Asset Prices and Intergenerational Risk Sharing: The Role of Idiosyncratic Earnings Shocks*

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May 2006

Abstract

In their seminal paper, Mehra and Prescott (1985), Rajnish Mehra and Edward Prescott were the first among many subsequent authors to suggest that non-traded labor-market risk may provide a resolution to the equity-premium puzzle. The most direct demonstration of this was Constantinides and Duffie (1996), who showed that, under certain conditions, cross-sectionally uncorrelated unit-root shocks which become more volatile during economic contractions can resolve the puzzle. We examine the robustness of this to life-cycle effects. Retired people, for instance, do not face labor-market risk. If we incorporate them, to what extent will the equity premium be resurrected? Our answer is "not very much."

*Prepared for the *Handbook of Investments: Equity Risk Premium*, edited by Rajnish Mehra. This chapter builds on the work in Storesletten, Telmer and Yaron (2006). We thank Rajnish Mehra and Stan Zin for detailed comments.

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Our model, with realistic life cycle features, can still account for about 75% of the average equity premium and the Sharpe ratio observed on the U.S. stock market.

1 Introduction

This chapter analyzes the channels by which idiosyncratic earning shocks affect asset prices. The findings and framework are largely built on Storesletten, Telmer, and Yaron (2006). We show that the asset pricing effects of idiosyncratic shocks depend on the relative magnitude of human capital shocks to financial risk. The exposure to this dimension changes dramatically over the life cycle – making life cycle consideration an important element of our framework. More specifically, we show that individuals' portfolio choices are sensitive to idiosyncratic earning shocks when (i) they display persistence and counter-cyclical volatility (CCV) – features documented using earnings data, and that (ii) the shocks to human capital are large relative to financial risk when young. Young agents are more susceptible to human capital shocks since they hold little financial capital, and because the shocks are persistent, they have a large impact on discounted earnings (i.e., human capital). As agents age, the exposure to financial (human) capital increases (declines). With that logic, agents with no labor income – retirees – can accommodate the largest exposure to aggregate risk. However, since they have less labor income than the middle-aged, they still prefer to reduce their exposure to financial risk somewhat. Moreover, with counter-cyclical volatility of earnings shocks, agents who are fully exposed to human capital risk – the youngest agents – would like the smallest exposure to aggregate risk. As a consequence, our model displays average portfolio rules which are hump-shaped in age. That is, agents choose to hold very little equity when young, levered equity positions when middle-aged, and intermediate equity positions when retired.

We show that in equilibrium these portfolio choices manifest themselves into asset prices that depend on how much intergenerational risk sharing takes place across cohorts. Without idiosyncratic shocks and with constant aggregate wages (and thus return on human capital), the old would prefer to issue debt while the young would prefer to be levered in equity. Such intergenerational risk sharing would tend, *ceteris paribus*, to lower the equity premium. However, the presence of counter-cyclical idiosyncratic shocks dissuades the young from holding equity, which curtails intergenerational risk sharing.

Following the work in Storesletten, Telmer, and Yaron (2006) we show that quantitatively the effects of idiosyncratic risk are significant. Idiosyncratic risk inhibits intergenerational risk sharing, imposing a disproportionate share of aggregate risk on the wealthy middle-aged cohorts who demand an equity premium for their exposure to this risk. We use a stationary overlapping-generations model to show how life-cycle portfolio choices interact with intergenerational risk sharing to accentuate the equity premium. For a risk aversion of 8, our model is able to account for about 75% of the average equity premium and the Sharpe ratio observed on the U.S. stock market.

It is important to note that the driving force in our model is a concentration of aggregate risk and equity ownership on middle-aged and old agents. The model of Constantinides, Donaldson, and Mehra (2002) shares a similar feature. Where their model is driven by portfolio constraints, ours is driven by portfolio choices made in light of how nontradeable and tradeable risks interact. We discuss these issues further below.

An advantage of our model relates to risk-sharing behavior. U.S. data on income and consumption indicate that, while complete markets may not characterize the world, neither does a distinguishing feature of the Constantinides and

Duffie (1996) framework: autarky.[1] The cross-sectional standard deviation of U.S. consumption, for instance, is roughly 35 percent smaller than that of non-financial earnings.[2] However, our framework shows that in spite of the autarky dimension and the lack of realistic life cycle features, the Constantinides-Duffie model is still able to provide useful quantitative asset pricing results. Papers by Balduzzi and Yao (2005), Brav, Constantinides, and Geczy (2002), Cogley (2002) Ramchand (1999) and Sarkissian (2003) investigate the Constantinides-Duffie model's consumption Euler equation restrictions directly and find mixed evidence. Our approach, emphasizing endogenous asset pricing, seem to be consistent with the channels put forth in their paper (e.g., persistent and time varying idiosyncratic risk).

The idea that market incompleteness may contribute to the equity premium is not new, and by and large, most of the quantitative findings have been 'negative' in terms of the ability of the proposed models to generate a viable equity premium. The agents in these models tend to be 'very efficient' in insuring themselves against the relatively transitory income shocks they face; thus the resulting equity premium is essentially the one derived in Mehra and Prescott (1985) (e.g., Telmer (1993), Heaton and Lucas (1996)). To generate a sizeable equity premium such models usually had to resort to large transaction costs and/or tight borrowing constraints. The distinguishing aspect of our paper,

[1] By autarky we mean a situation where there is no trade amongst living agents and they are forced to consume their endowments every period.

[2] A large number of papers, including Altonji, Hayashi, and Kotlikoff (1991), Attanasio and Davis (1996), Attanasio and Weber (1992), Cochrane (1991), Deaton and Paxson (1994), Mace (1991) and Storesletten, Telmer, and Yaron (2004a) provide evidence which is suggestive of imperfect risk sharing. Altug and Miller (1990) find opposing evidence. The numerical value cited here is based upon evidence in both Deaton and Paxson (1994) and Storesletten, Telmer, and Yaron (2004a).

life-cycle, has also important implications for the ability to capture persistent (almost unit root) processes for individual earnings shocks while maintaining aggregate quantities that are characterized by relatively transitory processes. A number of studies have examined more specifically the quantitative implications of the Constantinides and Duffie (1996) model. The closest to our work is Krusell and Smith (1997) and Gomes and Michaelides (2004). The latter use a similar life cycle model, with fixed entry costs to the stock market and preference heterogeneity across stock and non-stockholders. They show that preference heterogeneity and not the fixed entry costs is the important dimension for matching the relative size of stockholders. Others include Aiyagari (1994), Aiyagari and Gertler (1991), Alvarez and Jermann (2001), den Haan (1994), Guvenen (2005), Heaton and Lucas (1996), Huggett (1993), Lucas (1994), Mankiw (1986), Marcet and Singleton (1999), Ríos-Rull (1994), Telmer (1993), Weil (1992), and Zhang (1997). The stationary OLG framework we develop owes much to previous work by Ríos-Rull (1994), Huggett (1996) and Storesletten (2000). More recent examples using life cycle economies to asses portfolio choice and equity returns include Olovsson (2004) and Benzoni, Collin-Dufresne, and Goldstein (2004).

The remainder of this chapter is organized as follows. In Section 2 we formulate a version of the Constantinides and Duffie (1996) model, calibrate it, and examine its quantitative asset pricing properties. In section 3 we introduce life-cycle savings by assuming that retirement income is equal to zero; that is, a model with trade with a more realistic distribution of human to financial wealth. In Section 4 we analyze the economic workings of this model using a class of computational experiments. Section 5 concludes the paper.

2 An Analytical Example of the Constantinides-Duffie Model

We begin with an analytical example of the Constantinides and Duffie (1996) model. There are two asset markets, a one-period riskless bond and an Arrow Debreu security. Agents live for $H \leq \infty$ periods and have standard CRRA preferences,

$$\max E\left(\sum_{t=1}^{H} \beta^t \frac{c_{it}^{1-\gamma}}{1-\gamma}\right) \tag{1}$$

Labor income for agent i in period t is given by

$$y_{it} = C_t \exp(z_{it}) \tag{2}$$

where C_t is aggregate consumption and z_{it} is agent i's share of aggregate consumption during period t. Aggregate consumption growth follows a two-state process and equals $1 + z$ in booms and $1 - z$ in recessions. The probability of remaining in an aggregate state is P.

Individual's share of consumption, z_{it}, follows a unit root process with heteroskedastic innovations,

$$z_{it} = z_{i,t-1} + \eta_{it} \tag{3}$$

$$z_{i0} = 0 \tag{4}$$

$$\eta_{it} \sim \mathrm{N}\left(-\frac{\sigma_t^2}{2}, \sigma_t^2\right) \tag{5}$$

where the time-varying conditional variance depends on consumption growth, z_t, according to

$$\sigma_{t+1}^2 = \begin{cases} a - b \cdot z_t & \text{in recessions} \\ a + b \cdot z_t & \text{in booms} \end{cases} \tag{6}$$

where the coefficient b is the sensitivity of the cross-sectional variance to aggregate growth rate. That is b defines the heteroskedasticity of the process σ_t. For example, the Constantinides and Duffie intuition by which the cross sectional variation rises during bad times entail a b coefficient that is negative (i.e., the cross sectional variance grows during negative aggregate growth).

