus on economies in which the interest rate is low relative to the discount factor.[2] We contrast three planning allocations subject to different constraints. First, we analyze the Ramsey allocation with commitment when agents cannot leave negative bequests. Second, while continuing to preclude negative bequests, we study the politico-economic equilibrium with repeated voting. Formally, this is the Markov equilibrium of a dynamic game between a sequence of planners who set the current fiscal policy without commitment. Third, we relax, in addition, the constraint that bequests may not be negative.

The lack of commitment has dramatic implications. If the planner could commit the future fiscal policy, then the allocation would converge to public poverty in the long run, i.e., the government would accumulate a large debt and become unable to provide public goods. Yet, private consumption would not fall to zero since the tax rate would never exceed the top of the Laffer curve. In contrast, the time-consistent allocation (hence, the political equilibrium) features a lower debt, lower taxes, and positive public good provision in the long run. In other words, the lack of commitment empowers future generations – protecting them from predatory debt policies enacted by the earlier generations. The driving force of fiscal discipline is the concern among young voters to avoid a future situation of private affluence and public poverty. This conclusion contrast sharply with the earlier literature, in which the impossibility of sustaining time-inconsistent optimal policies is a burden on future generations. For instance,

[2] This can be shown to be the (stationary) equilibrium outcome of the model of a world economy comprising a continuum of small open economies with an endogenous interest rate. See SSZ12 for more discussion.

in Klein and Ríos-Rull (2003) limited commitment induces excessive capital taxation, causing low investments and low future capital stock.

The assumption that agents cannot leave private bequests is equally important. In an alternative environment where (a) the planner cannot control the private wealth dynamics, and (b) agents can leave negative bequests, then, even the political equilibrium would feature public poverty in the long run. Along transition, both private and public good consumption would be crowded out by ever-increasing private and public debt, leading to complete immiseration. We conclude that an institutional constraint on the intergenerational transmission of private wealth has important effects on public debt dynamics.

The contrasting results of our analysis under different bequest scenarios highlight the fundamental mechanism of the model. In the range of low interest rates that we consider, old agents would like to increase their consumption of both private and public goods at the expense of their heirs. If the old cannot pass private debt on to their descendants, the only way to extract transfers from them is through issuing more public debt. However, the fiscal policy must strike a compromise between the wishes of young and old voters. In the case where negative bequests are ruled out, as public debt accumulates, public goods are crowded out. This increases the ratio of private-to-public consumption, thereby increasing the relative marginal utility of public good provision. Hence, young agents become increasingly adamant to protect future public good provision, and more and more opposed to the accumulation of government debt. This results in the slowdown of debt expansion. In contrast, if the old can bequeath private debt, future generations suffer a proportional reduction in both private and public good consumption. Thus, the demand for fiscal discipline does not increase as debt accumulates.

The benchmark analysis hinges on separable preferences between private and public consumption. We generalize the analysis to non-separable preferences. This is potentially important: for instance, if agents could substitute private for public health services, they would be less concerned for future public good provision and, hence, more prone to accumulate public debt. We show that when private and public consumption are substitutes, fiscal discipline is weaker, as agents can cope better with public good impoverishment by substituting it by private consumption. The opposite occurs when private and public goods are complements. In this case, both private and public consumption are higher in the long run than in the case of separable utility.

Among the earlier contributions to the political economy of government debt and intergenerational redistribution, Cukierman and Meltzer (1989) construct a model where agents who are heterogenous in altruism and cannot leave negative bequests vote on public debt, transfers and taxation. Different from our theory, in their model agents are myopic and take future fiscal policy as given. In their benchmark model, highly altruistic agents are indifferent with respect to debt policy, due to Ricardian equivalence. Thus, fiscal policy is chosen according to the preferences of the most bequest-constrained agent.

Our paper is also related to our previous research in SSZ12. Apart from the model differences discussed above, SSZ12 focuses on a different set of questions, emphasizing the implications of cross-country heterogeneity in the efficiency of public good provision. In addition, SSZ12 lays out a general equilibrium model of the world economy with an exogenous interest rate, whereas here, for simplicity, we restrict attention to the decision of an individual small open economy. Finally, from a methodological standpoint, in the current setup the political equilibrium can be given a recursive presentation which is equivalent to the sequential choice of a benevolent social planner endowed with quasi-geometric discounting. This aspect is absent in the model with no altruism.

Our paper is more generally related to the politico-economic literature studying the determinants of government debt. This includes Persson and Svensson (1989) and Alesina and Tabellini (1990a, 1990b), who emphasized political conflict as a driving factor for public debt in models without intergenerational conflict. The contrasting fiscal policy dynamics under commitment and lack of commitment are analyzed by De Bortoli and Nunes (2013) and Rohrs (2014) in closed infinite-horizon models with endogenous interest rates, *à la* Lucas and Stokey (1983). Other recent politico-economic models of public debt and intergenerational redistribution include Bassetto (2008), Battaglini and Coate (2008 and 2011), Gonzalez Eiras and Niepelt (2008 and 2012), Mateos-Plana (2010), D'Amato and Galasso (2010), Yared (2010), Acemoglu, Golosov and Tsyvinski (2011), Arawatari and Ono (2011), Azzimonti Renzo (2011), Song (2011 and 2012), Esslinger and Müller (2013), Lancia and Russo (2013), Chen and Song (2014), and Ono (2014).

The analysis of dynamic choices with time-inconsistent preferences builds on Harris and Laibson (2001) who consider consumers with hyperbolic discounting. Time inconsistency in government policy is emphasized by Krusell, Kuruscu and Smith (2015), and Bisin, Lizzeri and Yariv (2013). In the latter model, voters have self-control problems, whereas in our theory voters have standard preferences, and the time inconsistency arises from the aggregation of individual preferences through a probabilistic voting mechanism *à la* Lindbeck and Weibull (1987). Amador (2003) and Tsur (2014) emphasize reasons why policy makers may have shorter time horizons than those of ordinary market participants. This is also different from our model, where the time inconsistency does not arise from political failures, and actually ends up being benign towards future generations by restraining debt accumulation.[3]

The paper is organized as follows. In section 2 we describe the model environment. Section 3 characterizes the commitment solution and the political equilibrium. Section 4 solves for the politico-economic equilibrium in a calibrated economy. Section 5 extends the analysis to the case in which agents can leave negative bequests. Section 6 concludes. The proofs of the Lemmas and Propositions are contained in Appendix A and in online Appendix B.

2 Model Economy

The model economy is populated by overlapping generations of two-period lived agents who work in the first period and live off their savings in the second period. Each agent has one child, implying a constant population size. Agents consume two goods: a private good (c) and a public good (g), provided by the government. We assume preferences to be time separable. In the benchmark case, we assume utility to be additively separable between the consumption of the public and private good.

Successive generations are linked by altruistic ties: the old care about their children with an altruistic factor λ. We also extend the model to allow the young to care about the old. The discounted utility of the old and young agents at t can be written recursively as follows:

$$\begin{aligned} U_{O,t} &= \tilde{u}\left(c_{O,t}\right) + u\left(g_t\right) + \lambda U_{Y,t}, \\ U_{Y,t} &= \tilde{u}\left(c_{Y,t}\right) + u\left(g_t\right) + \beta U_{O,t+1}, \end{aligned}$$

where β is the individual discount factor, and the functions $\tilde{u}$ and u are assumed to be strictly increasing and strictly concave, with $\lim_{x\to 0}\tilde{u}'(x) = \infty$ and $\lim_{x\to 0} u'(x) = \infty$.

Standard algebra yields the following sequential representations of the discounted utilities at time zero:

$$U_{O,0} = \tilde{u}\left(c_{O,0}\right) + \sum_{t=0}^{\infty} \left(\beta\lambda\right)^t \left(\lambda\phi\left(c_{Y,t}, c_{O,t+1}\right) + \left(1+\lambda\right) u\left(g_t\right)\right),$$

[3] Calvo and Obstfeld (1988) show that a planner endowed with an utilitarian social welfare function over current and future generations of finitely lived agents can yield time inconsistent preferences. This is different from our paper, since our agents are linked by altruistic ties, and the planner has no independent social welfare function, but care about the future generations exclusively through the altruistic discount factor of the currently living. Saez-Marti and Weibull (2005) show that time-inconsistent decision making arises when agents have a direct altruistic concern (*pure altruism*) not only for their children, but also more distant descendants (e.g., grandchildren).

and

$$U_{Y,0} = -\frac{1}{\lambda}u(g_0) + \frac{1}{\lambda}\sum_{t=0}^{\infty}(\beta\lambda)^t(\lambda\phi(c_{Y,t}, c_{O,t+1}) + (1+\lambda)u(g_t)),$$

where β is the discount factor and $\phi(c_{Y,t}, c_{O,t+1}) = \tilde{u}(c_{Y,t}) + \hat{\beta}\tilde{u}(c_{O,t+1})$, i.e., $\phi(c_{Y,t}, c_{O,t+1})$ is the lifetime utility of an individual born at t derived from the consumption of the private good.

2.1 Representation of the social planning problem

2.1.1 Benchmark Model

Consider a decision maker (e.g., a benevolent social planner) wishing to maximize the weighted average discounted utility of young and old agents, with Pareto weights $1-\omega$ and ω, respectively. In section 3.2 below, we interpret the planner's objective function as the outcome of a politico-economic voting equilibrium. Since the representation of preferences is invariant to affine transformations, we can write the utility accruing to the planner at time zero as:

$$\hat{U}_t = \frac{1+\lambda}{1+\omega\lambda} \times (\omega U_{O,t} + (1-\omega)U_{Y,t}),$$

where the multiplicative constant is introduced for analytical convenience and does not alter any properties of the model. Evaluating $\hat{U}_t$ at $t=0$, we obtain

$$\hat{U}_0 = \frac{\delta\lambda\omega}{1-\omega(1-\lambda)}\tilde{u}(c_{O,0}) + \delta\lambda\phi(c_{Y,0}, c_{O,1}) + (1+\lambda)u(g_0) + \delta\sum_{t=1}^{\infty}(\beta\lambda)^t(\lambda\phi(c_{Y,t}, c_{O,t+1}) + (1+\lambda)u(g_t)),$$

where

$$\delta \equiv 1 + \frac{1-\omega}{\lambda(1+\lambda\omega)} \geq 1.$$

Suppose (as in the problems studied below) that $\tilde{u}(c_{O,0})$ is determined by past choices, and cannot be affected by the planner. Thus, the maximization of $\hat{U}_0$ is identical to maximizing

$$U_0 = \delta\lambda\phi(c_{Y,0}, c_{O,1}) + (1+\lambda)u(g_0) + \delta\sum_{t=1}^{\infty}(\beta\lambda)^t(\lambda\phi(c_{Y,t}, c_{O,t+1}) + (1+\lambda)u(g_t)). \quad (1)$$

Note that, if $\omega = 1$, then, $\delta = 1$. Then, we can write:

$$U_0|_{\omega=1} = \sum_{t=1}^{\infty} (\beta\lambda)^t \left(\lambda\phi\left(c_{Y,t}, c_{O,t+1}\right) + (1+\lambda)\, u\left(g_t\right)\right),$$

which admits a recursive representation, of the form:

$$U_0 = \lambda\phi\left(c_{Y,0}, c_{O,1}\right) + (1+\lambda)\, u\left(g_0\right) + \beta\lambda U_1.$$

However, if $\delta > 1$, the problem is non-recursive and the decision maker is subject to time-inconsistency. In particular, *ex-post*, she would attach a lower value to the current public good provision than in the *ex-ante* program. Conversely, she would attach a greater weight to current private consumption and future utility than in the ex-ante plan. We will show below that this time-inconsistency yields a "conservative" fiscal policy when the planner cannot set the entire future path of policy with commitment.

2.1.2 Two-Way Altruism

The model can be extended to allow for two-way altruistic ties. Let λ_Y denote the altruistic factor by which the young care about the old. The discounted utility of the young agents at t is modified as follows

$$U_{Y,t} = \tilde{u}\left(c_{Y,t}\right) + u\left(g_t\right) + \hat{\beta} U_{O,t+1} + \lambda_Y U_{O,t}.$$

Standard algebra yields the following sequential representations of the discounted utilities at time zero:

$$\begin{aligned} U_{O,0} &= \frac{1}{1-\lambda\lambda_Y}\tilde{u}\left(c_{O,0}\right) + \\ &\quad \frac{1}{1-\lambda\lambda_Y}\sum_{t=0}^{\infty}\left(\frac{\beta\lambda}{1-\lambda\lambda_Y}\right)^t \left(\lambda\phi\left(c_{Y,t}, c_{O,t+1}\right) + (1+\lambda)\, u\left(g_t\right)\right), \end{aligned}$$

and

$$\begin{aligned} U_{Y,0} &= \frac{\lambda_Y}{1-\lambda\lambda_Y}\tilde{u}\left(c_{O,0}\right) - \frac{1}{\lambda}u\left(g_0\right) \\ &\quad + \frac{1}{\lambda}\frac{1}{1-\lambda\lambda_Y}\sum_{t=0}^{\infty}\left(\frac{\beta\lambda}{1-\lambda\lambda_Y}\right)^t \left(\lambda\phi\left(c_{Y,t}, c_{O,t+1}\right) + (1+\lambda)\, u\left(g_t\right)\right), \end{aligned}$$

For the discounted utilities to be well-defined, we must assume, here, that $\lambda\lambda_Y < 1$ and $\beta\lambda/\left(1-\lambda\lambda_Y\right) < 1$.

Standard algebra shows that the analogue of equation (1) can be written as

$$\begin{aligned} \tilde{U}_0 &= \tilde{\delta}\lambda\phi\left(c_{Y,0}, c_{O,1}\right) + (1+\lambda)\, u\left(g_0\right) \\ & +\tilde{\delta}\sum_{t=1}^{\infty}\left(\tilde{\beta}\lambda\right)^t\left(\lambda\phi\left(c_{Y,t}, c_{O,t+1}\right) + (1+\lambda)\, u\left(g_t\right)\right), \end{aligned} \tag{2}$$

where $\tilde{\beta} \equiv \beta/\left(1-\lambda\lambda_Y\right)$ and

$$\tilde{\delta} \equiv 1 + \frac{(1-\omega)\left(1-\lambda\lambda_Y\right)}{\lambda\left(1+\lambda\omega+\lambda_Y(1-\omega)\right)} \geq 1.$$

The problem is isomorphic to the case of one-sided altruism. In particular, unless $\omega = 1$, the decision maker exhibits time-inconsistent preferences. A larger λ_Y implies a larger $\tilde{\beta}$ and a lower $\tilde{\delta}$. Therefore, our model can capture two-sided altruism, up to a reinterpretation of the parameters β and δ.

2.2 Technology

We introduce distortionary taxation by assuming that taxes crowd out market production in favor of home production. More formally, we assume that the private good can be produced via two technologies – market and household production. Market production is subject to constant returns, and agents earn a pre-tax hourly wage w. Wages are subject to a linear tax rate, $\tau \in [0,1]$.[4]

The household production technology is represented by the production function $y_H = F(h)$, where the total time endowment is unity, $h \in [0,1]$ is the market labor supply, and $1-h$ is the time devoted to household production. The function F has the following properties: $F'(h) < 0$, $F''(h) \leq 0$, $F'''(h) \leq 0$, $F(1) = 0$, and $-F'(1) > w$. Since the government cannot tax household production, taxation distorts the time agents work in the market. Agents choose the allocation of their time so as to maximize total after-tax labor income, denoted by $A(\tau)$:

$$A(\tau) \equiv \max_{h\in[0,1]} \left\{(1-\tau)\, wh + F(h)\right\}. \tag{3}$$

This program defines the optimal market labor supply $H(\tau) = -\left(F'\right)^{-1}\left((1-\tau)\, w\right)$, where $H'(\tau) \leq 0$ and $H''(\tau) \leq 0$, and where the envelope theorem implies $A'(\cdot) = -wH(\tau)$. Let $e(\tau) \equiv -\left(dH(\tau)/d\tau\right)\left(\tau/H(\tau)\right)$ denote the tax elasticity of labor supply. The assumptions on F ensure that $e'(\tau) \geq 0$. Moreover, let $\bar{\tau}$ denote the tax rate corresponding to the top of the Laffer curve: $\bar{\tau} \equiv \arg\max_\tau \tau \cdot H(\tau)$. Standard algebra shows that $e(\bar{\tau}) = 1$, hence, $e(\tau) < 1$ for all $\tau < \bar{\tau}$.

