Frictional Wage Dispersion in Search Models: A Quantitative Assessment

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“Residual” wage distribution for janitors in Philadelphia

Mean / first percentile = 2.24
Why are similar workers paid differently?

- **Unobserved heterogeneity**
  
  - Human capital  
    (e.g., “innate” ability)
Why are similar workers paid differently?

• Unobserved heterogeneity
  ▶ Human capital
    (e.g., “innate” ability)

• Compensating differentials
  ▶ Job characteristics
    (e.g., non-pecuniary amenities)
Why are similar workers paid differently?

• Unobserved heterogeneity
  ▶ Human capital
    (e.g., “innate” ability)

• Compensating differentials
  ▶ Job characteristics
    (e.g., non-pecuniary amenities)

• Labor market luck
  ▶ Search/matching frictions ⇒ “frictional wage dispersion”
Outline of the talk

1. **New tool** to study frictional wage dispersion in search models

   **Mean-min ratio**
Outline of the talk

1. New tool to study frictional wage dispersion in search models
   - Mean-min ratio

2. Calibrate the “textbook” model $\Rightarrow$ frictional dispersion is very small

3. Data detour: residual wage dispersion is larger by a factor of 20!

4. Extensions of the textbook model: risk-aversion, volatile wages, on-the-job search

5. Relation to empirical search literature and to Shimer-Hall puzzle
McCall-Mortensen search model (continuous time)

- Homogeneous workers, infinitely lived, risk-neutral
- Discount rate $r$, flow value of unemployment $b$
- Wage remains constant on the job
- Exogenous job separation (into unemployment) at rate $\sigma$
- Search only during unemployment
- At rate $\lambda_u$, unemployed workers encounter wage offers drawn from the exogenous distribution $F(w)$
Solution of the model

- Flow values of employment and unemployment

\[ rW(w) = w - \sigma [W(w) - U] \]

\[ rU = b + \lambda_u \int_{w^*}^{w_{\text{max}}} [W(w) - U] dF(w) \]
Solution of the model

• Flow values of employment and unemployment

\[ rW(w) = w - \sigma [W(w) - U] \]

\[ rU = b + \lambda_u \int_{w^*}^{w_{\text{max}}} [W(w) - U] dF(w) \]

• The reservation wage equation is:

\[ w^* = rU = b + \frac{\lambda_u}{r + \sigma} \int_{w^*}^{w_{\text{max}}} [w - w^*] dF(w) \]
Solution of the model

- WLOG, let \( b \equiv \rho \bar{w} \), with \( \bar{w} = E[w|w \geq w^*] \), then:

\[
\begin{align*}
    w^* &= \rho \bar{w} + \frac{\lambda_u [1 - F(w^*)]}{r + \sigma} \int_{w^*}^{w_{\text{max}}} [w - w^*] \frac{dF(w)}{1 - F(w^*)} \\
    &= \rho \bar{w} + \frac{\lambda_u^*}{r + \sigma} [\bar{w} - w^*]
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\]

where \( \lambda_u^* \equiv \lambda_u [1 - F(w^*)] \)
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Last step eliminates \( F(w) \) which is unobservable through the job finding rate \( \lambda_u^* \).
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where $\lambda_u^* \equiv \lambda_u [1 - F(w^*)]$

- Last step eliminates $F(w)$ which is unobservable through the job finding rate $\lambda_u^*$

- Equation relates average wage $\bar{w}$ and lowest wage paid $w^*$ only through observables $(r, \sigma, \lambda_u^*, \rho)$
Mean-min ratio (Mm)

\[ Mm \equiv \frac{\bar{w}}{w^*} = \frac{\frac{\lambda_u}{r+\sigma}} + 1 + \frac{\lambda_u}{r+\sigma} + \rho \]
Mean-min ratio (Mm)

\[
Mm \equiv \frac{\bar{w}}{w^*} = \frac{\lambda_u^*}{r+\sigma} + \rho
\]