Agents trade two assets in zero net supply – a one-period bond and a one-period Arrow-Debreu security paying one unit of consumption in booms and zero in recessions. Following Constantinides and Duffie (1996), it is straightforward to show that the equilibrium consumption allocation is autarky, that is $c_{it} = y_{it}$. The pricing kernel for this economy satisfies the standard pricing Euler equation,

$$E_t[M_{t+1}R_{i,t+1}] = 1 \tag{7}$$

where $M_{t+1} = \hat{\beta}\left(\frac{C_{t+1}}{C_t}\right)^{-\hat{\gamma}}$ and $\hat{\gamma}$ and $\hat{\beta}$ are respectively the risk-aversion and discount factor of the aggregate 'mongrel' consumer (see equations (17)-(18) in Constantinides and Duffie (1996)), and are given by

$$\hat{\gamma} = \gamma - \frac{\gamma(\gamma+1)}{2} \cdot b \tag{8}$$

$$\hat{\beta} = \beta \cdot \exp\left(\frac{(\gamma+1)\gamma}{2}a\right) \tag{9}$$

Equilibrium prices in this economy are now very easy to compute and follow the standard pricing formulas using CRRA preferences with the adjusted time discount and risk aversion. In particular, as shown in detail in the Appendix, in the simple case of $i.i.d$ aggregate shocks (namely, $P = 1/2$), the Sharpe ratio for the excess return on the risky asset, denoted r_{t+1}, is given by the familiar formula,

$$\frac{E(r)}{std(r)} = \frac{(1-z)^{-\hat{\gamma}} - (1+z)^{-\hat{\gamma}}}{(1+z)^{-\hat{\gamma}} + (1-z)^{-\hat{\gamma}}} \approx z \cdot \hat{\gamma} = std(\log(C_{t+1}/C_t))\hat{\gamma} \tag{10}$$

where the approximation exploits that $\log(1+z) \approx z$ for small z.

As is clear from the denominator of the equation above, the return on the risky Arrow-Debreu asset can have a large standard deviation. However, by combining the risky Arrow-Debreu asset with the bond, it is straightforward to construct a portfolio that looks like a an 'equity stock', i.e. an asset whose return has a standard deviation of about, 10%. Note however, that the Sharpe ratio on the stock will be the same as that of the risky Arrow–Debreu asset — making the computation above informative.

2.1 Calibration of the Constantinides-Duffie economy

To quantify the implications of the model above we need to specify the process for consumption growth as well as the countercyclical volatility. We now ask if values of a and b implied by labor market data help the model account for the equity premium. We use estimates from Storesletten, Telmer, and Yaron (2004b) which are based on annual PSID data, 1969-1992. They show that (a) idiosyncratic shocks are highly persistent and that a unit root is plausible, (b) the conditional standard deviation of idiosyncratic shocks is large, averaging 17%, and (c) the conditional standard deviation is countercyclical, increasing by roughly 68% from expansion to contraction (from 12.5% to 21.1%). In Appendix A we show that these estimates map into values $a = 0.0143$ and $b = -0.1652$.

We use a stochastic process for z_t which is essentially the same as that in Mehra and Prescott (1985): a two-state Markov chain with mean, and a standard deviation of aggregate consumption growth of 0.018, 0.033, and a transition matrix in which $P = 2/3$ approximately matching an autocorrelation of -0.14. We choose the 'effective' discount factor, β^*, to match the average

U.S. annual risk-free interest rate 1.3%.

We vary the risk aversion γ from 3 to 8.[3] We also vary the degree of counter cyclical variation in idiosyncratic income process. In addition to the counter cyclical variability (CCV) estimated in Storesletten, Telmer, and Yaron (2004b), we also examine a time-invariant process in which the variance of idiosyncratic risk is constant across recessions and expansions and is set to the unconditional volatility. Finally we also examine as a benchmark the 'Complete Markets' economy, one in which the idiosyncratic volatility is set to zero. Table 1 reports the results. In the column 'SR%' we report the percent Sharpe ratio (which is slightly different from equation (10) because $P > 1/2$). We set '$std\,(r_t)$', the standard deviation of the excess return on the risky security, to be 10% by changing the weights of the portfolio of the risky Arrow-Debreu asset and the bond. Table 1 clearly demonstrates that as risk aversion is increased, the Sharpe ratio rises. Not surprisingly, in order to maintain the risk free rate at 1.3%, β needs to be lowered. It appears that in order to generate a large Sharpe ratio one requires both a sizeable risk aversion (i.e., risk aversion of 8) and the presence of CCV. Furthermore, the presence of CCV seems to have a much more dramatic effect in the case of risk aversion of 8. In that case the introduction of CCV raises the Sharpe ratio from 23 to 37 percent, relative to a modest rise in the Sharpe ratio of only 3% in the case of low risk aversion of 3.

[3]Cogley (2002) formulates an asset-pricing model with idiosyncratic risk (to individuals' consumption) and uses the empirical time-varying cross-sectional moments of consumption growth from the Survey of Consumer Expenditures (CEX) to ask what level of risk aversion would be required to account for the empirical equity premium. Interestingly, this approach delivers a risk aversion of 8 (assuming a plausible level of measurement error).

2.2 Model Implications

There are several lessons to be taken from the special structure of the Constantinides-Duffie economy described above. First, the examples above have zero capital. By increasing aggregate wealth to some positive level, agents are more likely be able to self insure some fraction of their shocks, even if these shocks are permanent. The reason for this is the following. If agents have some positive financial wealth in addition to their human capital (defined as discounted future labor earnings), then a permanent shock to earnings implies a less-than-proportional change in total wealth (as financial wealth is unaffected). Consequently, financial wealth helps agents to partially smooth even permanent shocks (see e.g., Storesletten, Telmer, and Yaron (2004a)). Since agents in this economy all have the same normalized 'target wealth' (see Carroll (2004) for details), a positive amount of aggregate wealth introduces a motive for trade and an endogenous wealth distribution – a key feature of the calibrated economy in Section 3.

An economy with trade and a non-trivial distribution of wealth is in sharp contrast to the Constantinides-Duffie economy above in which the wealth distribution is degenerate (all agents have zero financial wealth). Since agents are able to better smooth consumption in economies with positive wealth (and trade), it is intuitive that countercyclical risk should have a smaller impact on asset prices. Thus, the Sharpe ratio in the above Constantinides-Duffie economy represents an 'upper bound' on the Sharpe ratio in economies with trade.

Another important shortcoming of the Constantinides-Duffie economy presented above, is that agents receive labor income all periods of life. Therefore, asset prices (e.g., equation (10)) are derived from a 'worker' with labor income who is bearing all of aggregate risk. A more realistic economy should have some retirement years with no labor income risk. However, since these retirees do not face any labor income risk, their attitude towards risk should be the same as a standard Mehra-Prescott representative agent. One might therefore suspect that the presence of retirees will reduce the price of risk in the model.

To analyze the effect of retirement, Storesletten, Telmer, and Yaron (2006) extend the no-trade Constantinides-Duffie model above to include retirees. The key insight of that analysis is that in order for the retirees to be content with no trade, one has to endow them with a 'leveraged' claim to aggregate consumption. Consequently, in the autarkic equilibrium, retirees take over some of aggregate risk from the workers. In contrast, in the complete markets case the workers and the retirees are equally exposed to aggregate risk. In this sense, countercyclical income risk tends to mitigate the inter-generational risk sharing. Moreover, the equilibrium price of risk will, intuitively, lie in between the Constantinides-Duffie benchmark above and that of the complete markets model.

Overall the analysis above shows that the Constantinides-Duffie model is quite successful at accounting for significant component of the equity premium given a realistic parametrization of idiosyncratic risk. The model has several counterfactual features (such as excess volatility of the risk free rate) which we discuss in more detail in Storesletten, Telmer, and Yaron (2006) and which in principle can be rectified with a richer process for σ_t. However, for the remainder of this paper we focuses on the implications of the issues discussed above regarding age, retirement, and risk sharing by quantitatively analyzing a calibrated model which includes these features.