[4] In SSZ12, the technology uses also physical capital as an input. Here, we abstract from it. Following SSZ12, we also abstract from taxes on consumption and returns to savings.

Each cohort maximizes utility subject to the budget constraint

$$c_{Y,t} + \frac{c_{O,t+1}}{R} = x_{Y,t} + A(\tau) - \frac{x_{O,t+1}}{R}. \tag{4}$$

where $x_{Y,t}$ is the wealth inherited by the young agent at time t and $x_{O,t+1}$ are the bequests the old agent leaves to her offspring. In the first part of the paper, we abstract from intergenerational transfers (e.g., bequests or inter-vivo transfers), and set $x_Y = x_O = 0$. We show later that for sufficiently small λ and λ_Y, agents find it optimal to give no positive transfers. In section 5 we allow agents to leave negative bequests.

2.3 Fiscal policy

The planner can affect the intergenerational distribution of resources and the consumption of public and private goods by choosing a fiscal policy sequence. This policy is the only instrument at her disposal; she cannot dictate transfers of private resources between generations.

Given an inherited debt b, the planner chooses the tax rate (τ), the public good provision (g) and the debt accumulation (b'), subject to the following dynamic budget constraint:[5]

$$b' = g + Rb - \tau w H(\tau). \tag{5}$$

Both private agents and the planner have access to an international capital market providing borrowing and lending at the constant gross interest rate R. The planner is committed to not repudiate the debt. This implies that debt cannot exceed the present discounted value of the maximum tax revenue that can be collected:

$$b \leq \frac{\bar{\tau} w H(\bar{\tau})}{R-1} \equiv \bar{b}, \tag{6}$$

where $\bar{b}$ denotes the natural debt limit and, recall, $\bar{\tau}$ is the tax rate attaining the top of the Laffer curve. The constraint (6) rules out government Ponzi schemes. Throughout the paper we restrict attention to the increasing portion of the Laffer curve, i.e., $\tau \leq \bar{\tau}$, since larger taxes would never be chosen by the planner. We restrict debt to lie in a compact set, $b \in \left[\underline{b}, \bar{b}\right]$.[6] This restriction, together with the government budget constraint (5), implies that also g and τ are bounded: $\tau \in [0, \bar{\tau}]$, and $g \in [0, \bar{g}]$.

[5] Hereafter, unless specified otherwise, we omit time indexes and switch to a recursive notation with primes denoting next-period variables.

[6] The lower bound on b simplifies the analysis since it avoids uninteresting corner solutions in taxes and market labor supply when the government is very rich. This restriction is innocuous since $\underline{b}$ can be chosen to be so small that it will not bind in the political equilibrium.

2.4 Altruism

We maintain throughout the paper that $\lambda\beta R < 1$, emphasizing the case of a low interest rate relative to people's altruism. In the benchmark case we abstract from bequests by assuming that (i) agents *cannot* leave negative bequests, and (ii) λ is sufficiently small so that agents *do not want to* leave positive bequests.

Assumption 1 *Negative bequests are not allowed.*

Lemma 1 *Let $\hat{x} < \infty$ denote the bequest of the young in the first period. Then, $\exists\ \bar{\lambda}(\hat{x}) > 0$ (defined in the proof) such that, for all $\lambda < \bar{\lambda}(\hat{x})$, the desired bequest would be negative for all feasible tax sequence $\{\tau_t\}_{t=0,\ldots,\infty}$ such that $\tau_t \in [0, \bar{\tau}]$. Then, given Assumption 1, $x_{Y,t} = x_{O,t} = 0$ for all $t \geq 1$.*

The intuition of the proof of Lemma 1 is simple. Absent constraints, old agents would choose bequests so that $\tilde{u}'(c_{O,t}) = \lambda \tilde{u}'(c_{Y,t})$, so desired bequests increase with λ. However, these could be negative, since young agents also have a labor income. Labor income, in turn, has a natural lower bound, since no rational government would ever tax beyond the top of the Laffer curve. The existence of a Laffer curve guarantees that the consumption of the young be positive and its marginal utility be finite for *any* feasible (and rational) fiscal policy. Thus, one can always find a range of low λ's such that the old would like to grab resources from their children rather than leave bequests. An extreme example is the canonical OLG model without altruism, where, if they could, old agents would set infinitely negative bequests. Motivated by Lemma 1, we focus on low λ and abstract from bequests in our main analysis.[7] However, since the case of unconstrained bequests helps highlight the mechanism behind our results, we relax Assumption 1 in section 5. Absent bequests, the optimal consumption in both periods is only a function of total after-tax labor income: $c_Y = c_Y(A(\tau))$ and $c_O = c_O(A(\tau))$, where $c_Y'(\cdot) > 0$, $c_O'(\cdot) > 0$, and $\tilde{u}'(c_Y)/\tilde{u}'(c_O) = \beta R$.

3 Equilibrium

We first characterize the (Ramsey) policy sequence that the planner would choose in the first period, if she could commit the entire future path of fiscal policy. Then, we move to the Markov equilibrium with no commitment (i.e., the political equilibrium) which is the main contribution of this paper.

[7] A number of empirical studies document that only a small fraction of the population leave significant bequests (see, e.g., Hurd, 1989, Leitner and Ohlsson, 2001). Moreover, part of these bequests are involuntary. These observations motivate the focus of our analysis on low altruism.

3.1 The Commitment Solution

Consider, first, the commitment problem. Since $\delta \geq 1$, the maximization of (1) subject to (6) and a sequence of government budget constraints, (5), does not admit a standard recursive representation. However, it admits the following two-stage recursive formulation (proof omitted).

Lemma 2 *The commitment problem is characterized as follows;*

(i) After the initial period, policies are the solution to the recursive problem

$$V_O^{comm}(b) = \max_{\{\tau,g,b'\}} \left\{ v(\tau,g) + \beta\lambda V_O^{comm}(b') \right\}, \tag{7}$$

subject to (5)-(6), where

$$v(\tau,g) \equiv (1+\lambda)\, u(g) + \lambda\phi(A(\tau)). \tag{8}$$

(ii) In the initial period, policies solve the following problem, subject to (5)-(6):

$$\{\tau_0, g_0, b_1\} = \arg\max_{\{\tau_0,g_0,b_1\}} \{v(\tau_0,g_0) + (\delta-1)\lambda\phi(\tau_0) + \delta\beta\lambda V_O^{comm}(b_1)\}. \tag{9}$$

Consider, first, the particular case in which $\omega = \delta = 1$, i.e., the planner cares only about the old. Then, the planner's objective function is time consistent, and the commitment solution coincides with the allocation that would be chosen sequentially by the planner. Standard arguments establish that the program is a contraction mapping, and, hence, a solution exists and is unique (see Lemma 7 in Appendix B). To solve the program, we combine the First Order Conditions with respect to τ and g, and invoke the result that $A'(\tau) = -wH(\tau)$ to obtain the following intratemporal optimality condition:

$$-\lambda\phi'(A(\tau)) = (1+\lambda)(1-e(\tau))\, u'(g), \tag{10}$$

Since $e'(\tau) > 0$, and $\phi(\cdot)$ and $u(\cdot)$ are concave, higher g is associated with lower τ and, hence, higher private consumption. Intuitively, the planner equates the marginal cost of taxation (foregone utility from private consumption of the young) to its marginal benefit (marginal utility of public good consumption, adjusted by the marginal cost of raising public funds). In addition, standard analysis leads to an Euler equation for public consumption:

$$\frac{u'(g)}{u'(g')} = \beta\lambda R. \tag{11}$$

Consider next the general case when also the young have political influence ($\omega < 1$). After the initial period, the optimality conditions (10)-(11) continue to characterize the commitment solution, irrespective of ω.

As mentioned above, we focus on an equilibrium where $\beta\lambda R < 1$. Under this assumption, (11) implies that public good provision declines asymptotically to zero. Then, (10) implies that taxes converge to the top of the Laffer curve. Finally, (5) implies that debt converges asymptotically to the natural limit, $\bar{b}$.

Proposition 1 *The commitment solution converges to public poverty:*
$\lim_{t\to\infty} b_t = \bar{b}$, $\lim_{t\to\infty} g_t = 0$ *and* $\lim_{t\to\infty} \tau_t = \bar{\tau}$*.*

Note that, in the general case the initial-period problem is different from that of the following periods. The young demand less government expenditure and more fiscal discipline (i.e., lower taxation and debt accumulation) than do the old. As a result, the commitment solution is time inconsistent: if at some future point the living voters were allowed to re-optimize, they would deviate from the policy sequence dictated by the initial generation.

3.2 The Political Equilibrium

In this section we consider the planning problem without commitment. The planner maximizes the objective function (1), but policies are chosen sequentially.

In SSZ12, we prove that the planning equilibrium without commitment is equivalent to the politico-economic equilibrium of a probabilistic voting model *à la* Lindbeck and Weibull (1987) in which fiscal policy is determined every period through repeated elections. This model is discussed in detail in Persson and Tabellini (2000) and its application to a dynamic model is outlined in the online appendix. Here, we summarize the main features: Agents cast their votes on one of two office-seeking candidates. Voters' preferences may differ not only over fiscal policy, but also over other orthogonal policy dimensions about which the candidates cannot make binding commitments. In a probabilistic voting equilibrium, both candidates propose the same fiscal policy, which turns out to maximize a weighted sum of individual utilities where the weights may differ between young and old agents. The weights (ω and $1-\omega$) capture differences between the young and the old in their respective population size, and in their relative political clout (i.e., the relative proportion of "swing voters", or their ability to lobby the government). Thus, the political equilibrium can be represented as a set of fiscal policy rules maximizing a weighted average indirect utility of young and old households, given b. With this equivalence in mind, we label the planning problem without commitment as the "political equilibrium" and refer to the planner without commitment as the "government".

We characterize the Nash equilibrium of the dynamic game between successive governments. The set of equilibria is potentially large. We restrict attention to Markov Perfect Equilibria where strategies can only be conditioned on pay off-relevant state variables. In the current environment, b is the only pay off-relevant state variable. This hinges on the assumptions that (i) preferences are separable between private and public goods consumption; (ii) private wealth is not bequeathed. In the analysis of Sections 4.3 and 5, we relax each of these assumptions. Then, the state vector must also include the private wealth.

In the rest of the paper, primes denote next period's variables and boldface variables are vectors, defined as follows: $\boldsymbol{x} = [x, x', x'', ...] = [x, \boldsymbol{x}']$. It is also convenient to define the planner's indirect utility function as $U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$. Namely, $U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$ is identical to (1), but is expressed as a function of the policy sequence instead of consumption allocations.

Definition 1 *A Markov Perfect Political Equilibrium (MPPE) is defined as a 3-tuple $\langle B, G, T\rangle$, where $B : [\underline{b}, \bar{b}] \to [\underline{b}, \bar{b}]$ is a debt rule, $b' = B(b)$, $G : [\underline{b}, \bar{b}] \to [0, \bar{g}]$ is a government expenditure rule, $g = G(b)$, and $T : [\underline{b}, \bar{b}] \to [0, \bar{\tau}]$ is a tax rule, $\tau = T(b)$, such that:*

1. *$\langle B(b), G(b), T(b)\rangle = \arg\max_{\{b' \in [\underline{b}, \bar{b}], g \in [0, \bar{g}], \tau \in [0, \bar{\tau}]\}} U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$, subject to (5) and (6), where $\boldsymbol{b} = [b, b', B(b'), B(B(b')), ...]$, $\boldsymbol{\tau} = [\tau, T(b'), T(B(b')), T(B(B(b'))), ...]$, and $\boldsymbol{g} = [g, G(b'), G(B(b')), G(B(B(b'))), ...]$.*

2. *The government budget constrained is satisfied:*

$$B(b) = G(b) + Rb - T(b) \cdot w \cdot H(T(b)) \tag{12}$$

Definition 2 *A MPPE is said to be differentiable (DMPPE) if the equilibrium functions $\langle B, G, T\rangle$ are continuously differentiable in the interior of their domain, $(\underline{b}, \bar{b})$.*

In words, the government chooses the current fiscal policy (taxation, expenditure and debt accumulation) subject to the budget constraint, under the expectation that future fiscal policies will follow the equilibrium policy rules, $\langle B(b), G(b), T(b)\rangle$. Furthermore, the vector of policy functions must be a fixed point of the system of functional equations in parts 1 and 2 of the definition, where part 2 requires the equilibrium policies to be consistent with the resource constraint. The following Lemma is a useful step to characterize the MPPE.

Lemma 3 *The MPPE (part 1 of Definition 1) admits the following two-stage formulation:*

$$\langle B(b), G(b), T(b)\rangle = \arg \max_{\{b' \in [\underline{b}, \bar{b}], g \in [0, \bar{g}], \tau \in [0, \bar{\tau}]\}} \left\{ v(\tau, g) + (\delta - 1)\lambda\phi(A(\tau)) + \delta\beta\lambda V_O(b') \right\}, \tag{13}$$

where $v(.)$ is defined as in (8), subject to (5) and (6), and V_O satisfies the functional equation;

$$V_O(b') = v(T(b'), G(b')) + \beta\lambda V_O(B(b')). \tag{14}$$

The difference between the commitment solution and the political equilibrium can be appreciated by comparing (7) with (14). In the political equilibrium, the government in period t cannot choose the entire future policy sequence, but takes the mapping from the state variable into the (future) policy choices as given. For this reason, there is no maximization operator in the definition of V_O. The two programs are identical if and only if $\omega = \delta = 1$, i.e., the planner cares only about the old, or in the politico-economic interpretation only the old vote. In this case, the commitment solution is time-consistent.

Why does the commitment solution differ from the MPPE? Recall that the planning problem is inherently time-inconsistent, with time inconsistency taking the form of a (repeated) low weight on the current public good consumption (see equation (1)). In the commitment solution, this low weight features only in the first period. In contrast, it is recurrent in the MPPE, as a new generation of young voters enters the stage in each election. As a result, the political equilibrium delivers more fiscal discipline.

If a differentiable MPPE exists, it can be characterized by applying standard recursive methods to the First Order Conditions of (13)-(14). The results are summarized by the following Proposition.

Proposition 2 *A DMPPE is fully characterized by a system of two functional equations:*

1. *A trade-off between private and public good consumption*

$$-\delta\lambda\phi'(A(\tau)) = (1+\lambda)(1-e(\tau))\,u'(g). \tag{15}$$

where $g = G(b)$ and $\tau = T(b)$.

2. *A Generalized Euler Equation (GEE) for public good consumption*

$$\frac{u'(g)}{u'(g')} = \beta\lambda R - \underbrace{(\delta-1)\,\beta\lambda G'(b')}_{\text{the disciplining effect}}, \tag{16}$$

where $g = G(b)$, $g' = G(b')$, $\tau = T(b)$ and $b' = g + Rb - \tau w H(\tau) \equiv B(b)$.

Consider, first, equation (15). The only difference between (15) and (10) lies in the δ term appearing in the left-hand side of (15): increasing the weight of the young (i.e., increasing δ) increases the marginal disutility of taxation (LHS) in the planner's objective function. Thus, the MPPE features lower taxes than does the commitment solution.

The GEE, (16), is the key equilibrium condition. The ratio between the marginal utilities of public good consumption in two consecutive periods consists of two terms. The first, $\beta\lambda R$, is the standard Euler-equation term appearing in the commitment solution, (11). The second, which we label the *disciplining effect*, arises from the dynamic game between successive governments, and is absent from the commitment solution. The government anticipates that an increase in the future debt will prompt a fiscal adjustment. This effect hinges on the forward-looking voting of the young, and vanishes when the old have full political power. The derivative $G'(b')$ describes the effect of the future fiscal adjustment on next-period government expenditure. Although a global characterization of G' is not available – except in particular cases discussed below – we can establish that in a neighborhood of any steady state $G' < 0$, i.e., higher debt is associated with lower public spending.[8] Consequently, the disciplining effect increases the growth rate of public expenditure. As in a standard Euler equation, high growth of g is attained by reducing expenditure and increasing public savings today.

Consider the comparative statics of the influence of the young. Increasing δ magnifies the disciplining effect, thereby restraining debt accumulation. Moreover, conditional on b, it reduces taxes and government expenditure; see equation (15). The case of majority voting with a majority of young is a limit case of Proposition 2 where $\delta = 1 + \lambda^{-1}$, maximizing fiscal discipline.

The commitment solution coincides with the MPPE when only the old have political influence. In this case, a standard contraction mapping argument ensures the existence and uniqueness of the MPPE (proof omitted).