- New measure of wage dispersion in search models
- “Distribution-free” measure [i.e., does not depend on \( F(w) \)]
- Derived naturally from reservation wage equation
- Increasing in \((\sigma, r)\), decreasing in \((\rho, \lambda_u^*)\)
Equilibrium search models

- Lucas-Prescott (1974) island-model
  - Random search across islands
  - Competitive labor market on each island
Equilibrium search models

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- Pissarides (1990) matching model
  - Aggregate matching function
  - Free entry of firms
  - Nash bargaining
Equilibrium search models

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  ▶ Random search across islands
  ▶ Competitive labor market on each island

• Pissarides (1990) matching model
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  ▶ Nash bargaining

• Both models imply that same expression for $M m$
Quantitative implications for $Mm$

\[ Mm = \frac{\frac{\lambda^*_u}{r+\sigma}}{\frac{\lambda^*_u}{r+\sigma} + \rho} + 1 \]

- **Calibration** of model to U.S. economy (monthly frequency)
  - Interest rate: $r = 0.0041$ (Cooley, 1995)
  - Separation rate: $\sigma = 0.020$ (Shimer, 2005a)
  - Job finding rate: $\lambda^*_u = 0.39$ (Shimer, 2005a)
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  - **Flow value of unemployment:** $\rho = 0.40$ (Shimer, 2005b)
Quantitative implications for $Mm$

$$Mm = \frac{\lambda^*_u}{r + \sigma} + 1 \left( \frac{\lambda^*_u}{r + \sigma} + \rho \right)$$

- **Calibration** of model to U.S. economy (monthly frequency)
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  - Flow value of unemployment: $\rho = 0.40$ (Shimer, 2005b)

$$Mm = 1.036$$
Some unpleasant search arithmetic

\[ Mm = \frac{\lambda_u^*}{r+\sigma} + 1 \]

\[ = \frac{0.39}{0.0041+0.02} + 1 \]

\[ = 16.2 + 1 \]

\[ = \frac{17.2}{16.2} = 1.062 \]

• ... if we are willing to accept that \( \rho \geq 0 \)
Interpretation: “Good things come to those who wait”

• Unemployed workers search longer (and turn down jobs) if there is a large option value of waiting for better offers

• Option value is determined by wage dispersion

• Short unemployment duration, as in U.S. data, reveals that dispersion is small!!
Interpretation: “Good things come to those who wait”

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- Option value is determined by wage dispersion.

- Short unemployment duration, as in U.S. data, reveals that dispersion is small!!

- How large is frictional wage dispersion in the data?
Measurement of residual wage dispersion

• Empirical counterpart of theory: wage observations for ex-ante similar workers searching in the same labor market

• Three key data issues:

  1. Definition of labor market: occupation/geographical area
Measurement of residual wage dispersion

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- Three key data issues:
  1. Definition of labor market: occupation/geographical area
  2. Control for ex-ante individual heterogeneity
     - observable: e.g., education, experience, gender, race
     - unobservable: e.g., innate workers’ characteristics
Measurement of residual wage dispersion

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  3. Measurement error in hourly wages
Three sources of micro data

1. 5% sample of 1990 US Census (∼ 4,600,000 obs.)
   - 487 occupations, 799 geographical areas
   - controls: gender, race, edu, exp, marital status
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2. Occupational Employment Statistics (\(\sim 1,200,000\) establishments)
   - 637 occupations, 337 metropolitan areas
   - no controls available, but no reporting errors
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2. Occupational Employment Statistics (∼1,200,000 establishments)
   • 637 occupations, 337 metropolitan areas
   • no controls available, but no reporting errors

3. PSID (∼ 80,000 obs.)
   • controls: gender, race, edu, exp, mar. status, region, occ, occ × region
   • panel data ⇒ control for fixed individual effects
Empirical Findings