3 Incorporating the Life Cycle

The Constantinides-Duffie framework is useful in that it serves as a quantitative frame-of-reference for what we are ultimately interested in: models in which idiosyncratic risk motivates trade between heterogeneous agents. Why do we view the no-trade model as being insufficient? First, partial risk sharing is an undeniable aspect of U.S. data on labor earnings, consumption and labor supply (see Heathcote, Storesletten, and Violante (2005), Storesletten, Telmer, and Yaron (2004a)). Partial risk sharing is likely to mitigate the asset-pricing implications of idiosyncratic labor-market risk, so incorporating it is important for quantitative questions such as ours. Second, our emphasis is on how idiosyncratic risk interacts with life-cycle economics. That is, whereas the Constantinides-Duffie framework restricts the distribution of idiosyncratic shocks, we seek to incorporate the distribution of what is being shocked: human capital. Having a realistic distribution of capital necessarily means that we must incorporate trade. Finally, in a life-cycle model with trade we are able to incorporate certain aspects of reality into our calibration (*e.g.*, the demographic structure), thereby making for a more robust quantitative exercise. We proceed by summarizing the critical aspects of the model which is more formally laid out in Storesletten, Telmer, and Yaron (2006). There are H overlapping generations of agents, indexed by $h = 1, 2, \dots, H$, with a continuum of agents in each generation. Preferences are

$$U(c) = E_t \sum_{h=1}^{H} \beta^h (c_{it+h}^h)^{1-\gamma}/(1-\gamma) \tag{11}$$

where c_{it}^h is the consumption of the i^{th} agent of age h at time t and β and γ denote the discount factor and risk aversion coefficients, respectively. Newborn agents have zero financial wealth. Retirees receive zero labor income. Nontradeable endowments take the form of labor efficiency units which are inelastically supplied to firms in returns for a wage of w_t per unit. These labor efficiency units are denoted n_{it}^h, for agent i of age h at date t. They are governed by the stochastic process

$$\log n_{it}^h = \kappa_h + z_{i,t}^h \tag{12}$$

where κ_h is used to characterize the cross-sectional distribution of mean income across ages, and the idiosyncratic component $z_{i,t}^h$, follows

$$z_{it} = z_{i,t-1} + \eta_{it} \tag{13}$$

$$z_{i0} = 0 \tag{14}$$

$$\eta_{it} \sim \mathrm{N}\left(-\frac{\sigma_t^2}{2}, \sigma_t^2\right) \tag{15}$$

The time-varying conditional variance depends on an aggregate shock, Z_t, according to

$$\begin{aligned} \sigma_t^2 &= \sigma_E^2 \text{ if } Z \geq E(Z) \\ &= \sigma_C^2 \text{ if } Z < E(Z) \end{aligned}$$

Individual labor income is the product of labor supplied and the wage rate: $y_{it}^h = w_t n_{it}^h$.[4]

Firms are represented by an aggregate production technology to which agents rent capital and labor services. Labor is supplied inelastically and, in aggregate, is fixed at N. Denoting aggregate consumption, output and capital as Y_t, C_t and K_t respectively, the production technology is

$$Y_t = Z_t K_t^\theta N^{1-\theta} \tag{16}$$

[4]Storesletten, Telmer, and Yaron (2006) abstracts from bequest motives. The main difference in the economic outcomes would be that with bequests wealth would not fall sharply during retirement. We abstracted from these effects in order to focus exclusively on the main point of the paper – the effect of changes in the ratio of human capital to total wealth during working age.

$$
\begin{aligned}
K_{t+1} &= Y_t - C_t + (1-\delta_t)K_t & (17)\\
r_t &= \theta Z_t K_t^{1-\theta} N^{1-\theta} - \delta_t & (18)\\
w_t &= (1-\theta)\, Z_t K_t^{-\theta} N^{-\theta} & (19)
\end{aligned}
$$

where r_t is the return on capital (the risky asset), w_t is the wage rate, θ is capital's share of output, Z_t is an aggregate shock, and δ_t is the depreciation rate on capital. The depreciation rate is stochastic:

$$
\delta_t = \delta + (1 - Z_t)\frac{s}{Std(Z_t)} \ \ , \tag{20}
$$

where δ controls the average and s is, approximately, the standard deviation of r_t.[5]

Turning back to the household sector, agents can trade in a riskless one-period bond and in ownership of aggregate capital. An agent's decision problem is to maximize (11) subject to the following sequence of budget constraints. We omit the i and t notation and express things recursively. Budget constraints are

$$
\begin{aligned}
c_h + k'_{h+1} + b'_{h+1} q(\mu, Z) &\leq a_h + n_h w(\mu, Z) & (21)\\
a_h &= k_h r(\mu, Z) + b_h &\\
k'_{H+1} &\geq 0 &\\
b'_{H+1} &\geq 0 &
\end{aligned}
$$

where a_h denotes beginning-of-period wealth, k_h and b_h are beginning-of-period capital and bond holdings, and k'_{h+1} and b'_{h+1} are end-of-period holdings. We do

[5] Greenwood, Hercowitz, and Krusell (1997) have used a similar production technology in a business cycle context. Boldrin, Christiano, and Fisher (2001) have done so in an asset pricing context. We view our technology is essentially a reduced-form representation of, for instance, Greenwood, Hercowitz, and Krusell (1997), equation (B3).

not impose any portfolio restrictions over and above restricting terminal wealth to be non-negative (the third and fourth restriction).

The competitive equilibrium for this economy follows what is by now standard in a production economy with heterogeneous agents (e.g., Ríos-Rull (1994) , Krusell and Smith (1997)). The specific details are given in Storesletten, Telmer, and Yaron (2006). In essence, equilibrium requires market clearing for bonds, rental rates for capital and labor being given by their respective marginal productivity, and an equilibrium law of motion for the cross-sectional distribution of wealth. The main difficulty is that the wealth distribution enters as a state variable. To solve for an approximate equilibrium (in lieu of a multi-dimensional distribution of wealth), we use the computational methods developed by Krusell and Smith (1997) and adapted to life cycle economies in Storesletten, Telmer, and Yaron (2006).

3.1 Calibration

The model time-period is one year. The aggregate shock in equation (18) follows a first-order Markov chain with values $Z \in \{0.9725, 1.0275\}$. The unconditional probabilities are 0.5 and the transition probabilities are such that the probability of remaining in the current state is 2/3 (so that the expected duration of a 'business cycle' is 6 years). Capital's share of output, θ from equation (18), is set to 0.40, and the average annual depreciation rate, δ, is set to match the average riskfree rate of 1.3 percent. This results in $\delta = 0.164$. The average wage rate, w, is set equal to $(1-\theta)\, E(K)^{\theta} N^{-\theta}$.

The magnitude of the depreciation shocks in equation (20) is set so that the standard deviation of aggregate consumption growth is 3.3 percent. We choose this (as opposed to matching the variability of equity returns) because, just as in representative agent models, realistic properties for aggregate consumption are the primary disciplinary force on asset-pricing models with heterogeneity. Equation (7) makes this clear. The resulting implications for the standard deviation of equity returns is reported in Table 3. The volatility of the theoretical equity premium is roughly 7%, 3 percentage points less than the U.S. sample value.

The persistent component of hours worked, z_{it}^h, follows a unit root process with innovations governed by a four-state Markov chain, two states corresponding to an expansion and the other two a contraction. The conditional variances, σ_E^2 and σ_C^2, are set to 0.0095 and 0.0467 respectively, which are taken from Storesletten, Telmer, and Yaron (2004b), Table 1, and then scaled down so that the unconditional variance matches that of the $\rho = 0.92$ process (a value of 0.0281). The parameters κ_h are chosen so as to match the PSID mean age profile in earnings.

Young agents are born with zero assets and retired agents receive zero labor income. This serves as the primary motive for trade. It also results in a realistic life-cycle distribution of human to financial capital — younger agents hold most of the former whereas older agents hold most of the latter — which, as we'll see, plays an important role in portfolio choice.

The demographic structure is calibrated to correspond to several simple properties of the U.S. work force. Agents are 'born' at age 22, retire at age 65 and are dead by age 85. 'Retirement' is defined as having one's labor income drop to zero and having to finance consumption from an existing stock of assets. Retired agents comprise roughly 20 percent of the population.

Risk aversion is set to target the Sharpe ratio (discussed below, in reference to Table 1), and the discount factor, β, is chosen so that the average capital to output ratio is 3.3.

Table 2 illustrates the aggregate properties of our economy. The sample size for the U.S. data is chosen to be the same as that used by Mehra and Prescott (1985)

As is discussed in the text, the production side of our economy is unrealistic. Aggregate consumption variability, however, matches the data, as does the variability of the risky asset return (discussed above). Our model does not resolve the well-known problems with production-based asset pricing models. It is best viewed in the same way one views any endowment economy: a model with realistic properties of aggregate consumption which can be supported by some, potentially unrealistic, production technology. Alternatively, one can view our economy as featuring a linear technology — commonplace in the finance literature — where we are explicit about the implications of from the production side of the model — not very commonplace in the finance literature.

4 Quantitative Results

Ultimately, we are interested in the asset-pricing implications of idiosyncratic labor-market risk. These implications are manifest in how the heterogeneous-agent portfolio rules interact in general equilibrium. Therefore, we begin by describing portfolio choice.

Figure 1 graphs the portfolio rules in levels and Figure 2 graphs the (age-specific average) share of stocks as a fraction of financial wealth. For expositional reasons, we show results from our benchmark economy — that which we call the "CCV economy" — as well as results from an analogous complete-markets economy.