Lemma 4 *Assume that $\omega = \delta = 1$. Then, the MPPE induces the same allocation as the commitment solution. Consequently, the MPPE exists and is unique.*

In Appendix B, we provide sufficient conditions for the equilibrium policy functions to be continuous and differentiable (namely, for the equilibrium to be a DMPPE) in the $\omega = 1$ case (see Lemma 8). The crux is to impose restrictions on preferences and on the household technology that guarantee the concavity of the return function in the contraction mapping.

Extending the proof of existence and uniqueness of the MPPE to the general case of $\omega < 1$ is not straightforward. This is a common problem, as dynamic games generally do not admit a contraction-mapping formulation. However, Judd (2004) provides a strategy for proving local existence and uniqueness in such environments. He proposes to perturb the GEE in the neighborhood of a particular parameter configuration for which the problem is a contraction

[8] In steady state, $u'(g) = u'(g')$. Thus, the GEE (16) reduces to $G'(b^*) = -(1 - \beta\lambda R)/((\delta - 1)\beta\lambda) < 0$, where $G'(b^*)$ is independent of b^*. It can also be established that G is concave in the neighborhood of a steady state, as long as b converges monotonically to the steady state.

mapping. Here, we exploit the same strategy, by perturbing the equilibrium around the $\omega = 1$ case. The following proposition establishes local existence and uniqueness of the DMPPE.

Proposition 3 *Let $\left\langle \bar{B}(b), \bar{G}(b), \bar{T}(b) \right\rangle$ denote equilibrium policies when $\omega = 1$. Assume that $\bar{B}(b), \bar{G}(b), \bar{T}(b)$ are continuously differentiable. Suppose that*

$$\left| \frac{u''(g)}{\beta \lambda R u''(g')} - \bar{G}'(b') \left(1 - \frac{\phi'(A(\tau)) u''(g) wH(\tau)(1 - e(\tau)) / u'(g)}{\phi''(A(\tau)) A'(\tau) + \phi'(A(\tau)) e'(\tau) / (1 - e(\tau))}\right)\right| > 1, \tag{17}$$

where $g = \bar{G}(b)$, $g' = \bar{G}(b')$, $\tau = \bar{T}(b)$ and $b' = \bar{B}(b)$. Then, for δ close to unity, there exists a unique DMPPE.

The proof, which follows Judd (2004), is in Appendix B. Note that condition (17) is imposed on the equilibrium functions of the case with $\omega = \delta = 1$, for which existence and uniqueness are guaranteed (see Lemma 4). Thus, condition (17) can be verified numerically.

4 A Calibrated Economy

The equilibrium characterized by the functional equations (15)-(16) can be solved numerically, applying a standard projection method with Chebyshev collocation (Judd, 1992) to approximate T and G.[9] Judd, Kubler, and Schmedders (2003) review the literature applying this method to models featuring time inconsistency.

We parameterize the utility to be logarithmic, $\tilde{u}(c) = \log(c)$ and $u(g) = \theta \log(g)$, where $\theta > 0$ is a parameter describing the intensity of preferences for public good consumption. The household production technology is assumed to be $F(h) = X \cdot \left(1 - h^{1+\xi}\right) / (1 + \xi)$, where $\xi > 0$ is the inverse of the Frisch elasticity. In the benchmark calibration we abstract from the altruism of the young ($\lambda_Y = 0$). We discuss as a robustness check how the results change when $\lambda_Y > 0$.

We calibrate the parameters as follows. Since agents live for two periods, we let a period correspond to thirty years. Accordingly, we set $\beta = 0.985^{30}$ and $R = 1.025^{30}$, implying a 1.5% annual discount rate and a 2.5% annual interest rate, consistent with the average real long-term rate on U.S. government bonds between 1960 and 1990. As we have no strong prior on ω, we simply assume equal political weights on the young and old ($\omega = 0.5$).[10] We normalize

[9] In the special case of inelastic labor supply the system of functional equations (15)-(16) can be solved analytically by a guess-and-verify method. The equilibrium policy functions in that case are linear. See SSZ12 for details.

[10] Proposition 3 establishes existence and uniqueness in the neighborhood of $\omega = 1$. Standard caveats apply as we extend a local result to lower ω's. However, we have solved for a range of economies holding constant the parameters of Table 1 and varying ω. The numerical routine always converges to a set of policy functions satis-

the wage to unity and set X to target a ratio of market consumption to total consumption (including the value of home production) to 0.3, following Apps and Rees (1996, Table 2). Since $\bar{\tau} = \xi / (1 + \xi)$, we set $\xi = 1.5$ to target a labor tax associated with the top of the Laffer curve of 60% (in line with Trabandt and Uhlig, 2011). This implies a Frisch elasticity of 2/3. In the sensitivity analysis we show that our results are robust to a wide range of values for this elasticity.

The two remaining parameters θ and λ are set so that in steady state the model matches the empirical debt-to-output ratio and the ratio of labor tax revenue to labor income. The average labor income tax rate was about 27% in the US in the period 1970-2005, so we set $\tau^* = 0.27$.[11] We target a steady-state annualized debt-GDP ratio of 120%. This is the GDP-weighted average for the eight largest OECD economies (source: IMF). Our production function abstracts from capital. Since one period is thirty years, and the empirical labor share of output is ca. 0.67, we target a steady-state level of debt to labor earnings of $b/wH = 120\% \times 0.67/30 = 6\%$. Table 1 summarizes the parameters.

Table 1: Calibration

Target observation		*Parameter*	
Annual discount rate	1.5%	β	0.985^{30}
Annual interest rate	2.5%	R	1.025^{30}
Average tax on labor	27%	θ	0.092
Tax rate at the top of the Laffer Curve	60%	ξ	1.5
Debt-GDP ratio	120%	λ	0.674
Ratio of market-to-total consumption	30%	X	2.722
Relative political weight young-old	equal	ω	0.5

Figure 1 plots the equilibrium functions of our calibrated economy. Taxes are increasing in b (panel a) and public expenditure is decreasing in b (panel b). However, the debt policy is now a strictly convex function of b which crosses the 45-degree line twice: at an interior steady-state level ($b = 0.025$) and at the natural debt limit. Only the interior steady state is stable. Thus, for any initial $b < \bar{b}$, the economy converges to the internal steady state (see panel d). The steady-state government expenditure is $g^* = 0.086$, implying a 19.9% ratio of

fying (up to numerical approximations) the equilibrium conditions. Moreover, the equilibrium policy functions change with continuity, namely, small changes in ω lead to small changes in the equilibrium functions (details available upon request). In none of the simulations have we found more than one equilibrium for each parameter configuration.

[11] In the period 1970-2005, the aggregate tax revenue minus corporate taxes minus social security contributions in the US was on average 18% (source: OECD). With a labor share of 0.67, this implies an average tax rate on labor of 27%. Klein and Ríos-Rull (2003) report an average income tax rate of 24% for the period 1947-90.

public expenditure to private market consumption.[12] Panel d shows the transition of debt towards the steady state.

FIGURE 1 HERE

The tax function is non-decreasing and concave, while the expenditure function is decreasing and concave. As b increases, the tax function, $T(b)$ becomes less steep, whereas the expenditure function, $G(b)$, becomes steeper. Namely, at high debt levels, the government responds to debt accumulation by cutting expenditure more than by increasing taxes. Hence, the ratio of public-to-private consumption falls as b increases. This fall in relative government expenditure is what deters voters from demanding more debt in steady state.

4.1 Robustness to parameter changes

The qualitative findings of an internal steady state are robust to a large range of parameter values.[13] We verify that an internal steady state exists for even very low Frisch elasticities. For instance, if $\xi = 3$ (low Frisch elasticity), the steady state features a tax rate of 46% and debt-earnings ratio $b/wH = 23\%$ (an internal steady state actually exists for as low a Frisch elasticity as 0.01). If $\xi = 1$ (high Frisch elasticity), the corresponding figures are 17% and -2% (i.e., a government surplus). Thus, the more distortionary taxes are, the lower the debt is and the lower taxes are in steady state.

As far as the altruism of the old is concerned, an interior steady state may cease to exist if λ is too low. For instance, if we fix all other parameters as in Table 1 and vary λ, a steady state with $b < \bar{b}$ is sustained only if $\lambda > 0.55$. For lower λ's the economy converges to the maximum debt. An equilibrium with an interior steady state can be sustained even for $\lambda = 0$, but only as long as we increase either the interest rate or the Frisch elasticity.

We also consider the altruism of the young towards their parents. Recall that a larger altruism of the young increases the *de facto* discount factor β and decreases δ. The former strengthens fiscal discipline while the latter weakens it. The net effect on long-run debt is therefore ambiguous. Figure A1 in the Appendix reports the simulated policy functions if one assumes $\lambda_Y = 0.05$ while keeping all other parameters as in Table 1. Conditional on the debt level, the altruism of the young yields higher taxes, lower public spending and, hence, lower debt accumulation. This induces a lower steady-state debt level than in the benchmark

[12] The corresponding number for the U.S. over the period 1970-2005 happens to be precisely 19.9% when public goods are measured as expenditures on defense, highways, and a number of public goods provided on the state and local level (see Appendix B for a comprehensive list of items).

[13] In all experiments, we checked that the sufficient condition of Lemma 1 continues to be satisfied.

economy. In our parameterization, the steady-state debt level is negative, $b = -0.045$. The corresponding tax and public good provision are $\tau = 0.13$ and $g = 0.23$, namely, in the long run this economy delivers both higher private and higher public good consumption. Intuitively, the effect on β dominates in our example, and the altruism of the young strengthens the fiscal discipline.

The results are also robust to changes in ω. If we fix all other parameters as in Table 1 and vary ω, a steady state with $b < \bar{b}$ is sustained only if $\omega < 0.77$. If we set $\omega = 0.7$ while keeping the other parameters as in Table 1, the steady-state tax rate is 47% and the debt-earnings ratio is 63%. If $\omega = 0.3$, the corresponding figures are 13% and -12%. As expected, increasing the weight of the young strengthens fiscal discipline.

Finally, we investigate the robustness of the results to changes in the utility functions by considering u being CRRA; $u(g) = \left(g^{1-\sigma} - 1\right) / (1-\sigma)$, while maintaining $\tilde{u}(c) = \log(c)$ (the specification of $\tilde{u}$ has hardly any effect on the sustainability of an internal steady state). Note that preferences are still separable between c and g, an assumption we relax in section 4.3. The results are qualitatively similar to the log-log case discussed above. The steady-state debt level is decreasing with σ, and an internal steady state can be sustained as long as $\sigma \geq 0.48$. Intuitively, a larger σ makes agents more concerned about reduced future public good provision, thereby strengthening fiscal discipline. Figure A2 in the Appendix reports the simulated policy functions for $\sigma = 1.5$ and $\sigma = 0.75$.

4.2 Commitment vs. Markov Equilibrium

It is instructive to compare the DMPPE in the benchmark calibration of Figure 1 with the corresponding commitment (Ramsey) solution. We already know that under commitment debt converges to $\bar{b}$ and g converges to zero (Proposition 1). In Figure 2, we compare the transitional dynamics for economies starting with zero debt.

FIGURE 2 HERE

In the first period, the commitment solution features slightly lower taxes (τ_0) and higher government spending (g_0) than the DMPPE. Consequently, b_1 is higher under commitment. Government expenditure is significantly larger in the second period (g_1) than in the first (g_0). This comes at the expense of a larger increase in the debt inherited by agents born in period two. Thereafter, debt accumulates at a higher rate in the commitment solution where taxes and spending converge, respectively, to the top of the Laffer curve (panel b) and to zero (panel c) and debt converges to $\bar{b}$ (panel a). All generations born in period two or later are strictly worse off in the commitment solution, while the agents who are alive at the time of the initial vote are better off.

4.3 Non-Separable Utility

In this section, we generalize the analysis to non-separable preferences between private and public consumption. We assume $u(c,g) = \log\left(((1/(1+\theta))\, c^{\rho} + ((\theta/(1+\theta)))\, g^{\rho})^{1/\rho}\right)$ where $\rho < 1$. This specification encompasses the benchmark separable utility as $\rho \to 0$. Private and public good consumption are substitutes if $\rho > 0$ and complements if $\rho < 0$. This generalization has interesting implications. For instance, if agents can substitute private for public health services they may be less concerned for future public good provision and, hence, be less averse to public debt.

The analysis must take two new features into account. First, private savings now depend not only on current taxes but also on the current and next-period public good provision. Second, the private wealth of the old is a pay off-relevant state variable, since it affects the marginal utility of public expenditure of the old and, hence, the probabilistic voting equilibrium outcome. Formally, the equilibrium policy functions depend now on a two-dimensional state vector: $\tau = T(s_{-1}, b)$, $g = G(s_{-1}, b)$, and $b' = B(s_{-1}, b)$.

FIGURE 3 HERE

A formal characterization of the DMPPE, in terms of a system of functional equations analogous to those in Proposition 2, is deferred to Appendix B (Proposition 8).[14] Figure 3 shows the equilibrium time paths of taxes (panel a), public spending (panel b), private savings (panel c) and debt (panel d), with non-separable preferences. The initial conditions (b and s) are set equal to the steady-state levels in the model with separable utility. The solid and dotted lines correspond to $\rho = 0.1$ (substitutes) and $\rho = -0.1$ (complements), respectively. The constant dashed lines correspond to the case of separable utility. Otherwise the parameter values are as in Table 1.

Taking stock, when c and g are substitutes ($\rho > 0$), the demand for public good falls and fiscal discipline is weaker. Taking the steady-state debt level of Figure 1 as the initial condition, voters support an initial tax cut and a decrease in public good provision. The weaker fiscal discipline induces debt accumulation, causing a further decline in public goods and increasing taxes over time. Debt converges to a higher steady-state level with higher taxes and lower

[14] Proposition 8 provides a characterization for any quasi-concave utility function, $u = u(c,g)$, where $u_c > 0$, $u_g > 0$, of which the CRRA is a particular case.

public good provision than in the benchmark economy with separable utility. An internal steady-state debt level bounded away from the debt limit is sustained as long as $\rho < 0.45$.

The opposite is true when c and g are complements ($\rho < 0$). In this case, the dynamics have the opposite sign. Upon impact, taxes and public good provision increase. Government debt falls over time, and in the long run both private and public consumption are higher than in the benchmark case of separable utility.

5 Unconstrained Bequests

In the analysis thus far, we have ruled out negative bequests. In this section, we relax this constraint.[15] Although we do not view negative bequests as realistic, solving the model without constraints sheds light on the mechanism of the theory and allows us to derive additional implications about the dynamics of debt.

We assume that the old bequeath before the young decide their savings. This rules out the possibility for the young to choose their savings strategically in order to attract more bequests.[16] For simplicity, we restrict attention to log utility and separable preferences. In this environment, it is useful to define as a *household* the young and old members of a dynasty who are alive in the same period.[17] By controlling bequests, the old *de facto* dictate the allocation of private consumption of each member of the *household*. In the appendix we show that under log utility the consumption of the young and old agents are, respectively, $c_{Y,t} = \lambda (1+\lambda)^{-1} c_t$, and $c_{O,t} = (1+\lambda)^{-1} c_t$, where $c_t = c_{Y,t} + c_{O,t}$ denotes total household consumption. Substituting these conditions into the expressions of U_Y and U_O, respectively, yields the following sequential representation of the planner's objective function:

Lemma 5 *The planner's objective function can be written as:*

$$U_0 = \log(c_0) + \theta \log(g_0) + \delta \sum_{t=1}^{\infty} (\beta\lambda)^t (\log(c_t) + \theta \log(g_t)) . \tag{18}$$

The objective function (18) can be compared with its analogue in the case of no bequests, (1). Now, the weight of the young (captured by δ) has no effect on the intratemporal trade

[15] The analysis of this section does not require any restriction on altruism. However, for comparison, we focus on the case in which $\beta\lambda R < 1$.

[16] Another potential source of strategic behavior could be labor supply. However, in our model there is no pure leisure, so the allocation of time between market and household production is affected by neither wealth nor bequests.