Table 1: Dispersion measures for hourly wage from various data sources

<table>
<thead>
<tr>
<th></th>
<th>Min. obs. per cell</th>
<th>Number of labor mkts</th>
<th>Ratio of mean wage to min.</th>
<th>1st pct.</th>
<th>5th pct.</th>
<th>10th pct.</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel Study of Income Dynamics 1990-1996</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd stage residuals</td>
<td>–</td>
<td>–</td>
<td>3.11</td>
<td>1.90</td>
<td>1.46</td>
<td>1.32</td>
<td>0.25</td>
</tr>
<tr>
<td>Occupational Employment Statistics 2000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Occ./Geog. Area</td>
<td>–</td>
<td>106,278</td>
<td>–</td>
<td>2.07</td>
<td>1.66</td>
<td>1.48</td>
<td>–</td>
</tr>
<tr>
<td>5% sample of 1990 US Census</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Occ./Geog. Area (N≥50)</td>
<td>13,246</td>
<td>2.94</td>
<td>2.66</td>
<td>2.04</td>
<td>1.76</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td>Full time/Full year (N≥50)</td>
<td>7,195</td>
<td>2.74</td>
<td>2.49</td>
<td>1.92</td>
<td>1.66</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>Unskilled Occ. (N≥50)</td>
<td>1,152</td>
<td>2.51</td>
<td>2.37</td>
<td>1.98</td>
<td>1.77</td>
<td>0.45</td>
<td></td>
</tr>
</tbody>
</table>

- We target $Mm = 1.70$ and $CV = 0.30$
Distribution of Mp5 across US labor markets (Census)

13,246 cells
Median Mm=2.04

Hornstein-Krusell-Violante, "Frictional Wage Dispersion" – p. 16/45
Distribution of individual Mm (PSID)

14,572 observations
Median Mm = 1.57
### Private Firm Hourly Wage Dispersion Measures

**Danish IDA Data, 1994-1995**

<table>
<thead>
<tr>
<th>Occupation</th>
<th># of firms</th>
<th>1pct</th>
<th>5pct</th>
<th>10pct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>113,325</td>
<td>1.62</td>
<td>1.39</td>
<td>1.29</td>
</tr>
<tr>
<td>Management</td>
<td>49,667</td>
<td>2.09</td>
<td>1.65</td>
<td>1.45</td>
</tr>
<tr>
<td>Salaried</td>
<td>57,513</td>
<td>1.60</td>
<td>1.38</td>
<td>1.30</td>
</tr>
<tr>
<td>Skilled</td>
<td>44,527</td>
<td>1.56</td>
<td>1.32</td>
<td>1.27</td>
</tr>
<tr>
<td>Unskilled</td>
<td>70,886</td>
<td>1.66</td>
<td>1.40</td>
<td>1.30</td>
</tr>
</tbody>
</table>
How far is the model from the data?

Pairs of \((\rho, r)\) consistent with \(M_m=1.70\)

Reasonable pairs

Net value of leisure as a fraction of \(w\) \((\rho)\)

Monthly interest rate \((r)\)
Iso-$Mm$ curves for the benchmark model

Net value of leisure as a fraction of $w$ ($\rho$)

Monthly interest rate ($r$)

- $Mm=1.2$
- $Mm=1.7$
- $Mm=2.2$

Hornstein-Krusell-Violante, "Frictional Wage Dispersion" – p. 20/45
Why is $\rho = -3$ implausible?

- In order to avoid a week of unemployment, a worker would be willing to:
  
  - work for free for a week
  
  - pay $1,500 (3 times the average weekly salary)
  
  - and, at the end of the week, draw a wage offer from the same distribution he would face if unemployed
Resolving the data-model discrepancy

1. Nothing wrong with the model $\Rightarrow$ frictional wage dispersion is **negligible** compared to unobserved workers’ heterogeneity

   - Bad news for search theory?
Resolving the data-model discrepancy

1. Nothing wrong with the model $\Rightarrow$ frictional wage dispersion is negligible compared to unobserved workers’ heterogeneity
   - Bad news for search theory?

2. Nothing wrong with the model + workers truly hate unemployment $\Rightarrow$ frictional wage dispersion is large
   - Bad news for the rest of macro-labor?
Resolving the data-model discrepancy

1. Nothing wrong with the model ⇒ frictional wage dispersion is negligible compared to unobserved workers’ heterogeneity
   - Bad news for search theory?