We start with the complete-markets economy. In this case aggregate consumption growth equals that of individual consumption growth, cohort-by-cohort. However, the portfolio rules which support this allocation differ substantially across ages. Figure 2 shows that portfolio rules are hump-shaped, with the middle-aged workers holding the largest position in equities. The reasons are as follows. First, retirees have zero labor income. So, in order to replicate aggregate consumption risk, they hold diversified portfolios of stocks and bonds, the reason being that stock returns are much more volatile than aggregate consumption growth (10.9% versus 3.3%). Second, the older workers hold relatively much of their financial wealth in stocks because, analogously to the well-known model of Bodie, Merton, and Samuelson (1992) (BMS) model, their labor income has bond-like properties. In BMS this means that labor income is deterministic. In our model it just means that it's a lot less volatile than stock returns, which bear the brunt of the depreciation-rate shocks from equation (20). The upshot, nevertheless, is the same; older workers hold more stocks than retirees because their labor income serves as a partial hedge against their stock portfolio. Finally, young workers have negative financial wealth and, as a result, the youngest agents in the complete market economy actually short-sell stocks! The result is a *hump-shaped* portfolio profile over the entire life-cycle. What drives this, vis-a-vis BMS, is a combination of negative financial wealth and risky wages. Average wage rates are risky and perfectly correlated with stock returns. With negative financial wealth it is as if the agent is levered in aggregate risk.[6] Hence, by shorting stocks, young agents reduce their exposure

[6]Consider, for example, a worker who maintains a large debt invested in bonds. If con-

to aggregate risk, thus implementing the complete-market allocation.

Now consider the CCV economy. With CCV and incomplete markets, aggregate risk is no longer shared uniformly across different age cohorts. It gets shifted from the young to the old, because the young face the most human-capital risk and CCV affects only human-capital risk, not financial-capital risk. This implication is clearly borne out in Figure 1 where the stock profile of the CCV economy is, for the young, shifted to the right. All agents younger then 55, in the CCV economy, hold less stocks and more bonds than their complete-market counterparts. After age 55 the relationship is reversed. Similarly, Figure 2 shows that after 55, the share of stocks is higher in the CCV economy than in the complete-market economy. Further details are available in Storesletten, Telmer, and Yaron (2006).

The resulting hump-shaped pattern in equity ownership in the CCV economy (see Figure 2) is broadly consistent with U.S. data and has been the focus of recent work by Amerkis and Zeldes (2000) and Heaton and Lucas (2000). Brown (1990) shows that non-tradeable labor income can generate hump-shaped portfolio rules in age, and Amerkis and Zeldes (2000) discuss a similar phenomenon.

sumption equals wages net of the deterministic interest rate payments, consumption will be *more* volatile than wages. With sufficiently large debt, the agent's consumption will be more volatile than aggregate consumption, so she would want to reduce her exposure to aggregate risk. Consequently, the agent would short stocks as an insurance against aggregate risk: shorting stocks implies that in good (bad) times, when earnings growth is large (small), stock repayment is large (small).

4.1 Asset Pricing Implications

Table 3 reports the Sharpe ratio, the risk free rate, and the first two moments of the risky return for this class of models. Each row describes a different economy. In the case of complete markets, there are no idiosyncratic shocks to earnings and only aggregate shocks are operative. In the case of 'No CCV' there are idiosyncratic shocks but they are homoskedastic with respect to aggregate shocks. We think of our 'benchmark economy' as one with a risk aversion coefficient of 8, which is close to the value which delivers the sample Sharpe ratio in the Constantinides-Duffie model (see Table 1). We also report moments for a more standard value of 3.

The Sharpe ratio in our benchmark economy is about 33%.[7] Adding countercyclical variation to idiosyncratic shocks increases the Sharpe ratio by about 6 percentage points (an increase of about one fourth). This compares to the Constantinides-Duffie model's value of 37%, slightly less than the sample estimate of 41%. Life-cycle effects, thus, mitigate our model's ability to account for the equity premium, vis-a-vis the Constantinides-Duffie framework. This is due to the existence of retirees who do not face labor market risk. Nevertheless, our model still delivers a Sharpe ratio of 33%, which compares to the complete-market value of 27% and the Mehra-Prescott model's value (with risk aversion of 8) of 23%. In addition, the existence of retirees also dampens the effect of CCV. CCV delivers an additional 14 percentage points to the Sharpe ratio in

[7] We focus on the Sharpe ratio because the volatility of the risky return varies across our different economies. As described above, this is because the shock variability, from equation (20), is chosen so that aggregate consumption variability remains constant across economies. In Table 3, for instance, we see that higher CCV is associated with lower volatility in equity returns. This is because, *ceteris paribus*, higher CCV results in higher aggregate consumption variability which, then, necessitates us to reduce the variability of the depreciation shocks.

the Constantinides-Duffie model, but only 8 points in our model.[8]

4.2 Sensitivity analysis

Lowering risk aversion to 3 obviously lowers the Sharpe ratio and also the contribution of CCV-risk to the Sharpe ratio (over and above the Sharpe ratio with complete markets). However, the qualitative findings discussed above and documented in the figures (all of which pertain to economies with a risk aversion of 8), remain unchanged with a risk aversion of 3.

The age portfolio profiles in Figure 2 do not change much for other parts of the wealth distribution, or change much over the business cycle. As all the figures thus far were for the *average* investor, Figure 3 documents portfolio holdings of the top 10% of the wealth distribution. The portfolio holdings in this figure show that the portfolio choice patterns underlying the model's equity premium are relatively constant across the wealth distribution. Figures 4 and 5 provide the age profile of portfolio shares as a function of the aggregate state for both the complete market and the CCV economy respectively. The upshot is that these portfolio profiles do not change in any significant manner across recessions and expansions, although, as expected, there is a slight increase in equity investment during expansions.

[8] We have ignored frictions such as portfolio constraints and trading costs. It is quite likely that an important aspect of the overall impact of CCV involves its interaction with such frictions. See, for example, Alvarez and Jermann (2001), Gomes and Michaelides (2004) and Lustig (2004).

5 Conclusions

This chapter asks whether idiosyncratic labor market risk is quantitatively important for asset prices. This question is not new and often the answer has been 'no' (e.g, Telmer (1993), Heaton and Lucas (1994)). On the other hand Constantinides and Duffie (1996) show that with permanent shocks to marginal utility and infinitely lived agents the dynamic properties of idiosyncratic risk can crucially affect asset prices. A natural question arise is whether a realistic calibration, which takes into account the fact that workers and retirees face differential labor risk, will lead to very different conclusions.

The premise of our investigation is that idiosyncratic risk is naturally tied down to life cycle effects. Workers face earnings shocks while retirees don't, and the young have less financial assets to secure themselves against these shocks. This life cycle structure of idiosyncratic labor risk provides *prima facie* a role for intergenerational risk sharing by which the old will help the young smooth shocks. This presents a challenge to the asset pricing story by Constantinides and Duffie (1996). Our investigation shows, however, that introducing idiosyncratic risk within a life cycle context matters – that is the equity premium can be sizeable. Moreover, the model delivers an age 'hump-shaped' portfolio choice pattern — a feature consistent with the data.

The risk premium in the model reflects both the countercyclical-volatility risk emphasized by Constantinides and Duffie (1996), and a "concentration of aggregate risk" upon the middle-aged and old, alluded to by Mankiw (1986). These two risks are manifestation of two key features of the model. First, the life cycle nature of the ratio of human capital to financial wealth and the fact only the former is affected by idiosyncratic risk clearly makes such risks be more important for the young (as older agents have built a non-trivial financial wealth). It follows that the CCV effect is therefore less important for older agents who therefore are more content to hold equity. The second feature stems from the fact that returns are more volatile than wages, making older agents, whose wealth is mostly in the form of financial wealth, be less tolerant to holding equity. These two offsetting effects imply that young agents hold zero equity, retired agents hold diversified portfolios of equity and bonds, and middle-aged agents hold levered equity, issuing bonds to both the young and the old — resulting in the hump shape age profile for equity holdings.

In Constantinides, Donaldson, and Mehra (2002) (CDM), young agents are endowed with very small wealth and are barred from borrowing or shorting equity. The young therefore choose not to hold any assets. Consequently, the age profile of equity holdings is also hump shaped. Thus, the equity premium in CDM crucially depends on the concentration of aggregate risk on the middle age agents. However, the reasons for why the young do not hold equity in CDM are fundamentally different from those in our model. In our model the decision to avoid equity is driven by *risk*—avoidance of the countercyclical volatility risk. On the other hand, in the CDM framework young agents view equity as a desirable investment, that is any positive savings would have been channelled to equity. Which of these interpretations is more important, although it is quite plausible both can coexist, is something we leave for future research.

References

Aiyagari, S. R., (1994), Uninsured idiosyncratic risk and aggregate saving, *Quarterly Journal of Economics* 109, 659–684.

Aiyagari, S. R. and M. Gertler, (1991), Asset returns with transactions costs and uninsured individual risk, *Journal of Monetary Economics* 27, 311–331.

Altonji, J. G., F. Hayashi, and L. J. Kotlikoff, (1991), Risk sharing, altruism, and the factor structure of consumption, NBER working paper number 3834.

Altug, S. and R. A. Miller, (1990), Households choices in equilibrium, *Econometrica* 58, 543–70.

Alvarez, F. and U. Jermann, (2001), Quantitative asset pricing implications of endogeneous solvency constraints, *Review of Financial Studies* 14, 1117–1152.

Amerkis, J. and S. P. Zeldes, (2000), How do household portfolio shares vary with age?, Unpublished manuscript, Columbia University.