[17] Note that this is not a unitary household. The old and young members of the household retain separate consumption and have conflicting interests on fiscal policy.

off between c and g in the first period.[18] However, as before, δ affects the intertemporal trade off. Consequently, the planner's preferences are time-inconsistent and feature quasi-geometric discounting. We restrict attention to Markov equilibria (interpreted as a political equilibria) as in the rest of the paper. Private savings is now an additional state variable because the wealth of the old determines the bequests and, thus, the marginal utility of consumption of the young. This, in turn, affects the political preference over taxation. The planner maximizes utility, taking as given the policy rules governing her future selves' choices from the second period onward. The planning problem admits the following two-stage recursive representation:

$$V\left(s_{-1}, b\right) = \max_{\{c,\tau,g,b'\}} \left\{ \log\left(c\right) + \theta \log\left(g\right) + \delta\beta\lambda V^{b}\left(s, b'\right) \right\}, \tag{19}$$

where

$$V^{b}\left(s_{-1}, b\right) = \log\left(C\left(s_{-1}, b\right)\right) + \theta \log\left(G\left(s_{-1}, b\right)\right) + \beta\lambda V^{b}\left(s, b'\right). \tag{20}$$

$C\left(s_{-1}, b\right)$ and $G\left(s_{-1}, b\right)$ are equilibrium private consumption and public expenditure functions. The maximization is subject to the government budget constraint, (5), and to a budget constraint for private wealth, $s = Rs_{-1} + A\left(\tau\right) - c$, plus to the respective no-Ponzi game conditions.

It is useful to distinguish between two planners endowed with different instruments. An N-planner (*non-empowered planner*) chooses fiscal policy sequentially and without commitment, being subject to the *implementability* constraint that private consumption is chosen optimally by the households. The N-planner allocation is equivalent to the MPPE of Section 3, except that now agents can leave positive or negative bequests. In contrast, an E-planner (*empowered planner*) has an additional instrument: she controls the intertemporal allocation of private consumption. The E-planner serves as a pedagogical second-best benchmark intended to build the intuition for the main results.

5.1 The E-planner Allocation

We start by analyzing the E-planner's problem. The following Proposition follows from the First Order Conditions and envelope conditions of the program (19)-(20).

Proposition 4 *Assume that $s_{-1} \geq \underline{s}$ and $b \leq \bar{b}$. A differentiable time-consistent E-planner allocation satisfies the following system of functional equations:*

[18] This is a non-robust feature. With general utility, the weight of the young would also affect the planner's relative intratemporal weights of c and g. However, the main equilibrium features are similar to those obtained in the log case.

1. *A trade-off between private and public good consumption*

$$\Delta^{\tau}(s_{-1},b) \equiv (1-e(\tau))\frac{\theta}{g} - \frac{1}{c} = 0, \tag{21}$$

2. *A GEE for public good consumption*

$$\Delta^{g}(s_{-1},b) \equiv -\frac{\theta}{g} + \lambda\beta R\frac{\theta}{g'} - (\delta-1)\lambda\beta\left(\frac{\theta}{g'}G_2(s,b') + \frac{1}{c'}C_2(s,b')\right) = 0, \tag{22}$$

3. *A GEE for private consumption*

$$\Delta^{c}(s_{-1},b) \equiv -\frac{1}{c} + \lambda\beta R\frac{1}{c'} + (\delta-1)\lambda\beta\left(\frac{\theta}{g'}G_1(s,b') + \frac{1}{c'}C_1(s,b')\right) = 0, \tag{23}$$

where, in the three equations above, $g = G(s_{-1},b)$, $c = C(s_{-1},b)$, $\tau = T(s_{-1},b)$, $g' = G(s,b')$, $c' = C(s,b')$, $s = Rs_{-1} + A(\tau) - c$, *and* $b' = g + Rb - \tau wH(\tau)$.

Equations (21)-(22) are the analogues of equations (15)-(16). As in the benchmark case, time inconsistency vanishes when $\omega = 1$, in which case the GEE is a standard Euler equation. The third term in (22) is the strategic (*disciplining*) effect which now incorporates the additional term $C_2(s,b')/c'$ capturing the fact that a debt-financed increase in g affects future private consumption by reducing the planner's total wealth. In the no-bequest case, $C_2 = 0$ since there the future consumption of the current young was independent of b'. In the no-bequest case, there is no GEE for private consumption (see equation (23)), since there every agent was born with zero private wealth. The first two terms yield a standard Euler equation for private consumption. The third term is a strategic effect: by saving, the planner increases her future selves' wealth which in turn affects the future provision of c and g.

It is possible to provide a full analytical characterization of the E-planner allocation.

Proposition 5 *The E-planner allocation is characterized as follows:*

1. *The tax policy function,* $T(s_{-1},b)$, *is the unique solution to the following equation:*

$$\theta\left(1-e\left(T(s_{-1},b)\right)\right)\left(\frac{A(T(s_{-1},b))}{R-1} + s_{-1}\right) = \frac{w\tau H(T(s_{-1},b))}{R-1} - b. \tag{24}$$

The other equilibrium policy functions are given by:

$$G(s_{-1},b) = (R-\varphi)\left(\tfrac{w\tau H(\tau)}{R-1} - b\right), \qquad C(s_{-1},b) = (R-\varphi)\left(\tfrac{A(\tau)}{R-1} + s_{-1}\right),$$
$$B(s_{-1},b) = \varphi b + (1-\varphi)\tfrac{w\tau H(\tau)}{R-1}, \qquad S(s_{-1},b) = \varphi s_{-1} - (1-\varphi)\tfrac{A(\tau)}{R-1},$$

where $\tau = T(s_{-1},b)$ *and* $\varphi \equiv \left(1 + \frac{(\delta-1)(1-\beta\lambda)}{1+\beta\lambda(\delta-1)}\right)\beta\lambda R > \beta\lambda R.$

2. *Along the equilibrium path, the tax rate is constant, and equal to* $\tau = T\left(\hat{s}_{-1}, \hat{b}_0\right)$*, where* $\hat{s}_{-1}$ *and* $\hat{b}_0$ *denote initial conditions,* $g'/g = c'/c = \varphi$ *and* $g/c = \theta\,(1 - e(\tau))$.

The E-planner chooses a constant tax rate, and uses public debt to ensure that along transition the ratio between private and public consumption is kept constant, as required by condition 1 in Proposition 4. This can be viewed as an extension of the tax-smoothing result of Barro (1979). The growth rate of c and g is larger than the term $\beta\lambda R$, due to a version of the disciplining effect studied above. Thus, a positive wealth accumulation can be sustained even though $\beta\lambda R < 1$. Note that taxes remain constant even though the state vector (s_{-1}, b) changes over time during transition.

5.2 The N-planner Allocation (DMPPE)

With the aid of the E-planner allocation, we can now analyze the N-planner allocation, which is equivalent to the MPPE, i.e., the allocation we are ultimately interested in. We start from the households' saving decision, which provides the implementability constraint for the N-planner (proof omitted).

Lemma 6 *The growth rate of total consumption follows a standard Euler equation,*

$$\frac{c'}{c} = \beta\lambda R. \tag{25}$$

Moreover, consumption and savings satisfy:

$$\begin{aligned} c &= C(s_{-1}, b) = (1 - \beta\lambda)(Rs_{-1} + W(s_{-1}, b)) && (26)\\ s &= Rs_{-1} + A(\tau) - C(s_{-1}, b), && (27) \end{aligned}$$

where W *denotes the discounted future after-tax income satisfying the recursion:* $W(s_{-1}, b) = A(T(s_{-1}, b)) + R^{-1}W(s, b')$*,* s *is given by (27), and* $b' = B(s_{-1}, b)$*. The function* $W(s_{-1}, b)$ *exists and is unique.*

Note that private consumption growth in (25) is lower than in the E-planner allocation. Intuitively, the old – who dictate saving decisions – have a lower discount factor than the E-planner. $C(s_{-1}, b)$ is the consumption function along the equilibrium path. However, the planner needs to know how current consumption responds to fiscal policy deviations in the current period. To this aim, let $\tilde{C}(g, \tau, b', s_{-1})$ denote current consumption as a function of the current fiscal policy under the assumption that equilibrium policies apply from the next period onward. If g, τ and b' are evaluated at the equilibrium, we have

$\tilde{C}\left(G\left(s_{-1},b\right),T\left(s_{-1},b\right),B\left(s_{-1},b\right),s_{-1}\right)=C\left(s_{-1},b\right)$. Then, (26) implies that $\tilde{C}$ must satisfy the following functional equation:[19]

$$\tilde{C}\left(g,\tau,b',s_{-1}\right)=\left(1-\beta\lambda\right)\left(Rs_{-1}+A\left(\tau\right)+R^{-1}W\left(Rs_{-1}+A\left(\tau\right)-\tilde{C}\left(g,\tau,b',s_{-1}\right),b'\right)\right). \tag{28}$$

The N-planner allocation is the solution to program (19)-(20), subject to the same constraints as in the E-planner problem, plus the implementability constraints

$$c=\tilde{C}\left(g,\tau,b',s_{-1}\right),\ \ s=Rs_{-1}+A\left(\tau\right)-\tilde{C}\left(g,\tau,b',s_{-1}\right). \tag{29}$$

The next proposition characterizes the solution to this program.[20]

Proposition 6 *Assume that $s_{-1}\geq\underline{s}$ and $b\leq\bar{b}$. A differentiable time-consistent N-planner allocation satisfies the following system of functional equations:*

$$0 = \Delta^{g}\left(s_{-1},b\right)+\Delta^{\tau}\left(s_{-1},b\right)\cdot\frac{C_{2}\left(s,b'\right)}{\lambda\beta R} \tag{30}$$

$$0 = \Delta^{c}\left(s_{-1},b\right)-\Delta^{\tau}\left(s_{-1},b\right)+\beta\lambda R\cdot\Delta^{\tau}\left(s,b'\right)-\Delta^{\tau}\left(s_{-1},b\right)\cdot\frac{C_{1}\left(s,b'\right)}{\lambda\beta R} \tag{31}$$

where Δ^{g}, Δ^{c} and Δ^{τ} are defined in Proposition 4, $C\left(s,b'\right)=\lambda\beta R\cdot\tilde{C}\left(g,\tau,b',s_{-1}\right)$, and b' and s satisfy the government and household intertemporal budget constraints, respectively.

Proposition 4 defined three wedges, Δ^{g},Δ^{τ} and Δ^{c}. The E-planner would set all wedges to zero. In contrast, the N-planner does not control private consumption and, therefore, has only two independent fiscal policy instruments. As she cannot set all three wedges to zero, she must make trade-offs.

Consider, first, the GEE for public consumption, (30). On the one hand, setting $\Delta^{g}=0$ would require a low public consumption today and a high growth rate of g. On the other hand, setting $\Delta^{\tau}=0$ while holding τ constant (as the E-planner would do) would require that she keep constant the g/c ratio, letting c and g grow at the same rate. However, it is impossible for the N-planner to achieve both objectives because households are more impatient than she is, and they choose a low private consumption growth. When trading off these objectives, the

[19] From Lemma 6 off-equilibrium consumption and discounted future after-tax income must satisfy

$$\begin{aligned}\tilde{C}\left(g,\tau,b',s_{-1}\right) &= \left(1-\beta\lambda\right)\left(Rs_{-1}+\tilde{W}\left(g,\tau,b',s_{-1}\right)\right),\\ \tilde{W}\left(g,\tau,b',s_{-1}\right) &= A\left(\tau\right)+R^{-1}W\left(Rs_{-1}+A\left(\tau\right)-\tilde{C}\left(g,\tau,b',s_{-1}\right),b'\right).\end{aligned}$$

Substituting away $\tilde{W}$ yields expression (28).

[20] The Euler equation allows us to eliminate $\tilde{C}$ and its derivative and express all GEEs as functions of C. See the proof of Proposition 6 for details.

N-planner chooses a lower growth rate of g than would the E-planner, but one that exceeds the growth rate of private consumption. As a result, the MPPE features positive wedges Δ^g and Δ^τ as well as a g/c ratio that increases over time.[21]

Consider, next, the GEE for taxes, (31). The E-planner would set $\Delta^c = \Delta^\tau = 0$. However, the N-planner cannot achieve $\Delta^c = 0$ as private consumption is controlled by the old. Thus, the N-planner uses the tax sequence to trade off the two wedges. In general, taxes will not be constant since the wedges change over time.

Contrary to the E-planner allocation, a full analytical characterization of the equilibrium is not available. Nevertheless we can establish a key long-run property of the model.

Corollary 1 *Assume $\beta\lambda R < 1$. Then, the DMPPE of Proposition 6 features* $\lim_{t\to\infty} c_t = 0$ *and* $\lim_{t\to\infty} g_t = 0$.

In sharp contrast with the no-bequest economy of section 3 the DMPPE features both private and public poverty in the long run. The intuition is simple: On the one hand, old agents with low altruism control private consumption and choose a consumption sequence converging to zero. On the other hand, although the fiscal policy is subject to the disciplining influence of the young, a falling c would open an arbitrarily large gap between the marginal utility of private and public good consumption, unless it were accompanied by a fall in g. More formally, as the marginal utility of private consumption tends to infinity, that of public good consumption must also tend to infinity; or, otherwise, the intratemporal wedge would grow without bound. This tension was absent in the model of section 3. There, private consumption was protected by the existence of a Laffer curve and by the inability of the old to leave negative bequests, implying that the marginal utility of private consumption was bounded in equilibrium. This made it possible to sustain equilibria where g does not fall to zero.[22]

Figure 4 illustrates the equilibrium dynamics of the E-planner and N-planner allocations for the parameter values of Table 1. It displays the time path of private (*panel a*) and public good (*panel b*) consumption, tax rate (*panel c*), g/c (*panel d*), private savings (*panel e*) and public debt (*panel f*) in the E-planner and N-planner (DMPPE) allocations. The initial condition for b and private wealth is in all cases the steady state of the benchmark economy of Table 1. As shown in Proposition 5, the E-planner attains a permanently higher private consumption growth than does the N-planner. Moreover, the E-planner lets c and g grow at a common

[21] Note that $C_2(s, b') < 0$, since a higher debt decreases total wealth. Thus, the sign of the two wedges must be the same. Since the current c is too high for the taste of the planner, Δ^τ must be positive.

[22] In SSZ12, we show that in an OLG economy with an inelastic labor supply c/g remains constant along the DMPPE path, even though bequests are constrained to be non-negative. In that case, we obtain that $\lim_{t\to\infty} c_t = 0$ and $\lim_{t\to\infty} g_t = 0$ as in the N-planner allocation.

rate. Under this calibration, the disciplining effect is so strong that the E-planner accumulates both private and public wealth, inducing an ever-growing sequence of c and g.[23] In contrast, the N-planner allocation converges to private and public poverty, consistent with Corollary 1. While the E-planner chooses a constant tax rate that is higher than the steady state in the no-bequest economy, the tax rate falls over time in the N-planner allocation. Finally, the government accumulates assets in both the E-planner and the N-planner allocations, whereas b remains constant in the benchmark case. However, while in the E-planner allocation agents also accumulate private wealth, this is depleted in the N-planner allocation.

FIGURE 4 HERE

Taking stock, the analysis of this section, which allows negative bequests, shows that in a world of low altruism, if a small open economy decided to start enforcing debt passed on from parents to their heirs, this economy would accumulate both private and public debt and experience an increasingly negative foreign asset position.

6 Conclusion

In this paper, we construct a theory of intergenerational private and public wealth transmission. We do so under the maintained assumptions that fiscal policy is determined by an elected government, agents have finite lives and low altruism, and the government cannot default on its debt. We study the equilibrium dynamics in different environments. In the benchmark model, we maintain that agents cannot die with a negative private wealth, and we consider alternative assumptions about the government's ability to tie future governments' hands (commitment). In this environment, if the fiscal policy path were set on behalf of the first generation of voters with full commitment, the economy would fall into public poverty: future generations would end up being taxed at the top of the Laffer curve to repay the public debt accumulated by their ancestors. In contrast, if fiscal policy is decided sequentially by elected governments, the long-run equilibrium may feature positive private and public consumption.

We contrast the results with an environment in which agents can bequeath negative wealth to their heirs. We show that this possibility weakens fiscal discipline and eventually leads to immiseration, with both public and private consumption going to zero in the long run. The crux for the model to deliver fiscal discipline is, then, a combination of lack of commitment to future fiscal policy and an institutional constraint limiting parents' ability to pass on debt

[23] In the E-planner problem, we do not have to impose a lower bound on b since we have an analytical solution. To avoid uninteresting complications, we have allowed negative taxes in these simulations.

to their children. When these two elements are present, as debt accumulates public good consumption falls relative to private consumption. The coexistence of public poverty and private affluence (Galbraith 1958) increases voters' desire to limit public debt accumulation in order to prevent public good provision from falling further.

Our analysis also gives reason to caution against excessive optimism. In particular, if progress in the private provision of collective goods makes it easier to substitute public good for private consumption, this may erode the fiscal discipline.