2. Nothing wrong with the model + workers truly hate unemployment ⇒ frictional wage dispersion is large
   - Bad news for the rest of macro-labor?

3. Model fails quantitatively ⇒ it needs to be augmented
   - We take this route here, but first...
In defense of the textbook model

1. "Model designed to study unemployment, not wage dispersion"

- Given $Mm = 1.7$, model predicts expected unemployment duration of 91 months!!
1. "Model designed to study unemployment, not wage dispersion"

- Given $Mm = 1.7$, model predicts expected unemployment duration of 91 months!!

2. "The model will perform better on European data"

- In steady-state: $\lambda^*_u u = \sigma (1 - u)$

\[
Mm = \frac{\lambda^*_u}{r+\sigma} + 1 \quad s.s. \quad \frac{\sigma}{r+\sigma} \frac{1-u}{u} + 1 \sim \quad \frac{1-u}{u} + 1 \quad \rho > 0 \quad \frac{1}{1-u}
\]
3. "Mm ratio is an unconventional measure of dispersion"

- Assume **Gamma distribution**

\[
g(w; w^*, \beta, \gamma) = \left( \frac{w-w^*}{\beta} \right)^{\gamma-1} \exp\left( -\frac{w-w^*}{\beta} \right) \frac{1}{\beta \Gamma(\gamma)}
\]
In defense of the textbook model, continued

3. "Mm ratio is an unconventional measure of dispersion"

- Assume **Gamma distribution**

\[
g(w; w^*, \beta, \gamma) = \frac{(\frac{w-w^*}{\beta})^{\gamma-1} \exp(-\frac{w-w^*}{\beta})}{\beta \Gamma(\gamma)}
\]

- Then, we can show that:

\[
CV(w) = \frac{1}{\sqrt{\gamma}} \frac{Mm(w) - 1}{Mm(w)}
\]

- **Data:** \( Mm(w) = 1.70\) and \( CV(w) = 0.30 \Rightarrow \hat{\gamma} = 1.88\)
- **Search model:** \( Mm = 1.036 \Rightarrow CV = 0.025\)
In defense of the textbook model, continued

4. "Endogenous search effort: if it is costly to search, doesn’t the model work better?"

- We obtain, with \( c_u (\lambda_u) = \kappa_u \lambda_u^{1+1/\gamma} \),

\[
  w^* = b - c_u (\lambda_u^o) + \frac{\lambda_u^*}{r + \sigma} (\bar{w} - w^*)
\]

and, given the FOC \( c'_o (\lambda_u^o) = \frac{1}{r+\delta} \int_{w^*}^{w_{\text{max}}} (w - w^*) dF (z) \),

\[
  Mm = \frac{\lambda_u^*}{r+\sigma} \frac{1}{1+\gamma} + 1
\]

- For \( \gamma = 34.5 \), this works. But then \( \frac{b-c_u(\lambda_u^o)}{\bar{w}} = -1/6.1 \).
In defense of the textbook model, continued

5. "Ability differences will work"

- Two types, fraction \( \alpha \) of type 1, with wage distributions scaled by \( \mu \% \):

\[
F_1(w) = F_2(w/(1 + \mu)).
\]

As before, implies low (and equal!) \( Mm \) ratios within groups but

\[
Mm = (1 + \alpha\mu) \frac{\bar{w}_2}{w^*_2}
\]

among all workers: using \( \mu \), this can be made arbitrarily large.