Attanasio, O. and S. J. Davis, (1996), Relative wage movements and the distribution of consumption, *Journal of Political Economy* 104, 1227–1262.

Attanasio, O. and G. Weber, (1992), Consumption growth and excess sensitivity to income: evidence from U.S. micro data, Unpublished manuscript, Stanford University.

Balduzzi, P. and T. Yao, (2005), Testing heterogeneous-agent models: An alternative aggregation approach, Forthcoming, *Journal of Monetary Economics.*

Benzoni, L., P. Collin-Dufresne, and R. Goldstein, (2004), Portfolio choice over the life-cycle in the presence of 'trickle down' labor income, Working paper, University of Minnesota.

Bodie, Z., R. C. Merton, and W. F. Samuelson, (1992), Labor supply flexibility and portfolio choice in a life cycle model, *Journal of Economic Dynamics and Control* 16, 427–49.

Boldrin, M., L. J. Christiano, and J. D. Fisher, (2001), Habit persistence, asset returns and the business cycle, *American Economic Review* 91, 149–166.

Brav, A., G. M. Constantinides, and C. Geczy, (2002), Asset pricing with heterogeneous consumers and limited participation: Empirical evidence, *Journal of Political Economy* 110, 793–824.

Brown, D. P., (1990), Age clienteles induced by liquidity constraints, *International Economic Review* 31, 891–911.

Carroll, C., (2004), Theoretical foundations of buffer stock saving, Working paper, John Hopkins University.

Cochrane, J. H., (1991), A simple test of consumption insurance, *Journal of Political Economy* 99, 957–76.

Cogley, T., (2002), Idiosyncratic risk and the equity premium: Evidence from the consumer expenditure survey, *Journal of Monetary Economics* 29, 309–334.

Constantinides, G. M., J. B. Donaldson, and R. Mehra, (2002), Junior can't borrow: a new perspective on the equity premium puzzle, *Quarterly Journal of Economics* 117, 269–296.

Constantinides, G. M. and D. Duffie, (1996), Asset pricing with heterogeneous consumers, *Journal of Political Economy* 104, 219–240.

Deaton, A. and C. Paxson, (1994), Intertepmoral choice and inequality, *Journal of Political Economy* 102, 437–467.

den Haan, W., (1994), Heterogeneity, aggregate uncertainty and the short term interest rate: a case study of two solution techniques, Working paper, University of California at San Diego.

Gomes, F. and A. Michaelides, (2004), Asset pricing with limited risk sharing and heterogeneous agents, Working paper, LBS.

Graham, J. R., (2000), How big are the tax benefits of debt?, *Journal of Finance* 55, 1901–1941.

Greenwood, J., Z. Hercowitz, and P. Krusell, (1997), Long-run implications of investment-specific technological change, *American Economic Review* 87, 342–362.

Guvenen, F., (2005), A parsimonious macroeconomic model for asset pricing: Habit formation or cross-sectional heterogeneity?, Working Paper, University of Rochester.

Heathcote, J., K. Storesletten, and G. L. Violante, (2005), Two views of inequality over the life-cycle, *Journal of the European Economic Association (Papers and Proceedings)* 3 (2-3), 543–552.

Heaton, J. and D. J. Lucas, (1994), The importance of investor heterogeneity and financial market imperfections for the behavior of asset prices, *Carnegie Rochester Conferance Series on Public Policy* .

Heaton, J. and D. J. Lucas, (1996), Evaluating the effects of incomplete markets on risk sharing and asset pricing, *Journal of Political Economy* 104, 443–487.

Heaton, J. and D. J. Lucas, (2000), Portfolio choice and asset prices; the importance of entrepreneurial risk, *Journal of Finance* 55, 1163–1198.

Huggett, M., (1993), The risk free rate in heterogeneous-agents, incomplete insurance economies, *Journal of Economic Dynamics and Control* 17, 953–969.

Huggett, M., (1996), Wealth distribution in life-cycle economies, *Journal of Monetary Economics* 38, 469–494.

Krusell, P. and A. A. Smith, (1997), Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns, *Macroeconomic Dynamics* 1, 387–422.

Lucas, D. J., (1994), Asset pricing with undiversifiable risk and short sales constraints: Deepening the equity premium puzzle, *Journal of Monetary Economics* 34, 325–341.

Lustig, H., (2004), The market price of aggregate risk and the wealth distribution, Working Paper, Stanford University.

Mace, B. J., (1991), Full insurance in the presence of aggregate uncertainty, *Journal of Political Economy* 99, 928–56.

Mankiw, N. G., (1986), The equity premium and the concentration of aggregate shocks, *Journal of Financial Economics* 17, 211–219.

Marcet, A. and K. J. Singleton, (1999), Equilibrium assets prices and savings of heterogeneous agents in the presence of portfolio constraints, *Macroeconomic Dynamics* 3, 243–277.

Mehra, R. and E. Prescott, (1985), The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–61.

Olovsson, C., (2004), Social security and the equity premium puzzle, Working paper, IIES.

Ramchand, L., (1999), Asset pricing in open economies with incomplete markets: implications for foreign currency returns, *Journal of International Money and Finance*, 18, 871-890.

Ríos-Rull, J. V., (1994), On the quantitative importance of market completeness, *Journal of Monetary Economics* 34, 463–496.

Sarkissian, S., (2003), Incomplete consumption risk sharing and currency risk premiums, *Review of Financial Studies* 16(3), 983–1005.

Storesletten, K., (2000), Sustaining fiscal policy through immigration, *Journal of Political Economy* 108, 300–323.

Storesletten, K., C. I. Telmer, and A. Yaron, (2004a), Consumption and risk sharing over the life cycle, *Journal of Monetary Economics* 59(3), 609–633.

Storesletten, K., C. I. Telmer, and A. Yaron, (2004b), Cyclical dynamics in idiosyncratic labor market risk, *Journal of Political Economy* 112(3), 695–717.

Storesletten, K., C. I. Telmer, and A. Yaron, (2006), Asset pricing with idiosyncratic risk and overlapping generations, under review, *Review of Economic Dynamics.*

Telmer, C. I., (1993), Asset pricing puzzles and incomplete markets, *Journal of Finance* 48, 1803–1832.

Weil, P., (1992), Equilibrium asset prices with undiversifiable labor income risk, *Journal of Economic Dynamics and Control* 16, 769–790.

Zhang, H., (1997), Endogenous borrowing constraints with incomplete markets, *Journal of Finance* 52, 2187–2209.

A Calibration Appendix

This appendix first describes the calibration of the no-trade (Constantinides and Duffie (1996)) economies in Section 2 and Table 1, and then goes on to describe the calibration of the economies with trade, presented in Section 3 and Table 1. It also demonstrates the sense in which our specification for countercyclical volatility — heteroskedasticity in the innovations to the idiosyncratic component of log income — is consistent with the approach used by previous authors (*e.g.*, Heaton and Lucas (1996), Constantinides and Duffie (1996)). In each case, the cross sectional variance which matters turns out to be the variance of the *change* in the log of an individual's share of income and/or consumption.

Calibration of No-Trade Economies

Aggregate consumption growth follows an *i.i.d* two-state Markov chain, with a mean growth of 1.8% and standard deviation of 3.3%. This is essentially the process used in Mehra and Prescott (1985) with slightly more conservative volatility. The Constantinides and Duffie (1996) model is then 'calibrated' via a re-interpretation of the preference parameters of the Mehra and Prescott (1985) representative agent. Recall that we use β and γ to denote an *individual* agent's utility discount factor and risk aversion parameters, respectively. Constantinides and Duffie (1996) construct a representative agent (their equation (16)) whose rate of time preference and coefficient of relative risk aversion are (using our notation),

$$-\log \beta^* = -\log(\beta) - \frac{\gamma(\gamma+1)}{2}\, a \tag{22}$$

and

$$\gamma^* = \gamma - \frac{\gamma(\gamma+1)}{2}\, b \tag{23}$$

respectively. In these formulae, the parameters a and b relate the cross sectional variance in the change of the log of individual i's share of aggregate consumption (y_{t+1}^2, using Constantinides-Duffie's notation) to the growth rate of aggregate consumption:

$$Var(\log \frac{c_{i,t+1}/c_{t+1}}{c_{it}/c_t}) = a + b \log \frac{c_{t+1}}{c_t} \tag{24}$$

All that we require, therefore, are the numerical values for a and b which are implied by our PSID-based estimates in Table 1 of Storesletten, Telmer, and Yaron (2004b).

Our estimates are based on income, y_{it}. Because the Constantinides-Duffie model is autarkic, we can interpret these estimates as pertaining to individual consumption, c_{it}. Balduzzi and Yao (2005), Brav, Constantinides, and Geczy (2002), and Cogley (2002) take the alternative route and use microeconomic consumption data. While their results are generally supportive of the model, they each point out serious data problems associated with using consumption data. Income data is advantageous is this sense. In addition, our objective is just as much relative as it is absolute. That is, consumption is endogenous in the model of Section 3, driven by risk sharing behavior and the exogenous process for idiosyncratic income risk. What Table 1 asks is, "what would the Constantinides-Duffie economy look like, were its agents to be endowed with idiosyncratic risk of a similar magnitude?" Also, "how does our model measure up, in spite of its non-degenerate (and more realistic) risk sharing technology?" Using income data seems appropriate in this context. Accordingly, for the remainder of this appendix we set $c_{it} = y_{it}$.