We believe that our theory offers foundations for the analysis of important aspects of the intergenerational conflict that are not addressed in this paper. For instance, altruism, the social discount factor and lack of commitment are central issues pertaining to environmental sustainability. Our theory identifies politico-economic forces that may push governments to intervene, or fail to intervene with sufficient energy, against the depletion of natural resources.

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Appendix A: Proofs of Lemmas and Propositions

Proof of Lemma 1. The proof strategy is based on constructing a tax sequence such that – conditional on λ – the incentive to bequeath is maximum, and then finding the range of low λ's such that agents do not wish to leave positive bequests even in this case. Since the incentive to bequeath is maximum when the next generation has a low private consumption relative to the current generation, we construct a worst-case scenario in which agents born at time zero face zero taxes, whereas agents born at period one and later are taxed at the top of the Laffer curve, $\tau_t = \bar{\tau}$. Then, we show that for sufficiently low lambda, agents do not leave positive bequests even in this scenario. Suppose that $\tau_t = \bar{\tau}$ for all $t \geq 1$, and that $x_{Y,t} = 0$. Given Assumption 1 and the constant disposable income, $\lambda \leq (\beta R)^{-1} \Rightarrow x_{O,t} = 0$, for $t > 1$. Moreover, for $t > 1$, $c_{Y,t} = \underline{c_Y}$ and $c_{O,t+1} = \underline{c_O}$, where $\{\underline{c_Y}, \underline{c_O}\} \in (R^+)^2$ is the solution of the individual optimization, characterized by $\tilde{u}'\left(\underline{c_Y}\right)/\tilde{u}'\left(\underline{c_O}\right) = \beta R$ and $\underline{c_Y} + \underline{c_O}/R = A(\bar{\tau})$. Suppose, next, that $\tau_0 = 0$ and $x_{Y,0} = \hat{x}$. Let $\{\overline{c_Y}(\hat{x}), \overline{c_O}(\hat{x})\} \in (R^+)^2$ be the the solution of the individual optimization $(\tilde{u}'(\overline{c_Y}(\hat{x}))/\tilde{u}'(\overline{c_O}(\hat{x})) = \beta R$ and $\overline{c_Y}(\hat{x}) + \overline{c_O}(\hat{x})/R = A(0) + \hat{x})$. Let $\bar{\lambda}(\hat{x})$ be such that $\bar{\lambda}(\hat{x}) = \tilde{u}'(\overline{c_O}(\hat{x}))/\tilde{u}'\left(\underline{c_Y}\right)$. Since $\tilde{u}'(\overline{c_O}(\hat{x})) > 0$ and $0 < \tilde{u}'\left(\underline{c_Y}\right) < \infty$, then $\bar{\lambda}(\hat{x}) > 0$. An agent endowed with $\lambda = \bar{\lambda}(\hat{x})$ finds it optimal to leave zero bequests, whereas any agent with $\lambda < \bar{\lambda}(\hat{x})$ would strictly prefer to leave negative bequests. Since this is forbidden by Assumption 1, then $\lambda < \bar{\lambda}(\hat{x}) \Rightarrow x_{O,1} = 0$. Moreover, since $\overline{c_O}(\hat{x})$ is the upper bound to the consumption of an old agent who started with an inherited wealth $\hat{x}$, while $\underline{c_Y}$ is the lower bound to the consumption of a young agent, no agent with $\lambda \leq \bar{\lambda}(\hat{x})$ will choose positive bequest for any feasible tax sequences not exceeding the top of the Laffer curve. QED

Proof of Proposition 1. Since $\beta\lambda R < 1$, then equation (11) implies that $\lim_{t\to\infty} u'(g_t) = \infty$, and hence $\lim_{t\to\infty} g_t = 0$**.** Since $\lim_{t\to\infty} u'(g_t) = \infty$, then (10) implies that $\lim_{t\to\infty} e(\tau_t) = 1$, which in turn implies that $\lim_{t\to\infty} \tau_t = \bar{\tau}$. These two facts, together with (5), establish that $\lim_{t\to\infty} b_t = \bar{b}$. QED

Proof of Lemma 3. Recall the sequential formulation of the planner's objective function, (1):

$$U_t = v(\tau_0, g_0) + (\delta - 1)\lambda\phi(A(\tau_0)) + \delta \sum_{t=1}^{\infty} (\lambda\beta)^t v(g_t, \tau_t).$$

Along the equilibrium path $\langle g_t, \tau_t \rangle = \left\langle G\left(B^t(b)\right), T\left(B^t(b)\right)\right\rangle$. Given the policy rules $T(b)$, $G(b)$ and $B(b)$, the discounted utility of the old can then be written as $V_O(b) \equiv \sum_{t=0}^{\infty} (\lambda\beta)^t v\left(G\left(B^t(b)\right), T\left(B^t(b)\right)\right)$, where $V_O(b)$ satisfies the functional equation (14). Therefore, part 1 of the Definition 1 of MPPE can be rewritten as equation (13) subject to (5) and the function V_O solving (14). QED

Proof of Proposition 2. The FOCs of the program (13) with respect to τ and g (after substituting away b' using (5)) yield:

$$\begin{aligned} -\delta\lambda\phi'(A(\tau)) - \delta\beta\lambda V_O'\left(b'\right)(1 - e(\tau)) &= 0, & (32)\\ (1+\lambda)u'(g) + \delta\beta\lambda V_O'\left(b'\right) &= 0, & (33)\end{aligned}$$

where we have used the definition of $e(\tau)$ and the envelope condition, $A'(\tau) = -wH(\tau)$. Combining the two FOCs yields equation (15).

Next, consider (14). Differentiating $V_O(b)$ using (15) yields

$$V_O'(b) = u'(G(b))\left(\begin{array}{c}(1+\lambda)G'(b) - \\ \frac{1+\lambda}{\delta}(1 - e(T(b)))wH(T(b))T'(b)\end{array}\right) + \beta\lambda V_O'(B(b))B'(b). \quad (34)$$

Leading by one period equation (34) yields an expression for $V_O'(b')$ which can be used, together with (33), to eliminate $V_O'(b')$ and $V_O'(B(b'))$.[24] The resulting expression is:

$$\frac{1+\lambda}{\delta\beta\lambda}u'(g) = (1+\lambda)\,u'(G(b'))\left(\frac{R}{\delta} - \left(1-\frac{1}{\delta}\right)G'(b')\right).$$

Rearranging terms leads to the GEE, (16). QED

Characterization of the intrahousehold allocation of consumption (section 5). Denote by x the bequests that the young receive from the old. Then, the utility of the young can be written as

$$U_Y(\boldsymbol{x},\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}) = \log(c_Y) + \theta\log(g) + \beta\left(\log\left(c_O'\right) + \theta\log\left(g'\right) + \lambda U_Y\left(\boldsymbol{x}',\boldsymbol{b}',\boldsymbol{\tau}',\boldsymbol{g}'\right)\right).$$

They maximize U_Y subject to

$$\begin{aligned} c_Y &= x + A(\tau) - s, \\ c_O' &= Rs - x'. \end{aligned}$$

Differentiating $U_Y(\boldsymbol{x},\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g})$ w.r.t. x, plus the standard Envelope argument, yields

$$\frac{\partial U_Y(\boldsymbol{x},\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g})}{\partial x} = \frac{1+\beta}{x + A(\tau) - x'/R}. \tag{35}$$

Next, consider the optimal bequest problem. The utility of the old is

$$U_O(s_{-1},\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}) = \log(c_O) + \theta\log(g) + \lambda U_Y(\boldsymbol{x},\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}),$$

where

$$c_O = Rs_{-1} - x.$$

Given (35) and the fact that $c_Y = (x + A(\tau) - x'/R)/(1+\beta)$, the FOC of the above problem implies

$$\frac{c_Y}{c_O} = \lambda. \tag{36}$$

Therefore, $c_Y = \lambda(1+\lambda)^{-1}c$, and $c_O = (1+\lambda)^{-1}c$, where c denotes total household consumption.

Proof of Proposition 4. The FOCs of the program (19)-(20) w.r.t. τ, g and c yield:

$$\begin{aligned} \delta\beta\lambda\left(V_1^b(s,b')\,A'(\tau) - V_2^b(s,b')\,(1-e(\tau))\,wH(\tau)\right) &= 0, &(37)\\ \frac{\theta}{g} + \delta\beta\lambda V_2^b(s,b') &= 0, &(38)\\ \frac{1}{c} - \delta\beta\lambda V_1^b(s,b') &= 0, &(39) \end{aligned}$$

[24] $B'(b)$ is obtained by differentiating $B(b)$ from equation (12).

where subscripts denote partial derivatives. First, using (38)-(39) to eliminate $V_2^b(s,b')$ and $V_1^b(s,b')$ from (37), and recalling that $A'(\tau) = -wH(\tau)$, yields (21). Next, differentiating $V^b(s_{-1},b)$ w.r.t. its arguments, and applying the Envelope theorem, yields:

$$V_1^b(s_{-1},b) = \left(1-\frac{1}{\delta}\right)\frac{C_1(s_{-1},b)}{c} + \left(1-\frac{1}{\delta}\right)\frac{\theta G_1(s_{-1},b)}{g} + \beta\lambda R V_1^b(s,b'), \quad (40)$$

$$V_2^b(s_{-1},b) = \left(1-\frac{1}{\delta}\right)\frac{C_2(s_{-1},b)}{c} + \left(1-\frac{1}{\delta}\right)\frac{\theta G_2(s_{-1},b)}{g} + \beta\lambda R V_2^b(s,b'). \quad (41)$$

We now use (38)-(39) to eliminate $V_1^b(s_{-1},b)$, $V_1^b(s_{-1},b)$, $V_2^b(s,b')$ and $V_1^b(s,b')$ from (40)-(41), respectively, and lead the expressions by one period. This yields

$$-\frac{\theta}{\delta\beta\lambda g} = \left(1-\frac{1}{\delta}\right)\frac{C_2(s_{-1},b)}{c} + \left(1-\frac{1}{\delta}\right)\frac{\theta G_2(s_{-1},b)}{g} - \frac{R}{\delta}\frac{\theta}{g'},$$

$$\frac{1}{\delta\beta\lambda c} = \left(1-\frac{1}{\delta}\right)\frac{C_2(s_{-1},b)}{c} + \left(1-\frac{1}{\delta}\right)\frac{\theta G_2(s_{-1},b)}{g} + \frac{R}{\delta}\frac{1}{c'}.$$

After rearranging terms, this yields (22)-(23). QED

Proof of Proposition 5. The proof consists of guessing-and-verifying that the policy functions $T(s_{-1},b)$, $G(s_{-1},b)$, $C(s_{-1},b)$ and $B(s_{-1},b)$ given in the Proposition, and the ensuing equilibrium law of motion of c and g ($c'/c = g'/g = \varphi$) satisfy the equilibrium conditions of Proposition 4. First, using (24) and the guesses of G and C yields

$$\theta(1-e(\tau)) = \frac{\frac{w\tau H(\tau)}{R-1} - b}{\frac{A(\tau)}{R-1} + s_{-1}} = \frac{G(s_{-1},b)}{C(s_{-1},b)}$$

which verifies the intratemporal trade-off, (21), while proving that $T(s_{-1},b)$ must be constant along the equilibrium path (suppose not, then g/c would change over time, contradicting that $c'/c = g'/g$). To show that $T(s_{-1},b)$ is the unique solution to (24) note that the right-hand side of (24) is non-negative for $\tau = \bar{\tau}$ (since $b \le \bar{b}$) and is continuous and monotone increasing in τ for $\tau \le \bar{\tau}$ (since $A' \le 0$ and $e' \ge 0$). Moreover, the left-hand side of (24) is zero for $\tau = \bar{\tau}$ (since $e(\bar{\tau}) = 1$) and is continuous and monotone decreasing in τ for $\tau \le \bar{\tau}$ (since $e'(\tau) \ge 0$ and $A'(\tau) \ge 0$). A standard fixed-point argument establishes uniqueness. Next, differentiating the equilibrium policy function (recalling that $A' = -wH(\tau)$ and $e(\tau) \equiv -\tau H'(\tau)/H(\tau)$), and leading the expressions one period, yields the following partial derivatives: $G_1(s,b') = (R-\varphi)\frac{wH(\tau)}{R-1}(1-e(\tau))T_1(s,b')$, $G_2(s,b') = (R-\varphi)\left(\frac{wH(\tau)}{R-1}(1-e(\tau))T_2(s,b') - 1\right)$, $C_1(s,b') = -(R-\varphi)\left(\frac{wH(\tau)}{R-1}T_1(s,b') - 1\right)$, and $C_2(s,b') = -(R-\varphi)\frac{wH(\tau)}{R-1}T_2(s,b')$. Note that in all expressions $\tau = T(s_{-1},b)$. Next, rewrite the GEE for g, (22) as

$$\begin{aligned} 0 &= -\frac{g'}{g} + \lambda\beta R - (\delta-1)\lambda\beta\left(G_2(s,b') + \frac{g'}{\theta c'}C_2(s,b')\right) \\ &= -\varphi + \lambda\beta R - (\delta-1)\lambda\beta\left(G_2(s,b') + (1-e(\tau))C_2(s,b')\right) \end{aligned}$$

Plugging in the expressions of φ, G_2, and C_2, and simplifying terms, verifies the GEE, (22). Similarly, rewrite the GEE for c, (23) as

$$\begin{aligned} 0 &= -\frac{c'}{c}+\lambda\beta R+(\delta-1)\lambda\beta\left(\frac{c'\theta}{g'}G_1\left(s,b'\right)+C_1\left(s,b'\right)\right) \\ &= -\varphi+\lambda\beta R+(\delta-1)\lambda\beta\left(\frac{G_1\left(s,b'\right)}{1-e\left(\tau\right)}+C_1\left(s,b'\right)\right) \end{aligned}$$

Plugging in the expressions of φ, G_1, and C_1, and simplifying terms, verifies the GEE, (23). Finally, we must verify that the government and private budget constraints hold, i.e., $B\left(s_{-1},b\right)=G\left(s_{-1},b\right)+Rb-\tau wH\left(\tau\right)$ and $S\left(s_{-1},b\right)=Rs_{-1}+A\left(\tau\right)-C\left(s_{-1},b\right).$ Given the expressions of B, G, S and C, it is straightforward to verify that both conditions hold. QED

Proof of Proposition 6. First note that the household Euler equation (25) implies that

$$C\left(Rs_{-1}+A\left(\tau\right)-c,g+Rb-\tau wH\left(\tau\right)\right)=\lambda\beta Rc, \tag{42}$$

where $c=\tilde{C}\left(g,\tau,b',s_{-1}\right)$. Differentiating (42) yields

$$\begin{aligned} \tilde{C}_{s_{-1}}\left(g,\tau,b',s_{-1}\right) &= \frac{RC_1\left(s,b'\right)}{\lambda\beta R+C_1\left(s,b'\right)}, \qquad (43)\\ \tilde{C}_{\tau}\left(g,\tau,b',s_{-1}\right) &= \frac{C_1\left(s,b'\right)A'\left(\tau\right)-C_2\left(s,b'\right)\left(1-e\left(\tau\right)\right)wH\left(\tau\right)}{\lambda\beta R+C_1\left(s,b'\right)}, \qquad (44)\\ \tilde{C}_{g}\left(g,\tau,b',s_{-1}\right) &= \frac{C_2\left(s,b'\right)}{\lambda\beta R+C_1\left(s,b'\right)}. \qquad (45) \end{aligned}$$

Now, consider the program (19), subject to the implementability constraint (29). The FOCs w.r.t. τ and g are, respectively:

$$\begin{aligned} \left(\frac{1}{c}-\delta\beta\lambda V_1^b\left(s,b'\right)\right)\tilde{C}_{\tau}+\delta\beta\lambda\left(\begin{array}{c} V_1^b\left(s,b'\right)A'\left(\tau\right) \\ -V_2^b\left(s,b'\right)\left(1-e\left(\tau\right)\right)wH\left(\tau\right)\end{array}\right) &= 0, \qquad (46)\\ \left(\frac{1}{c}-\delta\beta\lambda V_1^b\left(s,b'\right)\right)\tilde{C}_{g}+\frac{\theta}{g}+\delta\beta\lambda V_2^b\left(s,b'\right) &= 0. \qquad (47) \end{aligned}$$

Differentiating $V^b\left(s_{-1},b\right)$ w.r.t. s_{-1} yields

$$\begin{aligned} V_1^b\left(s_{-1},b\right) &= \left(\frac{1}{c}-\beta\lambda\delta V_1^b\left(s,b'\right)\right)\frac{\tilde{C}_{s_{-1}}}{\delta}+\left(1-\frac{1}{\delta}\right)\frac{1}{c}C_1\left(s_{-1},b\right) \qquad (48)\\ &\quad +\left(1-\frac{1}{\delta}\right)\frac{\theta}{g}G_1\left(s_{-1},b\right)+R\beta\lambda V_1^b\left(s,b'\right), \end{aligned}$$

where we use (46)-(47), and the fact that $C_1\left(s_{-1},b\right)=\tilde{C}_{s_{-1}}+\tilde{C}_{\tau}T_1\left(s_{-1},b\right)+\tilde{C}_gG_1\left(s_{-1},b\right).$ Substituting out $V_2^b\left(s,b'\right)$ in (46) and (47), using (44)-(45) to eliminate $\tilde{C}_g$ and $\tilde{C}_\tau$, recalling that $A'\left(\tau\right)=-wH\left(\tau\right)$, and rearranging terms establish:

$$\delta\beta\lambda V_1^b\left(s,b'\right)=\theta\frac{e\left(1-\tau\right)}{g}+\frac{C_1\left(s,b'\right)}{\lambda\beta R}\left(-\frac{1}{c}+\frac{\theta e\left(1-\tau\right)}{g}\right). \tag{49}$$

Using (49) to substitute out $V_1^b(s,b')$ from the RHS of (48), then leading the resulting expression one period and applying (49) again to eliminate $V_1^b(s,b')$ from the LHS establishes (31).