- But this, of course, is not frictional wage dispersion!
Using the “Mm tool” in richer search models

1. Risk-aversion

2. Volatile wages during employment

3. On-the-job search

The critical “rule of the game” is not adding unobserved heterogeneity
Risk Aversion

• Let preferences be $u(c) = \frac{c^{1-\theta}}{1-\theta}$, and let $c = w$

• Upper bound for effects of risk-aversion: **no self-insurance**

• Second-order Taylor expansion of reservation wage equation:

$$Mm \simeq \left[ \left(1 + \frac{1}{2} \theta (\theta - 1) CV(w)^2 \right) \frac{\lambda^*_u}{r+\sigma} + \rho^{1-\theta} \right]^{\frac{1}{\theta-1}}$$

• **Calibration**: only new number we need is $CV(w) = 0.30$
Numerical results for the model with risk-aversion

Pairs \((\rho, \theta)\) consistent with \(Mm=1.70\)

\[
\theta = 8
\]
Iso-$M_m$ curves for the model with risk-aversion

The graph shows the net value of leisure as a fraction of $w$ against the relative risk aversion ($\theta$). The curves represent different values of $M_m$: $M_m=1.7$, $M_m=1.2$, and $M_m=2.2$. The graph illustrates how the net value of leisure changes with different levels of relative risk aversion and varying values of $M_m$. The x-axis represents the relative risk aversion ($\theta$), ranging from 0 to 45, while the y-axis shows the net value of leisure as a fraction of $w$, ranging from 0 to 1.

Hornstein-Krusell-Violante, “Frictional Wage Dispersion” – p. 30/45
Wage shocks during employment (M-P, 1994)

• Wages fluctuate randomly along the employment spell

• At rate $\delta$, wage changes and employees draw from $F(w)$

• Endogenous separations at rate $\sigma^* = \delta F(w^*)$
Wage shocks during employment (M-P, 1994)

- Wages fluctuate randomly along the employment spell
- At rate $\delta$, wage changes and employees draw from $F(w)$
- Endogenous separations at rate $\sigma^* = \delta F(w^*)$
- Solve the model and obtain...

$$Mm = \frac{\lambda_u^* - \delta + \sigma^*}{\lambda_u^* - \delta + \sigma^*} + 1 + \rho$$

- As $\delta \to \lambda_u$, $Mm \to 1/\rho$

$1 - \delta$: autocorrelation coefficient of the wage process
Numerical results for the model with stochastic wages

Pairs of $(1-\delta)$ and $\rho$ consistent with $Mm=1.70$

Net value of leisure as a fraction of $w\, (\rho)$

Reasonable pairs
Iso-$Mm$ curves for the model with stochastic wages

Annual autocorrelation coefficient of wage process $(1-\delta)$

Net value of leisure as a fraction of $w$ $(\rho)$

- $Mm=1.7$
- $Mm=1.2$
- $Mm=2.2$

Hornstein-Krusell-Violante, “Frictional Wage Dispersion” – p. 33/45
On-the-job search (Burdett, 1978)

- Workers draw wage offers from $F(w)$ at rate $\lambda_u$ if unemployed, and at rate $\lambda_w$ if employed

  $F(w)$ could be any wage offer distribution

- When employed, accept offer $w'$ if $w' > w$: $F(w) \Rightarrow G(w)$
On-the-job search (Burdett, 1978)

- Workers draw wage offers from $F(w)$ at rate $\lambda_u$ if unemployed, and at rate $\lambda_w$ if employed
  
  $\triangleright$ $F(w)$ could be any wage offer distribution

- When employed, accept offer $w'$ if $w' > w$: $F(w) \Rightarrow G(w)$

- Solve the model for the realized steady-state wage distribution $G(w)$ and, for $r$ small, obtain...

\[
Mm \approx \frac{\lambda_u^* - \lambda_w}{r + \sigma + \lambda_w} + 1
\]

\[
\frac{\lambda_u^* - \lambda_w}{r + \sigma + \lambda_w} + \rho
\]

$\triangleright$ As $\lambda_w \rightarrow \lambda_u$, $Mm \rightarrow 1/\rho$
Restricting the value of $\lambda_w$

- Average separation rate $\chi$ in the model is:

$$\chi = \sigma + \lambda_w \int_{w^*} [1 - F(w)] dG(w) = \frac{\sigma (\lambda_w + \sigma) \log \left( \frac{\sigma + \lambda_w}{\sigma} \right)}{\lambda_w}$$

- BLS (JOLTS): monthly separation rate $\chi = 0.04$
Restricting the value of $\lambda_w$

- Average separation rate $\chi$ in the model is:

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- BLS (JOLTS): monthly separation rate $\chi = 0.04$

- Look for $\lambda_w$ consistent with both wage dispersion and labor mobility facts
Numerical results for the model with on the job search

Reasonable pairs of $(\rho, \lambda_w)$ consistent with $M_m = 1.70$

JOLTS estimate of average separation rate
Reallocation shocks (Nagypal, 2005)

• Baseline OJS model, plus...