We need to establish the relationship between our specification for idiosyncratic shocks and the log-shares of aggregate consumption in equation (24).

Denote individual $i's$ share at time t as γ_{it}, so that,

$$\log \gamma_{it} \equiv \log c_{it} - \log \tilde{E}_t c_{it}$$

where the notation $\tilde{E}_t(\cdot)$ denotes the cross-sectional mean at date t, so that $\tilde{E}_t c_{it}$ is date t, per-capita aggregate consumption. The empirical specification in Storesletten, Telmer, and Yaron (2004b) identifies an idiosyncratic shock as the residual from a log regression with year-dummy variables:

$$z_{it} = \log c_{it} - \tilde{E}_t \log c_{it}$$

which have a cross-sectional mean of zero, by construction, and a sample mean of zero, by least squares. The difference between our specification and the log-share specification is, therefore,

$$\begin{aligned} \log \gamma_{it} - z_{it} &= \tilde{E}_t \log c_{it} - \log \tilde{E}_t c_{it} \\ &= \tilde{E}_t \log \gamma_{it} - \log \tilde{E}_t \gamma_{it} \end{aligned}$$

The share, γ_{it}, is *defined* so that its cross-sectional mean is always one. The second term is therefore zero. For the first term, note that in both our economy and the statistical model underlying our estimates, the cross sectional distribution is log normal, *conditional* on knowledge of current and past aggregate shocks. If some random variable x is log normal and $E(x) = 1$, then $E(\log x) = -Var(\log x)/2$. As a result,

$$\log \gamma_{it} - z_{it} = -\frac{1}{2} \tilde{V}_t(\log \gamma_{it})$$

where $\tilde{V}_t$ denotes the cross-sectional variance operator. Because lives are finite in our model, and because we interpret data as being generated by finite processes, this cross-sectional variance will always be well defined, irrespective of whether or not the shocks are unit root processes.

The quantity of interest in equation (24) can now be written as,

$$\begin{aligned}\log \frac{c_{i,t+1}/c_{t+1}}{c_{it}/c_t} &\equiv \log \gamma_{i,t+1} - \log \gamma_{it} \\ &= z_{i,t+1} - z_{it} - \frac{1}{2}\left(\tilde{V}_{t+1}(\log \gamma_{i,t+1}) - \tilde{V}_t(\log \gamma_{it})\right) \qquad (25)\end{aligned}$$

The term in parentheses — the difference in the variances — does not vary in the cross section. Consequently, application of the cross-sectional variance operator to both sides of equation (25) implies,

$$\tilde{V}_{t+1}\left(\log \frac{c_{i,t+1}/c_{t+1}}{c_{it}/c_t}\right) = \tilde{V}_{t+1}\left(z_{i,t+1} - z_{it}\right) \quad .$$

The process underlying our estimates is

$$z_{i,t+1} - z_{it} = (1-\rho) z_{it} + \eta_{i,t+1}$$

where the variance of $\eta_{i,t+1}$ depends on the aggregate shock. For values of ρ close to one the variance of changes in z_{it} is approximately equal to the variance of $\eta_{i,t+1}$. The left side of equation (24) is, therefore, approximately equal to the variance of innovations, $\eta_{i,t+1}$,

$$\tilde{V}_{t+1}\left(\log \frac{c_{i,t+1}/c_{t+1}}{c_{it}/c_t}\right) \approx \tilde{V}_{t+1}\left(\eta_{i,t+1}\right) \quad .$$

For unit root shocks — which we assume for most of Section 3, this holds exactly. The estimates of σ_E and σ_C in Storesletten, Telmer, and Yaron (2004b), Table 1, are therefore sufficient to calibrate the Constantinides-Duffie model.

All that remains is to map our estimates into numerical values for a and b from equation (24). Since aggregate consumption growth is calibrated to be an $i.i.d$ process with a mean and standard deviation of 1.8% and 3.3% respectively, aggregate consumption growth, the variable on the right hand side of equation

(24), takes on only two values, 5.1% and -1.5%. Computing the parameters a and b, then simply involves two linear equations:

$$\begin{aligned}\sigma_E^2 &= a + 0.051b \\ \sigma_C^2 &= a - 0.015b \ .\end{aligned}$$

Storesletten, Telmer, and Yaron's (2004b) estimates are $\sigma_E^2 = 0.0156$ and $\sigma_C^2 = 0.0445$. These estimates, however, are associated with $\rho = .952$. For our unit root economies, we scale them down so as to maintain the same average unconditional variance (across age). This results in $\sigma_E^2 = 0.0059$ and $\sigma_C^2 = 0.0168$. The resulting values for a and b are, $a = 0.0143$ and $b = -0.1652$.

B Asset Pricing

Following the Euler equation (7), and recalling that P is the probability of remaining in a given state, the price of the risky security is,

$$p_t = \begin{cases} \beta P (1+z)^{-\hat{\gamma}} & \text{if } t \text{ is a boom} \\ \beta (1-P)(1+z)^{-\hat{\gamma}} & \text{if } t \text{ is recession} \end{cases} \tag{26}$$

Similarly the price of the bond is

$$\begin{aligned} q_t &= E_t \beta M_{t+1} \\ &= \begin{cases} \beta \left(P (1+z)^{-\hat{\gamma}} + (1-P)(1-z)^{-\hat{\gamma}} \right) & \text{if } t \text{ is boom} \\ \beta \left((1-P)(1+z)^{-\hat{\gamma}} + P (1-z)^{-\hat{\gamma}} \right) & \text{if } t \text{ is recession} \end{cases} \end{aligned} \tag{27}$$

It then follows that the unconditional bond-price is simply

$$E(q_t) = \frac{1}{2} \beta \left((1+z)^{-\hat{\gamma}} + (1-z)^{-\hat{\gamma}} \right) \tag{28}$$

Consequently, the realized return on the excess risky asset, r_{t+1} can be expressed as,

$$r_{t+1} = \begin{cases} \frac{1}{p_j} - \frac{1}{q_j} & \text{if } t+1 \text{ is boom} \\ -\frac{1}{q_j} & \text{if } t+1 \text{ is recession} \end{cases} \tag{29}$$

where $j \in \{\text{boom}, \text{recession}\}$ denotes the aggregate state in period t.

Given the expressions in equations (27) and (29), it is easy to solve for (10) in the case of *i.i.d.* aggregate shocks ($P = 1/2$).

Table 1

Asset Pricing Properties – No Trade Economies

	Risk	Riskfree Rate		Equity Premium		Sharpe
	Aversion	Mean	Std Dev	Mean	Std Dev	Ratio
U.S. data		1.30	1.88	6.85	16.64	41.17
U.S. data, unlevered		1.30	1.88	4.11	10.00	41.17
Models Without Trade (Constantinides-Duffie):						
Complete Markets	3.0	1.30	–	0.87	10.0	8.7
No CCV	3.0	1.30	–	0.87	10.0	8.7
Estimated CCV	3.0	1.30	–	1.15	10.0	11.5
Complete Markets	8.0	1.30	–	2.26	10.0	22.6
No CCV	8.0	1.30	–	2.26	10.0	22.6
Estimated CCV	8.0	1.30	–	3.72	10.0	37.2

'Models Without Trade' correspond to a calibration of the Constantinides and Duffie (1996) model using the idiosyncratic risk estimates from Storesletten, Telmer, and Yaron (2004b), Table 1, and the aggregate consumption moments from Mehra and Prescott (1985). Details are given in Appendix A. Rows labelled 'Complete Markets' are economies with no idiosyncratic volatility (i.e., $\sigma_t = 0$). Rows labelled 'No CCV' represent an 'homoskedastic' economy, i.e. an economy with idiosyncratic risk with time-invariant variance. This conditional variance is set equal to the average conditional variance across recessions and booms, $var(\eta) = 0.0114$. The rows labelled 'Estimated CCV' correspond to economies with idiosyncratic risk equal to a unit-root version of the estimated process in Storesletten, Telmer, and Yaron (2004b). Further details are given in Appendix A. In all the models $K/Y = 0$.

U.S. sample moments are computed using non-overlapping annual returns, from end of January-to the end of January, 1956-1996. Estimates of means and standard deviations are qualitatively similar using annual data beginning from 1927, or a monthly series of overlapping annual returns. Equity data correspond to the annual return on the CRSP value weighted index, inclusive of distributions. Riskfree returns are based on the one month U.S. treasury bill. Nominal returns are deflated using the GDP deflator. All returns are expressed as annual percentages. Unlevered equity returns are computed using a debt to firm value ratio of 40 percent, which is taken from Graham (2000).