Similarly, differentiating $V^b(s_{-1},b)$ w.r.t. b yields

$$V_2^b(s_{-1},b) = \left(1-\frac{1}{\delta}\right)\frac{C_2(s_{-1},b)}{c} + \left(1-\frac{1}{\delta}\right)\frac{\theta G_2(s_{-1},b)}{g} - \frac{1}{\delta}\frac{\theta R}{g}, \tag{50}$$

where we use (46) and (47), and the fact that $C_2(s_{-1},b) = \tilde{C}_\tau T_2(s_{-1},b) + \tilde{C}_g(G_2(s_{-1},b)+R)$. Substituting (49) back into (47) yields:

$$\delta\beta\lambda V_2^b(s,b') = -\left(\left(1+\frac{C_1(s,b')}{\lambda\beta R}\right)\left(\frac{1}{c}-\frac{\theta e(1-\tau)}{g}\right)\right)\frac{C_2(s,b')}{\lambda\beta R + C_1(s,b')} - \frac{\theta}{g}, \tag{51}$$

where we use (45) to substitute out $\tilde{C}_g$. A combination of (50) and (51) establishes (30). QED

Proof of Corollary 1. That $\lim_{t\to\infty} c_t = 0$ follows immediately from (25). To prove that $\lim_{t\to\infty} g_t = 0$, rearrange (31):

$$\begin{aligned} &\frac{\theta\lambda\beta R(1-e(\tau'))}{g'} - \frac{\theta(1-e(\tau))}{g} + \beta\lambda(\delta-1)\frac{\theta}{g'}G_1(s,b') \\ = &\left(\frac{\theta(1-e(\tau))}{g} - \frac{1}{c}(1+\beta\lambda(\delta-1))\right)\frac{C_1(s,b')}{\lambda\beta R}. \end{aligned} \tag{52}$$

First, we claim that $\lim_{t\to\infty} G_1(s_t,b_{t+1}) > -\infty$ and $\lim_{t\to\infty} C_1(s_t,b_{t+1}) > 0$. The former follows from the assumptions that G is continuous and that $g \geq 0$. The latter follows from the fact that as $\lim_{t\to\infty} C(s_t,b_{t+1}) = 0$, then $\lim_{t\to\infty} C_1(s_t,b_{t+1}) > 0$. Next, suppose for contradiction that $\lim_{t\to\infty} g_t > 0$. Then, the LHS of equation (52) would be finite, while the RHS would go to $-\infty$. This yields a contradiction, and establishes then that $\lim_{t\to\infty} g_t = 0$. QED

Appendix Figures A1 and A2 Appendix Figures A1 and A2 (referred in the text) follow

FIGURE A1 HERE

FIGURE A2 HERE

Appendix B (not for publication)

6.1 The Probabilistic Voting Model

In this section of the online appendix, we reproduce, for the reader's convenience, the description of the probabilistic voting model, based on Lindbeck and Weibull (1987) and Persson and Tabellini (2000), and applied to a dynamic voting setting by SSZ12. This material is also covered in the online appendix of SSZ12, and does not represent an original contribution of this paper.

The population has a unit measure and consists of two groups of voters, young and old, of equal size (we discussed below the extension to groups of different sizes). The electoral competition takes place between two office-seeking candidates, denoted by A and B. Each candidate announces a fiscal policy vector, b', τ, and g, subject to the government budget constraint, $b' = Rb + g - w(R)\,\tau\,H(\tau)$, and to $b' \leq \bar{b}$.[25] Since there are new elections every period, the candidates cannot make credible promises over future policies (i.e., there is lack of commitment beyond the current period). Voters choose either of the candidates based on their fiscal policy announcements and on their relative *appeal*, where the notion of appeal is explained below. In particular, a young voter prefers candidate A over B if, given the inherited debt level b, preference parameter θ, the world interest rate, and the equilibrium policy functions $\langle B, G, T\rangle$ which apply from tomorrow and onwards,

$$U_Y\left(\tau_A, g_A, G\left(b'_A\right); b, \theta, R\right) > U_Y\left(\tau_B, g_B, G\left(b'_B\right); b, \theta, R\right).$$

Likewise, a young voter prefers candidate A over B if

$$U_O\left(g_A; b, \theta, R\right) > U_O\left(g_B; b, \theta, R\right).$$

σ^{iJ} (where $J \in \{Y, O\}$) is an individual-specific parameter drawn from a symmetric group-specific distribution that is assumed to be uniform in the support $[-1/\left(2\kappa^J\right), 1/\left(2\kappa^J\right)]$. Intuitively, a positive (negative) σ^{iJ} implies that voter i has a bias in favor of candidate B (candidate A). Note that the distributions have density κ^J and that neither group is on average biased towards either candidate. The parameter δ is an aggregate shock capturing the ex-post average success of candidate B whose realization becomes known after the policy platforms have been announced. δ is drawn from a uniform i.i.d. distribution on $[-1/(2\psi), 1/(2\psi)]$.[26] The sum of the terms $\sigma^{iJ} + \delta$ captures the relative appeal of candidate B: it is the inherent bias of individual i in group J for candidate B irrespective of the policy that the candidates propose. The assumption of uniform distributions is for simplicity (see Persson and Tabellini (2000), for a generalization).

Note that voters are rational and forward looking. They take into full account the effects of today's choice on future private and public-good consumption. Because of repeated elections, they cannot decide directly over future fiscal policy. However, they can affect it through their choice of next-period debt level (b'), which affects future policy choices through the equilibrium policy functions B, T, and G.

[25] Note that the announcement over the current fiscal policy raises no credibility issue, due to the assumption that the politicians are pure office seekers and have no independent preferences on fiscal policy.

[26] The realization of δ can be viewed as the outcome of the campaign strategies to boost the candidates' popularity. Such an outcome is unknown *ex ante*.

It is at this point useful to identify the "swing voter" of each group, i.e., the voter who is ex-post indifferent between the two candidates:

$$\begin{aligned}\sigma^Y\left(b'_A,\tau_A,g_A,b'_B,\tau_B,g_B;b,\theta,R\right) &= U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)-U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)-\delta\\ \sigma^O\left(g_A,g_B;b,\theta,R\right) &= U_O\left(g_A;b,\theta,R\right)-U_O\left(g_B;b,\theta,R\right)-\delta.\end{aligned}$$

Conditional on δ, the vote share of candidate A is

$$\begin{aligned}&\pi_A\left(b'_A,\tau_A,g_A,b'_B,\tau_B,g_B;b,\theta,R\right)\\ =\ & 1-\pi_B\left(b'_A,\tau_A,g_A,b'_B,\tau_B,g_B;b,\theta,R\right)\\ =\ & \frac{1}{2}\kappa^Y\left(\sigma^Y\left(b'_A,\tau_A,g_A,b'_B,\tau_B,g_B;b,\theta,R\right)+\frac{1}{2\kappa^Y}\right)\\ & +\frac{1}{2}\kappa^O\left(\sigma^O\left(g_A,g_B;b,\theta,R\right)+\frac{1}{2\kappa^O}\right)\\ =\ & \frac{1}{2}+\frac{1}{2}\left(\kappa^Y\times\left(U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)-U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)\right)-\delta\right)\\ & +\frac{1}{2}\left(\kappa^O\times\left(U_O\left(g_A;b,\theta,R\right)-U_O\left(g_B;b,\theta,R\right)\right)-\delta\right).\end{aligned}$$

Note that π_A and π_B are stochastic variables, since δ is stochastic. The probability that candidate A wins is then given by

$$\begin{aligned}p_A &= \mathrm{Prob}_\delta\left[\pi_A\left(b'_A,\tau_A,g_A,b'_B,\tau_B,g_B;b,\theta,R\right)\geq\frac{1}{2}\right]\\ &= \mathrm{Prob}\left[\begin{array}{c}\delta<\frac{\kappa^Y}{\kappa^Y+\kappa^O}\left(U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)-U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)\right)\\ +\frac{\kappa^O}{\kappa^Y+\kappa^O}\left(U_O\left(g_A;b,\theta,R\right)-U_O\left(g_B;b,\theta,R\right)\right)\end{array}\right]\\ &= \frac{1}{2}+\psi\left(1-\omega\right)\left(U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)-U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)\right)\\ &\quad +\psi\omega\left(U_O\left(g_A;b,\theta,R\right)-U_O\left(g_B;b,\theta,R\right)\right),\end{aligned}$$

where $\omega\equiv\kappa^O/\left(\kappa^Y+\kappa^O\right)$.

Since both candidates seek to maximize the probability of winning the election, the Nash equilibrium is characterized by the following equations:

$$\begin{aligned}\left(b'^*_A,\tau^*_A,g^*_A\right) &= \max_{b'_A,\tau_A,g_A}\left(1-\omega\right)\left(U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)-U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)\right)\\ &\quad +\omega\left(U_O\left(g_A;b,\theta,R\right)-U_O\left(g_B;b,\theta,R\right)\right),\\ \left(b'^*_B,\tau^*_B,g^*_B\right) &= \max_{b'_B,\tau_B,g_B}\left(1-\omega\right)\left(U_Y\left(\tau_B,g_B,G\left(b'_B\right);b,\theta,R\right)-U_Y\left(\tau_A,g_A,G\left(b'_A\right);b,\theta,R\right)\right)\\ &\quad +\omega\left(U_O\left(g_B;b,\theta,R\right)-U_O\left(g_A;b,\theta,R\right)\right).\end{aligned}$$

Hence, the two candidates' platform converge in equilibrium to the same fiscal policy maximizing the weighted-average utility of the young and old,

$$\left(b'^*_A,\tau^*_A,g^*_A\right)=\left(b'^*_B,\tau^*_B,g^*_B\right)=\max_{b'\tau,g}\left(\left(1-\omega\right)U_Y\left(\tau,g,G\left(b'\right);b,\theta,R\right)+\omega U_O\left(g;b,\theta,R\right)\right),$$

subject to the government budget constraint. This is the recursive version of the planner's objective function, (1), given in the body of the paper.

Note that the parameter ω has a structural interpretation: it is a measure of the relative variability within the old group of the candidates' appeal. As shown above, κ^Y/κ^O (and, hence, ω) affects the number of swing voters in each group. For instance, suppose that $\kappa^O > \kappa^Y$. Intuitively, this means that the old are more "responsive" in electoral terms to fiscal policy announcements in favor of or against them. An alternative interpretation is that $1/\kappa^J$ measures the extent of group J heterogeneity with respect to other policy dimensions that are orthogonal to fiscal policy. For example, the young might work in different sectors and cast their votes also based on the sectoral policy proposed by each candidate. As a result, the vote of the young is less responsive to fiscal policy announcements, and the young have effectively less political power than the old. This interpretation is consistent with Mulligan and Sala-i-Martin (1999) and Hassler *et al.* (2005). In the extreme case of $\omega = 1$, the old only care about fiscal policy ($\kappa^O \to 0$) and the distribution has a mass point at $\sigma^O = 0$. In this case, the young have no influence and the old dictate their fiscal policy choice (as in the commitment solution with $\alpha = 0$).

Suppose, next, that the groups have different relative size, and that there are N_O old voters and N_Y young voters. Proceeding as above, the planner's objective function is then modified to

$$\begin{aligned}\left(b_A'^{*}, \tau_A^{*}, g_A^{*}\right) &= \left(b_B'^{*}, \tau_B^{*}, g_B^{*}\right) = \\ &= \max_{b'\tau,g} \left\{(1-\omega) N_Y U_Y \left(\tau, g, G\left(b'\right); b, \theta, R\right) + \omega N_O U_O \left(g; b, \theta, R\right)\right\}\end{aligned}$$

We conclude by noting that the probabilistic voting outlined in this appendix applies equally to both static and dynamic models (under the assumption of Markov Perfect Equilibrium). The political model entails some important restrictions. First, agents only condition their voting strategy on the payoff-relevant state variable (here, debt). Second, the shock δ is i.i.d. over time – otherwise, the previous realization of δ becomes a state variable, complicating the analysis substantially. Third, although the assumption of uniform distributions can be relaxed, it is necessary to impose regularity conditions on the density function in order to ensure that the maximization problem is well behaved.

6.2 Statement and Proof of Lemma 7

Lemma 7 *The program (7) subject to (5) and (6) is a contraction mapping. Hence, a solution exists and is unique.*

Proof. Consider the intra-temporal FOC, (10), that is derived in the text. The condition solves

$$g = \Theta\left(\tau\right), \tag{53}$$

with $\Theta'(\cdot) < 0$. We can rewrite the government budget constraint as

$$b' - Rb = \Lambda\left(\tau\right) \equiv \Theta\left(\tau\right) - \tau w H\left(\tau\right),$$

where $\Lambda : [0, \bar{\tau}] \to [-\bar{\tau} w H\left(\bar{\tau}\right), \Theta\left(0\right)]$ is monotonic. Therefore, $\tau = \Lambda^{-1}\left(b' - Rb\right)$. Then, (7) can be rewritten as

$$V_O^{comm}\left(b\right) = \max_{b' \in [\underline{b}, \bar{b}]} \left\{\hat{v}\left(b' - Rb\right) + \beta\lambda V_O^{comm}\left(b'\right)\right\}, \tag{54}$$

where

$$\hat{v}\left(b' - Rb\right) \equiv (1+\lambda)\, u\left(\Theta\left(\Lambda^{-1}\left(b' - Rb\right)\right)\right) + \lambda\phi\left(A\left(\Lambda^{-1}\left(b' - Rb\right)\right)\right).$$

Since the function $\hat{v}$ is bounded and continuous, and $\beta\lambda < 1$, Theorem 4.6 in Stokey and Lucas (1989) establishes that (54) has a unique fixed point. ■

6.3 Statement and Proof of Lemma 8

Lemma 8 *Assume that $\lambda > 0$, and*

$$(1+\lambda)\left(\left(\left(\Lambda^{-1}\right)'\right)^2\left(u''\left(\Theta'\right)^2 + u'\left(\Theta''\right)\right) + u'\Theta'\left(\Lambda^{-1}\right)''\right)$$

$$+\lambda\left(\left(\left(\Lambda^{-1}\right)'\right)^2\left(\phi''\left(A'\right)^2 + \phi' A''\right) + \phi' A'\left(\Lambda^{-1}\right)''\right) < 0, \tag{55}$$

where Λ is defined in the proof of Lemma 7. Then, the unique MPPE of Lemma 7 is a DMPPE.

Proof. The proof is an application of Theorem 2.1 in Santos (1991).[27] The theorem states that the policy functions are differentiable if (i) the return function $\hat{v}$ is strictly concave and (ii) optimal paths are strictly interior. Consider the formulation of the problem used in the proof of Lemma 7. Standard differentiation shows that the function $\hat{v}$ is strictly concave if and only if assumption (55) holds.