Table 2

Aggregate Moments–Economies With Trade

Panel A: Population Moments of Growth Rates, Theoretical Economy

	Std Dev	Autocorrelation	Correlation with Output
Output	0.930	-0.307	1.000
Investment	1.214	-0.309	0.998
Consumption	0.038	-0.117	0.935

Panel B: Sample Moments of Growth Rates, U.S. Economy, 1929-1982

	Std Dev	Autocorrelation	Correlation with Output
Output	0.062	0.561	1.000
Investment	0.358	0.225	0.143
Consumption	0.036	0.353	0.471

U.S. sample moments are based on annual NIPA data, 1929-1982. Theoretical moments are computed as sample averages of a long simulated time series.

Table 3

Asset Pricing Properties – Economies with Trade

	Risk					Riskfree Rate	Equity Premium		Sharpe
	Aversion	β	K/Y	σ^2_E	σ^2_C	Mean	Mean	Std Dev	Ratio
Complete markets	3	0.965	3.3	0	0	4.4	0.63	7.0	9.0
no CCV	3	0.947	3.3	0.0114	0.0114	2.6	0.80	7.0	11.5
estimated CCV	3	0.948	3.3	0.0168	0.0059	2.3	0.86	6.7	12.8
Complete markets	8	0.96	3.3	0	0	4.7	2.48	9.3	26.8
no CCV	8	0.809	3.3	0.0114	0.0114	2.6	19.8	7.6	26.1
estimated CCV	8	0.801	3.3	0.0168	0.0059	1.3	2.32	7.1	32.6
large CCV	8	0.797	3.3	0.0204	0.0023	1.6	2.51	6.6	38.0

'Models with Trade,' are described in Section 3. The calibration procedure is discussed in the text. All economies are calibrated so that aggregate consumption volatility is 3.3%. The 'Homoskedastic Economy' is distinguished by the volatility of idiosyncratic shocks *not* varying with aggregate shocks. The idiosyncratic shocks are calibrated so the unit root economy has the same average volatility as that in an economy based on the estimates of Storesletten, Telmer, and Yaron (2004b).