We must show that the optimal paths are interior; i.e., that for any $b \in \left(\underline{b}, \bar{b}\right)$,

$$B(b) \in \left(\underline{b}, \bar{b}\right).$$

Since setting $B(b) = \bar{b}$ would imply zero public expenditure in the next period, then $\lambda > 0$ ensures that $B(b) < \bar{b}$. It remains to prove that $B(b) > \underline{b}$ for any $b \in \left(\underline{b}, \bar{b}\right)$. Suppose instead that there exists a $\hat{b} \in \left(\underline{b}, \bar{b}\right)$ such that $B\left(\hat{b}\right) = \underline{b}$. The Euler equation must then be (recall that $\hat{v}'' < 0$);

$$\hat{v}'\left(\underline{b} - R\hat{b}\right) \leq \beta\lambda R\hat{v}'\left(B\left(\underline{b}\right) - R\underline{b}\right). \tag{56}$$

By concavity of $\hat{v}$, $\hat{b} > \underline{b}$ implies $\hat{v}'\left(\underline{b} - R\underline{b}\right) < \hat{v}'\left(\underline{b} - R\hat{b}\right)$. Equation (56) then implies

$$\hat{v}'\left(\underline{b} - R\underline{b}\right) < \beta\lambda R\hat{v}'\left(B\left(\underline{b}\right) - R\underline{b}\right) \tag{57}$$

Thus, if the agent is constrained for some $\hat{b} > \underline{b}$, she must be constrained for $b = \underline{b}$. Hence, $B\left(\underline{b}\right) = \underline{b}$. However, $\beta\lambda R \leq 1$ implies $\hat{v}'\left(\underline{b} - R\underline{b}\right) \geq \beta\lambda R\hat{v}'\left(B\left(\underline{b}\right) - R\underline{b}\right)$, which contradicts (57). This concludes the proof. ■

[27] Santos, Manuel "Smoothness of the Policy Function in Discrete Time Economic Models," *Econometrica*, 59 (1991), 1365-1382.

6.4 Statement and Proof of Proposition 3

For convenience, we restate the Proposition 3 already contained in the text.

Proposition 7 *[RESTATEMENT OF Proposition 3] Let* $\langle \bar{B}(b), \bar{G}(b), \bar{T}(b) \rangle$ *denote equilibrium policies when* $\omega = 1$. *Assume that* $\bar{B}(b), \bar{G}(b), \bar{T}(b)$ *are continuously differentiable. If*

$$\left| \frac{u''(g)}{\beta\lambda R u''(g')} - \bar{G}'(b') \left(1 - \frac{\phi'(A(\tau)) u''(g) wH(\tau)(1 - e(\tau))/u'(g)}{\phi''(A(\tau)) A'(\tau) + \phi'(A(\tau)) e'(\tau)/(1 - e(\tau))}\right) \right| > 1,$$

where $g = \bar{G}(b)$, $g' = \bar{G}(b')$, $\tau = \bar{T}(b)$ *and* $b' = \bar{B}(b)$. *Then, for* ω *close to unity, there exists a unique DMPPE.*

Proof. The strategy of the proof is based on Judd (2004). Let $\langle \bar{B}(b), \bar{G}(b), \bar{T}(b) \rangle$, denote the equilibrium policies when $\omega = 1$. Time subscripts will denote partial derivatives. We rewrite the equilibrium conditions (12), (16) and (15) in the following form:

$$\frac{u'(\bar{G}(b))}{u'\left(\bar{G}\left(\hat{B}(\bar{G}(b), b)\right)\right)} = \beta\lambda R - \beta\lambda G'\left(\hat{B}(\bar{G}(b), b)\right) \frac{1-\omega}{\lambda(1+\lambda\omega)}, \tag{58}$$

$$\hat{B}(\bar{G}(b), b) = \bar{G}(b) + Rb - \hat{T}(\bar{G}(b)) \cdot w \cdot H\left(\hat{T}(\bar{G}(b))\right), \tag{59}$$

$$\phi'\left(A\left(\hat{T}(\bar{G}(b))\right)\right) = \frac{1+\omega\lambda}{1-\omega(1-\lambda)} \left(1 - e\left(\hat{T}(\bar{G}(b))\right)\right) u'(\bar{G}(b)). \tag{60}$$

Let $\varepsilon \equiv \frac{\beta(1-\omega)}{\lambda(1+\lambda\omega)}$ where $\lim_{\omega\to 1} \varepsilon = 0$. We prove that in the neighborhood of $\varepsilon = 0$ there exists a unique policy function $G(b, \varepsilon)$ that solves the GEE, (58). Note that $G(b, \varepsilon)$ involves some slight abuse of notation. We plug-in the candidate equilibrium function $G(b, \varepsilon)$ into (58), obtaining

$$\Pi(\varepsilon, G(b, \varepsilon)) \equiv \frac{u'(G(b, \varepsilon))}{u'\left(G\left(\hat{B}(G(b, \varepsilon), b), \varepsilon\right)\right)} - \beta\lambda R + \varepsilon G_1\left(\hat{B}(G(b, \varepsilon), b), \varepsilon\right) = 0, \tag{61}$$

where we define

$$\hat{B}(G(b, \varepsilon), b) = G(b, \varepsilon) + Rb - \hat{T}(G(b, \varepsilon)) H\left(\hat{T}(G(b, \varepsilon))\right). \tag{62}$$

Next, we differentiate (61) with respect to ε, and evaluate the resulting expression at $\varepsilon = 0$ (recalling that $G(b, 0) = \bar{G}(b)$ and $G_1(b, 0) = \bar{G}'(b)$).

$$\begin{aligned}
&\frac{u''(\bar{G}(b)) G_2(b, 0)}{u'\left(\bar{G}\left(\hat{B}(\bar{G}(b), b)\right)\right)} \\
&- \frac{u'(\bar{G}(b)) u''\left(\bar{G}\left(\hat{B}(\bar{G}(b), b)\right)\right)}{\left(u'\left(\bar{G}\left(\hat{B}(\bar{G}(b), b)\right)\right)\right)^2} \begin{pmatrix} \bar{G}'\left(\hat{B}(\bar{G}(b), b)\right) \hat{B}_1(\bar{G}(b), b) G_2(b, 0) \\ + G_2\left(\hat{B}(\bar{G}(b), b), 0\right) \end{pmatrix} \\
&+ \bar{G}'\left(\hat{B}(\bar{G}(b), b)\right) = 0
\end{aligned}$$

After rearranging terms and using the fact that $u'\left(\bar{G}(b)\right) = \beta\lambda R u'\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)$ as implied by the GEE when $\varepsilon = 0$, we obtain:

$$\begin{aligned}&-\frac{\beta\lambda R u''\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)}{u'\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)} G_2\left(\hat{B}\left(\bar{G}(b), b\right), 0\right) \\ &+\left(\begin{array}{c} -\frac{\beta\lambda R u''\left(\bar{G}\left(\hat{B}\left(\bar{G}(b),b\right)\right)\right)}{u'\left(\bar{G}\left(\hat{B}\left(\bar{G}(b),b\right)\right)\right)} \bar{G}'\left(\hat{B}\left(\bar{G}(b), b\right)\right) \hat{B}_1\left(\bar{G}(b), b\right) \\ +\frac{u''\left(\bar{G}(b)\right)}{u'\left(\bar{G}\left(\hat{B}\left(\bar{G}(b),b\right)\right)\right)}\end{array}\right) G_2(b, 0) \\ &+\bar{G}'\left(\hat{B}\left(\bar{G}(b), b\right)\right) = 0\end{aligned} \tag{63}$$

Therefore, (63) implies that:

$$G_2(b, 0) = J(b) \cdot G_2\left(\hat{B}\left(\bar{G}(b), b\right), 0\right) + Z(b), \tag{64}$$

where

$$\begin{aligned} J(b) &= \left(\frac{u''\left(\bar{G}(b)\right)}{\beta\lambda R u''\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)} - \bar{G}'\left(\hat{B}\left(\bar{G}(b), b\right)\right) \hat{B}_1\left(\bar{G}(b), b\right)\right)^{-1} \\ Z(b) &= J(b) \cdot \bar{G}'\left(\hat{B}\left(\bar{G}(b), b\right)\right)\left(-\frac{u'\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)}{\beta\lambda R u''\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)}\right)\end{aligned}$$

Note that (64) has an iterative nature. Define the mapping:

$$\left(\Upsilon\ G_2(b, 0)\right)(b) \equiv J(b) \cdot G_2\left(\hat{B}\left(\bar{G}(b), b\right), 0\right) + Z(b).$$

If $|J(b)| < 1$, then Υ is a contraction mapping. We now show $|J(b)| < 1$ if and only if assumption (17) holds. Differentiating equation (60), we solve

$$\hat{T}'\left(\bar{G}(b)\right) = \frac{\frac{1+\omega\lambda}{1-\omega(1-\lambda)}\left(1 - e\left(\bar{T}(b)\right)\right) u''\left(\bar{G}(b)\right)}{\phi''\left(A\left(\bar{T}(b)\right)\right) A'\left(\bar{T}(b)\right) + \frac{1+\omega\lambda}{1-\omega(1-\lambda)} e'\left(\bar{T}(b)\right) u'\left(\bar{G}(b)\right)}. \tag{65}$$

Differentiating equation (59), together with (65), leads to

$$\begin{aligned}\hat{B}_1\left(G(b), b\right) &= 1 - \hat{T}'\left(\bar{G}(b)\right) wH\left(\bar{T}(b)\right)\left(1 - e\left(\bar{T}(b)\right)\right) \\ &= 1 - \frac{\frac{1+\omega\lambda}{1-\omega(1-\lambda)}\left(1 - e\left(\bar{T}(b)\right)\right) u''\left(\bar{G}(b)\right) wH\left(\bar{T}(b)\right)\left(1 - e\left(\bar{T}(b)\right)\right)}{\phi''\left(A\left(\bar{T}(b)\right)\right) A'\left(\bar{T}(b)\right) + \frac{1+\omega\lambda}{1-\omega(1-\lambda)} e'\left(\bar{T}(b)\right) u'\left(\bar{G}(b)\right)}.\end{aligned}$$

Hence

$$|J(b)| = \left|\left(\frac{u''\left(\bar{G}(b)\right)}{\beta\lambda R u''\left(\bar{G}\left(\hat{B}\left(\bar{G}(b), b\right)\right)\right)} - \bar{G}'\left(\hat{B}\left(\bar{G}(b), b\right)\right) \cdot \hat{B}_1\left(\bar{G}(b), b\right)\right)^{-1}\right| < 1,$$

by assumption (17) and the intra-temporal FOC (15) when $\omega = 1$. This establishes that Υ is a contraction mapping. Therefore, in the neighbuorhood of $\omega = 1$, there exists a unique derivative $G_2(b, 0)$.

Finally, we must show that the existence of a unique derivative $G_2(b, 0)$ establishes the existence of a unique equilibrium policy function, $G(b, \varepsilon)$, that satisfies the GEE. Differentiating the functional equation (61) with respect to ε and evaluating the result at $\varepsilon = 0$ lead to the linear operator equation

$$\Pi_1(0, G(b, 0)) + \Pi_2(0, G(b, 0)) G_2(b, 0) = 0.$$

The existence and the uniqueness of $G_2(b, 0)$ imply that $\Pi_2(0, G(b, 0))$ is invertible at neighborhood of $\varepsilon = 0$. Therefore, we can apply implicit function theorem (Judd, 2004, pp. 10) to show that there are neighborhoods ε_0 of $\varepsilon = 0$ and for all $\varepsilon \in \varepsilon_0$, there is a unique $G(b, \varepsilon)$. ■

6.5 Measurement of Public Goods

Our empirical measure of public good provision in the U.S. (from footnote 12) encompasses the following expenditure items: defense, highways, libraries, hospitals, health, employment security administration, veterans' services, air transportation, water transport and terminals, parking facilities, transit subsidies, police protection, fire protection, correction, protective inspection and regulation, sewerage, natural resources, parks and recreation, housing and community development, solid waste management, financial administration, judicial and legal, general public buildings, other government administration, and other general expenditures, not elsewhere classified.

Consumption is total personal consumption expenditures. The data source is the Economic Report of the President, tables B1, B20, and B86.

6.6 Statement and Proof of Proposition 8

Claim 1 *The DMPPE is defined by the program:*

$$\langle G(s_{-1}, b), T(s_{-1}, b)\rangle = \arg\max_{\tau, g} \quad \tilde{\nu}(\tau, g, s; s_{-1}) + \left(\tilde{\delta} - 1\right) \lambda u(A(\tau) - s, g) + \tilde{\delta}\beta\lambda V_O\left(s, b'\right),$$

where $\tilde{\nu}(\tau, g, s; s_{-1}) = u(Rs_{-1}, g) + \lambda u(A(\tau) - s, g)$, $\tilde{\delta} = 1 + (1 - \omega)/(\lambda\omega)$, $g' = G(s, b')$, $s = \tilde{S}(\tau, g, g')$, *and* $b' = Rb + g - \tau w H(\tau)$. V_O *is a fixed point of the following functional equation:*

$$V_O(s_{-1}, b) = \tilde{\nu}(T(s_{-1}, b), G(s_{-1}, b), S(s_{-1}, b); s_{-1}) + \beta\lambda V_O(S(s_{-1}, b), B(s_{-1}, b)),$$

where S *satisfies the equation:*

$$S(s_{-1}, b) = \tilde{S}(T(s_{-1}, b), G(s_{-1}, b), G(S(s_{-1}, b), B(s_{-1}, b))).$$

Proof. We write the planner's objective function in a sequential formulation.

$$\frac{\lambda}{1 - \omega + \omega\lambda} U = \frac{\omega\lambda}{1 - \omega + \omega\lambda} u(Rs_{-1}, g_0) + \lambda U_Y(\boldsymbol{s}, \boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$$

$$= \frac{\omega\lambda}{1-\omega+\omega\lambda} u\left(Rs_{-1}, g_0\right) + \lambda \sum_{t=0}^{\infty} (\lambda\beta)^t \left(u\left(A\left(\tau_t\right) - s_t, g_t\right) + \beta u\left(Rs_t, g_{t+1}\right)\right)$$

$$\begin{aligned} = \quad & \frac{\omega\lambda}{1-\omega+\omega\lambda} u\left(Rs_{-1}, g_0\right) + \lambda u\left(A\left(\tau_0\right) - s_0, g_0\right) \\ & + \sum_{t=1}^{\infty} (\lambda\beta)^t \left(u\left(Rs_{t-1}, g_t\right) + \lambda u\left(A\left(\tau_t\right) - s_t, g_t\right)\right). \end{aligned}$$

Multiplying both sides by $\frac{1-\omega+\omega\lambda}{\lambda\omega}$ yields

$$= \tilde{\nu}\left(\tau_0, g_0, s_0; s_{-1}\right) + \left(\tilde{\delta} - 1\right) \lambda u\left(A\left(\tau_0\right) - s_0, g_0\right) + \tilde{\delta} \sum_{t=1}^{\infty} (\lambda\beta)^t \cdot \tilde{\nu}\left(\tau_t, g_t, s_t; s_{t-1}\right)$$

$$= \tilde{\nu}\left(\tau_0, g_0, s_0; s_{-1}\right) + \left(\tilde{\delta} - 1\right) \lambda u\left(A\left(\tau_0\right) - s_0, g_0\right) + \tilde{\delta}\lambda\beta \cdot V_O\left(s_0, b_1\right),$$

where

$$\begin{aligned} V_O\left(s_0, b_1\right) &= \sum_{t=1}^{\infty} (\lambda\beta)^t \tilde{\nu}\left(\tau_t, g_t, s_t; s_{t-1}\right) \\ &= \sum_{t=1}^{\infty} (\lambda\beta)^t \tilde{\nu}\left(T\left(s_{t-1}, b_t\right), G\left(s_{t-1}, b_t\right), s_t; s_{t-1}\right), \end{aligned}$$

$s_t = S\left(s_{t-1}, b_t\right)$ and $b_{t+1} = B\left(s_{t-1}, b_t\right)$. The second equality follows from the fact that future variables must follow the equilibrium policy rules. Note in particular that in equilibrium, saving satisfies

$$S\left(s_{-1}, b\right) = \tilde{S}\left(T\left(s_{-1}, b\right), G\left(s_{-1}, b\right), G\left(S\left(s_{-1}, b\right), B\left(s_{-1}, b\right)\right)\right).$$

Since $\beta\lambda < 1$, V_O can be expressed recursively as

$$V_O\left(s_{-1}, b\right) = \tilde{\nu}\left(T\left(s_{-1}, b\right), G\left(s_{-1}, b\right), S\left(s_{-1}, b\right); s_{-1}\right) + \beta\lambda V_O\left(S\left(s_{-1}, b\right), B\left(s_{-1}, b\right)\right).$$

This concludes the proof of the preliminaries. ■

Proposition 8 *Let $u = u(c, g)$, where $u_c > 0$, $u_g > 0$, and u is a quasi-concave function. Then, a DMPPE is characterized by a system of two functional equations:*

1. *A trade-off between private and public good consumption*

$$\begin{aligned} \tilde{\delta}\lambda u_1\left(c_Y, g\right) A'(\tau) &= \left(u_2\left(Rs_{-1}, g\right) + \tilde{\delta}\lambda u_2\left(c_Y, g\right)\right) \\ & \cdot \left(1 - e(\tau)\right) A'(\tau) \\ & + \beta\lambda\left(\tilde{\delta} - 1\right) \frac{u_2\left(Rs, g'\right) G_1\left(s, b'\right)}{1 - \tilde{S}_3\left(\tau, g, g'\right) G_1\left(s, b'\right)} \\ & \cdot \left(\tilde{S}_2\left(\tau, g, g'\right)\left(1 - e(\tau)\right) A'(\tau) - \tilde{S}_1\left(\tau, g, g'\right)\right). \end{aligned} \tag{66}$$

where subscripts denote partial derivatives, and the following equilibrium conditions hold

$$\begin{aligned}
c_Y &= A(\tau) - s, \; c'_Y = A(\tau') - s', \\
c'_O &= Rs, \; c''_O = Rs', \\
g &= G(s_{-1}, b), \; g' = G(s, b'), \; g'' = G(s', b'') \\
\tau &= T(s_{-1}, b), \; \tau' = T(s, b'), \\
s &= \tilde{S}(\tau, g, g'), \; s' = \tilde{S}(\tau', g', g''), \\
b' &= g + Rb - \tau w H(\tau) \equiv B(s_{-1}, b), \; b'' = B(s, b').
\end{aligned}$$

2. *A Generalized Euler Equation (GEE) for public good consumption:*

$$\begin{aligned}
&\frac{u_2(Rs_{-1}, g) + \tilde{\delta}\lambda u_2(c_Y, g)}{u_2(Rs, g')} \\
&+\beta\lambda\left(\tilde{\delta} - 1\right)\left(\frac{G_2(s, b') + G_1(s, b')\tilde{S}_2(\tau, g, g')}{1 - \tilde{S}_3(\tau, g, g') G_1(s, b')}\right) \\
= \; & R\beta\lambda\left(\begin{array}{c} 1 + \tilde{\delta}\lambda\frac{u_2(c'_Y, g')}{u_2(c'_O, g')} \\ +\beta\lambda\left(\tilde{\delta} - 1\right)\frac{u_2(c''_O, g'')}{u_2(c'_O, g')} G_1(s', b'') \frac{\tilde{S}_2(\tau', g', g'') + \tilde{S}_3(\tau', g', g'') G_2(s', b'')}{1 - \tilde{S}_3(\tau', g', g'') G_1(s', b'')} \end{array}\right).
\end{aligned} \tag{67}$$

where c_Y, c'_Y, c'_O, c''_O, g, g', g'', τ, τ', s, s', b' and b'' are equilibrium values defined as above.

Proof. We start the proof from an analysis of the effect of τ and g on private savings. Taking the total differential of the saving function, $s = \tilde{S}(\tau, g, g') = \tilde{S}(\tau, g, G(s, b'))$, with respect to τ and g yields, respectively,

$$\begin{aligned}
\frac{\mathrm{d}s}{\mathrm{d}\tau} &= \tilde{S}_1(\tau, g, g') + \tilde{S}_3(\tau, g, g')\left(G_1(s, b')\frac{\mathrm{d}s}{\mathrm{d}\tau} - G_2(s, b')(1 - e(\tau)) w H(\tau)\right) \Rightarrow \\
\frac{\mathrm{d}s}{\mathrm{d}\tau} &= \frac{\tilde{S}_1(\tau, g, g') - \tilde{S}_3(\tau, g, g') G_2(s, b')(1 - e(\tau)) w H(\tau)}{1 - \tilde{S}_3(\tau, g, g') G_1(s, b')},
\end{aligned}$$

$$\begin{aligned}
\frac{\mathrm{d}s}{\mathrm{d}g} &= \tilde{S}_2(\tau, g, g') + \tilde{S}_3(\tau, g, g')\left(G_1(s, b')\frac{\mathrm{d}s}{\mathrm{d}g} + G_2(s, b')\right) \Rightarrow \\
\frac{\mathrm{d}s}{\mathrm{d}g} &= \frac{\tilde{S}_2(\tau, g, g') + \tilde{S}_3(\tau, g, g') G_2(s, b')}{1 - \tilde{S}_3(\tau, g, g') G_1(s, b')},
\end{aligned}$$

where we note that

$$\frac{\frac{\mathrm{d}s}{\mathrm{d}\tau}}{(1 - e(\tau)) w H(\tau)} + \frac{\mathrm{d}s}{\mathrm{d}g} = \frac{\frac{\tilde{S}_1(\tau, g, g')}{(1 - e(\tau)) w H(\tau)} + \tilde{S}_2(\tau, g, g')}{1 - \tilde{S}_3(\tau, g, g') G_1(s, b')}. \tag{68}$$

Now consider the problem defined in Claim 1. The FOC w.r.t. τ is:

$$0 = \tilde{\delta}\lambda u_1(A(\tau) - s, g)\left(A'(\tau) - \frac{\mathrm{d}s}{\mathrm{d}\tau}\right) + \tilde{\delta}\beta\lambda V_{O1}(s, b')\frac{\mathrm{d}s}{\mathrm{d}\tau} - \tilde{\delta}\beta\lambda V_{O2}(s, b')(1 - e(\tau)) w H(\tau). \tag{69}$$

The FOC w.r.t. g is

$$0=\tilde{\delta}\lambda u_1\left(A\left(\tau\right)-s,g\right)\left(-\frac{\mathrm{d}s}{\mathrm{d}g}\right)+\tilde{\delta}\lambda u_2\left(A\left(\tau\right)-s,g\right)+u_2\left(Rs_{-1},g\right)+\tilde{\delta}\beta\lambda V_{O1}\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}g}+\tilde{\delta}\beta\lambda V_{O2}\left(s,b'\right). \tag{70}$$

We first derive (66), and then derive (67).

Derivation of (66). We claim (proof below):

$$V_{O1}\left(s_{-1},b\right)=u_1\left(Rs_{-1},g\right)R+\left(1-\frac{1}{\tilde{\delta}}\right)u_2\left(Rs_{-1},g\right)G_1\left(s_{-1},b\right). \tag{71}$$

Next, we combine (69) and (70) to substitute out $V_{O2}\left(s,b'\right)$:

$$\begin{aligned}&\frac{\tilde{\delta}\lambda u_1\left(A\left(\tau\right)-s,g\right)\left(-A'\left(\tau\right)+\frac{\mathrm{d}s}{\mathrm{d}\tau}\right)-\tilde{\delta}\beta\lambda V_{O1}\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}\tau}}{\left(1-e\left(\tau\right)\right)wH\left(\tau\right)}\\ =\;&\tilde{\delta}\lambda u_1\left(A\left(\tau\right)-s,g\right)\left(-\frac{\mathrm{d}s}{\mathrm{d}g}\right)+\tilde{\delta}\lambda u_2\left(A\left(\tau\right)-s,g\right)+u_2\left(Rs_{-1},g\right)\\ &+\tilde{\delta}\beta\lambda V_{O1}\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}g}\end{aligned}$$

$\Rightarrow$

$$\begin{aligned}\tilde{\delta}\lambda u_1\left(c_Y,g\right)A'\left(\tau\right)\;=\;&\left(u_2\left(Rs_{-1},g\right)+\tilde{\delta}\lambda u_2\left(c_Y,g\right)\right)\\ &\cdot\left(1-e\left(\tau\right)\right)A'\left(\tau\right)\\ &+\beta\lambda\left(\tilde{\delta}-1\right)\frac{u_2\left(Rs,g'\right)G_1\left(s,b'\right)}{1-\tilde{S}_3\left(\tau,g,g'\right)G_1\left(s,b'\right)}\\ &\cdot\left(\tilde{S}_2\left(\tau,g,g'\right)\left(1-e\left(\tau\right)\right)A'\left(\tau\right)-\tilde{S}_1\left(\tau,g,g'\right)\right).\end{aligned}$$

where the first step uses (71) and the second step follows from equation (68) and from the household Euler equation, $u_1\left(A\left(\tau\right)-s,g\right)=\beta Ru_1\left(Rs,g'\right)$. The last expression is the "trade-off between private and public good consumption", (66).

Derivation of (67). We claim (proof below):

$$V_{O2}\left(s_{-1},b\right)=\left(1-\frac{1}{\tilde{\delta}}\right)u_2\left(Rs_{-1},g\right)G_2\left(s_{-1},b\right)+\beta\lambda RV_{O2}\left(s,b'\right). \tag{72}$$

Next, we use (71) to substitute out $V_{O1}\left(s,b'\right)$ in use (70), and use the household Euler equation, $u_1\left(A\left(\tau\right)-s,g\right)=\beta Ru_1\left(Rs,g'\right)$, to simplify terms. This yields:

$$\begin{aligned}0\;=\;&\tilde{\delta}\lambda u_2\left(A\left(\tau\right)-s,g\right)+u_2\left(Rs_{-1},g\right)\\ &+\beta\lambda\left(\tilde{\delta}-1\right)u_2\left(Rs,g'\right)G_1\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}g}+\tilde{\delta}\beta\lambda V_{O2}\left(s,b'\right).\end{aligned}$$

(72) implies that

$$
\begin{aligned}
& \frac{\tilde{\delta}\lambda u_2\left(c_Y,g\right)+u_2\left(Rs_{-1},g\right)}{u_2\left(Rs,g'\right)} \\
& +\beta\lambda\left(\tilde{\delta}-1\right)\left(\frac{G_2\left(s,b'\right)+G_1\left(s,b'\right)\tilde{S}_2\left(\tau,g,g'\right)}{1-\tilde{S}_3\left(\tau,g,g'\right)G_1\left(s,b'\right)}\right) \\
= \; & R\beta\lambda\left(\begin{array}{c} 1+\tilde{\delta}\lambda\frac{u_2\left(c_Y',g'\right)}{u_2\left(c_O',g'\right)} \\ +\beta\lambda\left(\tilde{\delta}-1\right)\frac{u_2\left(c_O'',g''\right)}{u_2\left(c_O',g'\right)}G_1\left(s',b''\right)\frac{\tilde{S}_2\left(\tau',g',g''\right)+\tilde{S}_3\left(\tau',g',g''\right)G_2\left(s',b''\right)}{1-\tilde{S}_3\left(\tau',g',g''\right)G_1\left(s',b''\right)}\end{array}\right).
\end{aligned}
$$

This expression is the GEE for public good consumption, (66). ■

6.6.1 Derivation of equations (71) and (72)

Claim 2 *The partial derivatives $V_{O1}\left(s_{-1},b\right)$ and $V_{O2}\left(s_{-1},b\right)$ can be expressed as:*

$$
V_{O1}\left(s_{-1},b\right)=u_1\left(Rs_{-1},g\right)R+\left(1-\frac{1}{\tilde{\delta}}\right)u_2\left(Rs_{-1},g\right)G_1\left(s_{-1},b\right).
$$

$$
V_{O2}\left(s_{-1},b\right)=\left(1-\frac{1}{\tilde{\delta}}\right)u_2\left(Rs_{-1},g\right)G_2\left(s_{-1},b\right)+\beta\lambda RV_{O2}\left(s,b'\right).
$$

Proof. Differentiating $V_O\left(s_{-1},b\right)$ w.r.t. s_{-1} yields

$$
\begin{aligned}
V_{O1}\left(s_{-1},b\right) = \; & \lambda u_1\left(A\left(\tau\right)-s,g\right)\left(A'\left(\tau\right)-\frac{\mathrm{d}s}{\mathrm{d}\tau}\right)T_1\left(s_{-1},b\right) \\
& +\lambda\left(u_1\left(A\left(\tau\right)-s,g\right)\left(-\frac{\mathrm{d}s}{\mathrm{d}g}\right)+u_2\left(A\left(\tau\right)-s,g\right)\right)G_1\left(s_{-1},b\right) \\
& +u_1\left(Rs_{-1},g\right)R+u_2\left(Rs_{-1},g\right)G_1\left(s_{-1},b\right) \\
& +\beta\lambda V_{O1}\left(s,b'\right)\left(\frac{\mathrm{d}s}{\mathrm{d}\tau}T_1\left(s_{-1},b\right)+\frac{\mathrm{d}s}{\mathrm{d}g}G_1\left(s_{-1},b\right)\right) \\
& +\beta\lambda V_{O2}\left(s,b'\right)\left(G_1\left(s_{-1},b\right)-\left(1-e\left(\tau\right)\right)wH\left(\tau\right)T_1\left(s_{-1},b\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
= \; & \left(\begin{array}{c}\lambda u_1\left(A\left(\tau\right)-s,g\right)\left(A'\left(\tau\right)-\frac{\mathrm{d}s}{\mathrm{d}\tau}\right)+\beta\lambda V_{O1}\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}\tau} \\ -\beta\lambda V_{O2}\left(s,b'\right)\left(1-e\left(\tau\right)\right)wH\left(\tau\right)\end{array}\right)T_1\left(s_{-1},b\right) \\
& +u_1\left(Rs_{-1},g\right)R+u_2\left(Rs_{-1},g\right)G_1\left(s_{-1},b\right) \\
& +\left(\begin{array}{c}-\lambda u_1\left(A\left(\tau\right)-s,g\right)\frac{\mathrm{d}s}{\mathrm{d}g}+\lambda u_2\left(A\left(\tau\right)-s,g\right) \\ +\beta\lambda V_{O1}\left(s,b'\right)\frac{\mathrm{d}s}{\mathrm{d}g}+\beta\lambda V_{O2}\left(s,b'\right)\end{array}\right)G_1\left(s_{-1},b\right).
\end{aligned}
$$

$$
=u_1\left(Rs_{-1},g\right)R+\left(1-\frac{1}{\tilde{\delta}}\right)u_2\left(Rs_{-1},g\right)G_1\left(s_{-1},b\right),
$$

which is equation (71).
Similarly, differentiating $V_O(s_{-1}, b)$ w.r.t. b yields

$$\begin{aligned}
V_{O2}(s_{-1}, b) &= \lambda u_1(A(\tau) - s, g)\left(A'(\tau) - \frac{\mathrm{d}s}{\mathrm{d}\tau}\right) T_2(s_{-1}, b) \\
&\quad + \lambda\left(u_1(A(\tau) - s, g)\left(-\frac{\mathrm{d}s}{\mathrm{d}g}\right) + u_2(A(\tau) - s, g)\right) G_2(s_{-1}, b) \\
&\quad + u_2(Rs_{-1}, g)\, G_2(s_{-1}, b) \\
&\quad + \beta\lambda V_{O1}(s, b')\left(\frac{\mathrm{d}s}{\mathrm{d}\tau} T_2(s_{-1}, b) + \frac{\mathrm{d}s}{\mathrm{d}g} G_2(s_{-1}, b)\right) \\
&\quad + \beta\lambda V_{O2}(s, b')\left(-(1 - e(\tau))\, wH(\tau)\, T_2(s_{-1}, b) + G_2(s_{-1}, b) + R\right)
\end{aligned}$$

$$\begin{aligned}
&= \left(\begin{array}{c} \lambda u_1(A(\tau) - s, g)\left(A'(\tau) - \frac{\mathrm{d}s}{\mathrm{d}\tau}\right) + \\ \beta\lambda V_{O1}(s, b')\frac{\mathrm{d}s}{\mathrm{d}\tau} - \beta\lambda V_{O2}(s, b')(1 - e(\tau))\, wH(\tau) \end{array}\right) T_2(s_{-1}, b) \\
&\quad + \beta\lambda R V_{O2}(s, b') \\
&\quad + \left(\begin{array}{c} \lambda u_1(A(\tau) - s, g)\left(-\frac{\mathrm{d}s}{\mathrm{d}g}\right) + \lambda u_2(A(\tau) - s, g) \\ + \beta\lambda V_{O1}(s, b')\frac{\mathrm{d}s}{\mathrm{d}g} + \beta\lambda V_{O2}(s, b') + u_2(Rs_{-1}, g) \end{array}\right) G_2(s_{-1}, b)
\end{aligned}$$

$$= \left(1 - \frac{1}{\tilde{\delta}}\right) u_2(Rs_{-1}, g)\, G_2(s_{-1}, b) + \beta\lambda R V_{O2}(s, b'),$$

which is equation (72). ■