Figure 1

Quantity of Bonds and Stocks, by Age

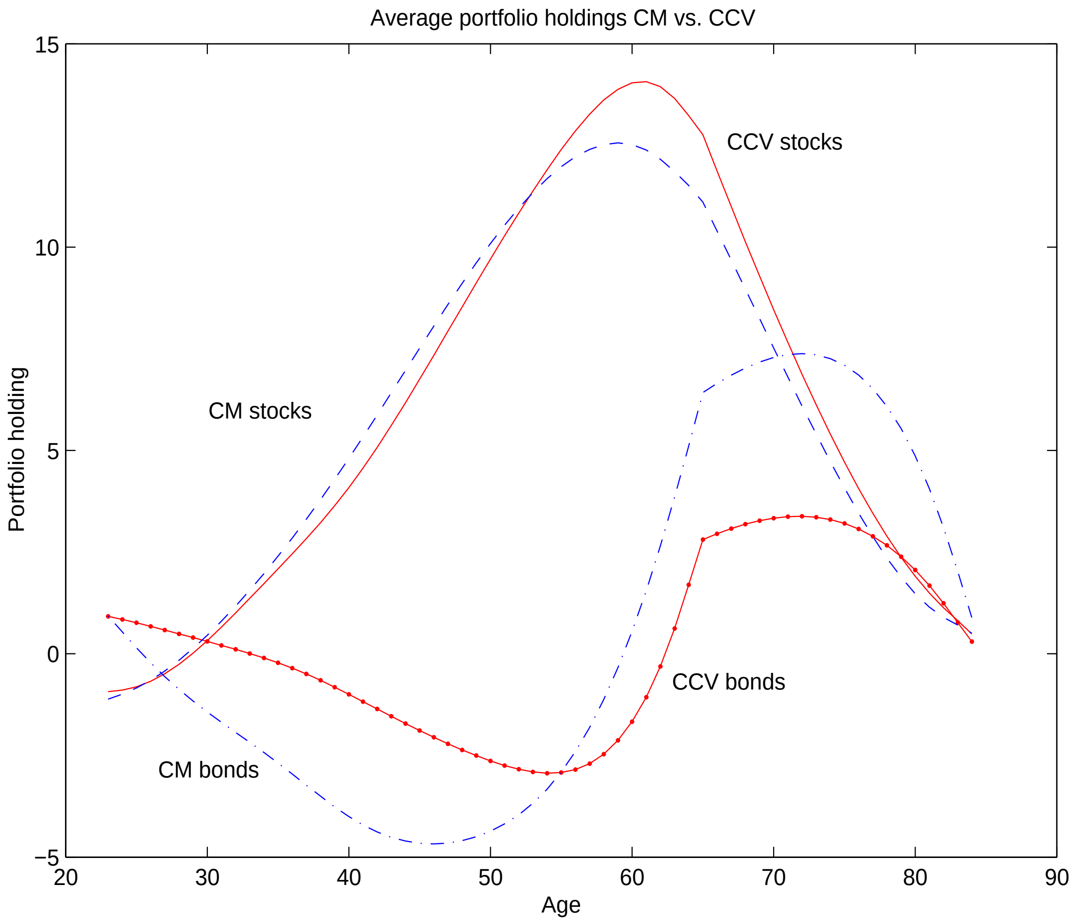


Figure 2

Bond and Stock Portfolio Shares, by Age

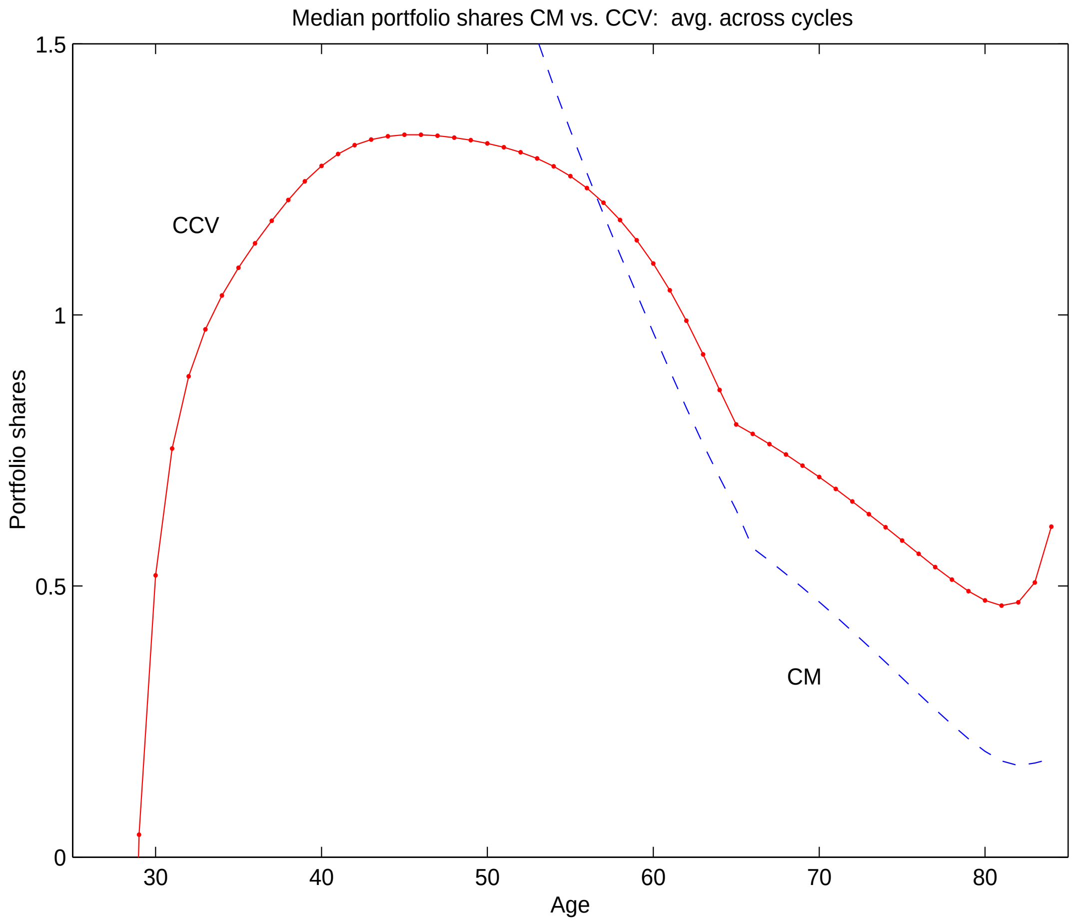


Figure 3

Portfolio Shares for Top 10 Earnings Percentile, by Age

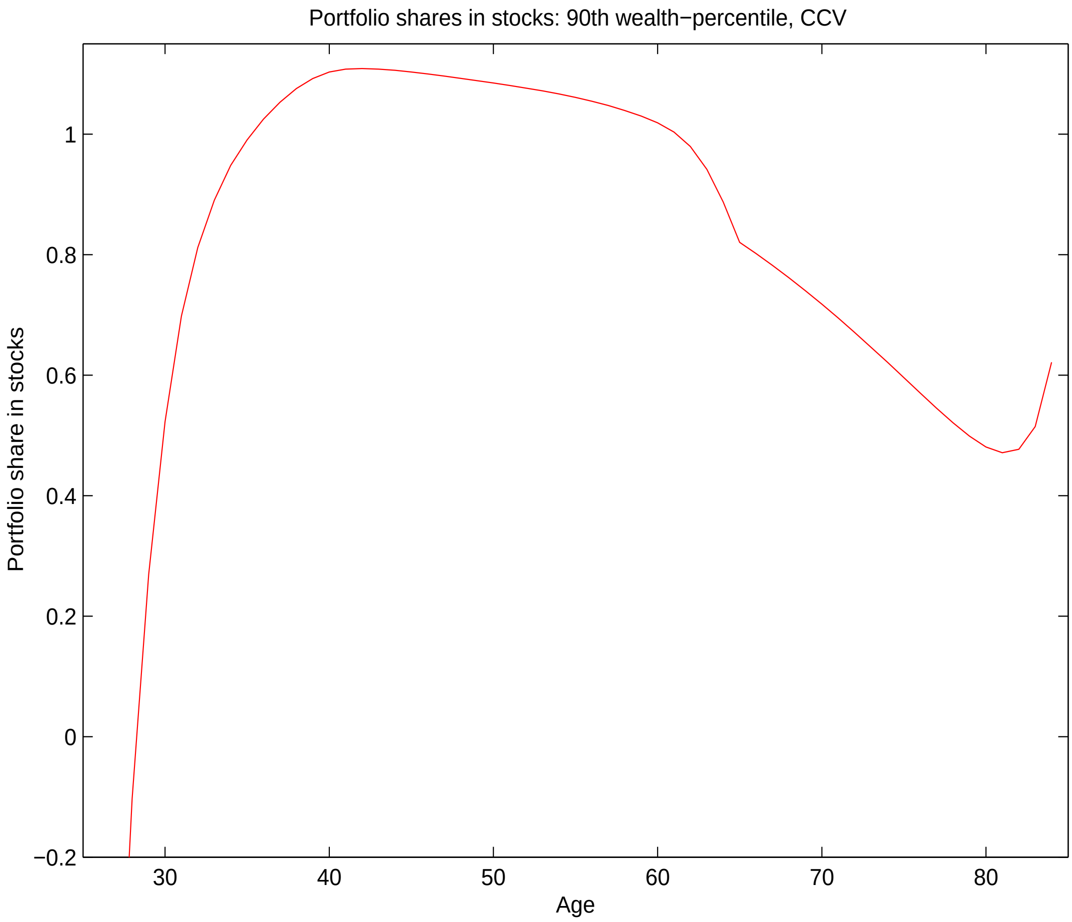
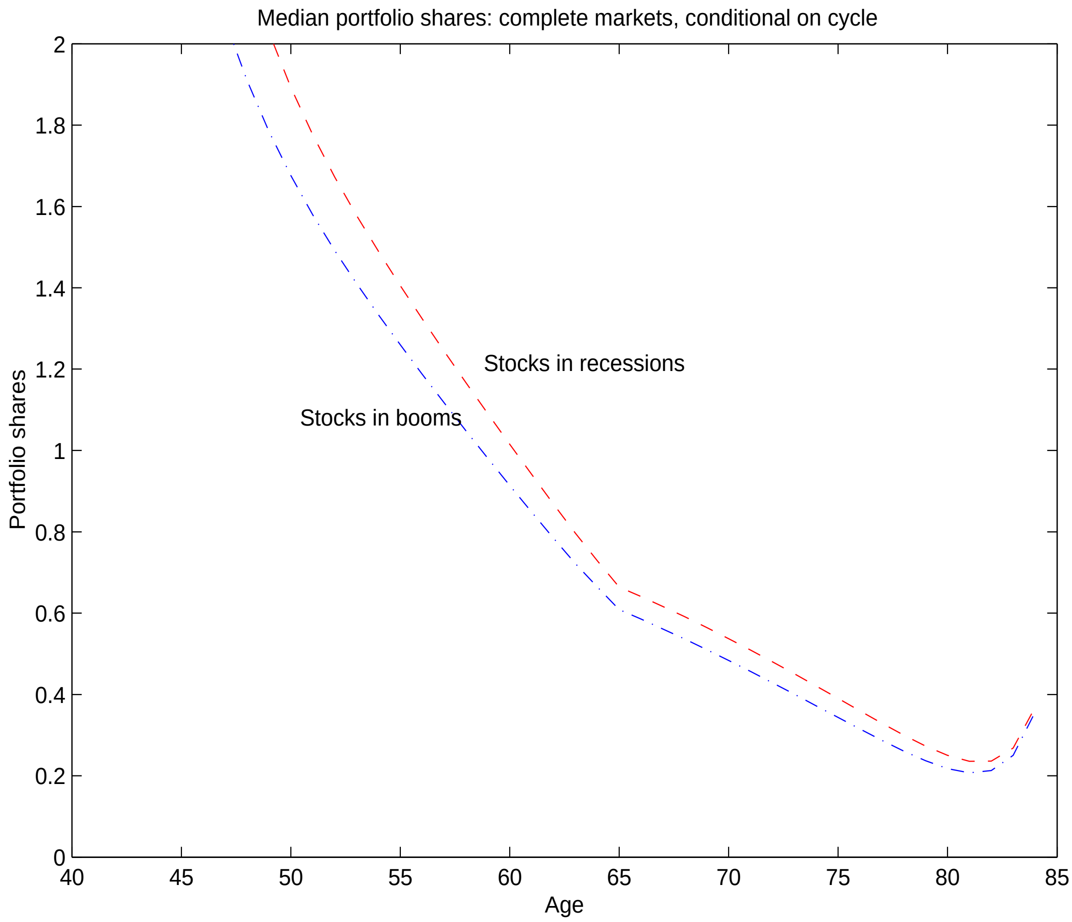


Figure 4

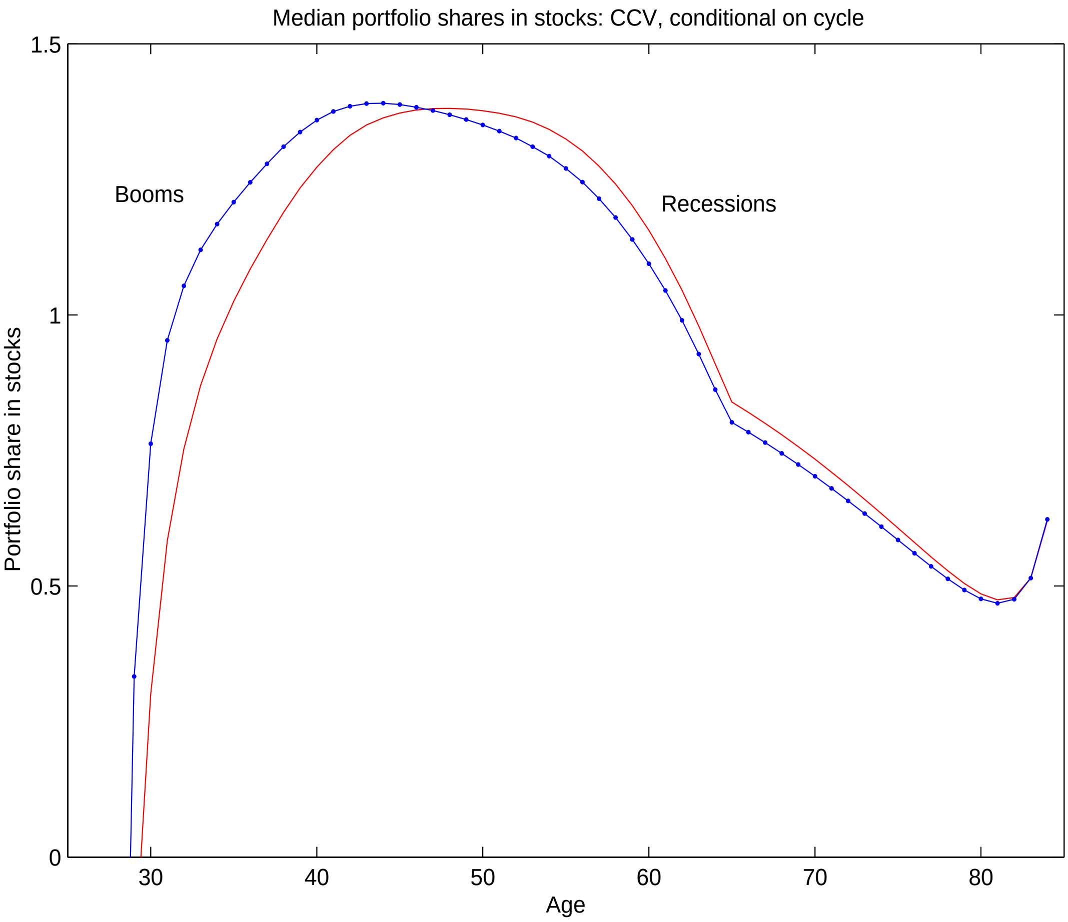
Portfolio Shares conditional on Business Cycle, by Age



The solid line conditions on aggregate expansions. The dashed line conditions on aggregate contractions.

Figure 5

Portfolio Shares conditional on Business Cycle, by Age



The solid line conditions on aggregate expansions. The dashed line conditions on aggregate contractions.