• At rate $\phi$, employed workers make a wage draw from $F(w)$ which either they accept, or they separate

$$Mm \simeq \frac{\lambda^*-\lambda_w-\phi}{r+\sigma+\lambda_w+\phi} + 1$$

• Restrict $(\lambda_w, \phi)$ to match: (i) $Mm = 1.70$ and (ii) sep. rate of 4%
Reallocation shocks (Nagypal, 2005)

- Baseline OJS model, plus...

- At rate $\phi$, employed workers make a wage draw from $F(w)$ which either they accept, or they separate

\[
Mm \simeq \frac{\lambda^*_u - \lambda_w - \phi}{r + \sigma + \lambda_w + \phi} + 1 + \rho \frac{\lambda^*_u - \lambda_w - \phi}{r + \sigma + \lambda_w + \phi} + \rho
\]

- Restrict $(\lambda_w, \phi)$ to match: (i) $Mm = 1.70$ and (ii) sep. rate of 4%

- Monthly rate at which workers are subject to a wage cut is:

\[
\kappa = \phi \int F(w) dG(w) = 0.095
\]
Numerical results for the model with on the job search

Iso-Mm curves

Net value of leisure as a fraction of $w (\rho)$

Monthly offer arrival rate on the job $(\lambda_w)$
New generation of models with OJS

- Endogenous search effort (Christensen et al., 2005)
  - Implied net value of leisure still negative
New generation of models with OJS

• Endogenous search effort (Christensen et al., 2005)
  ▶ Implied net value of leisure still negative

• OJS with counteroffers (Postel-Vinay and Robin, 2002)
  ▶ Plausible model for labor market at large?
  ▶ Hiring fee paid by workers
New generation of models with OJS

- **Endogenous search effort** (Christensen et al., 2005)
  - Implied *net* value of leisure still *negative*

- **OJS with counteroffers** (Postel-Vinay and Robin, 2002)
  - Plausible model for labor market at large?
  - Hiring fee paid by workers

- **Wage-tenure contracts** (Stevens, 1999; Burdett and Coles, 2002)
  - More reasonable and quantitatively successful
Endogenous search effort (CLMNW, 2005)

• Unemployed and employed workers choose search effort, i.e., contact rate $\lambda$

• Workers face constant elasticity search effort cost functions, $c_i \lambda^{1+1/\gamma}$, contingent on (un)employment state, $i = u, w$
Endogenous search effort (CLMNW, 2005)

- Unemployed and employed workers choose search effort, i.e., contact rate $\lambda$

- Workers face constant elasticity search effort cost functions, $c_i \lambda^{1+1/\gamma}$, contingent on (un)employment state, $i = u, w$

- Derive upper and lower bounds for the steady-state wage distribution $G(w)$ and, for $r$ small, obtain bounds for Mm-ratio,

$$\frac{1 + \kappa(w^{max})}{\rho + \kappa(w^{max})} \leq Mm \leq \frac{1 + \kappa(w^*)}{\rho + \kappa(w^*)} \quad \text{with} \quad \kappa(w) = \frac{\lambda_u - \lambda_w^*}{(1 + \gamma)(\sigma + \lambda_w)}$$

- Conditional on the CLMNW estimates, $\gamma = 1.2$, $\sigma = 0.3$, $\rho = 0.4$, and $\lambda_u = 1$, this implies Mm-ratios between 1.7 and 2
Bounds on wages net of search costs

• The lower bounds implied for the search costs are

\[
\frac{c_u}{\bar{w}} \geq \frac{\lambda_u}{\sigma + \lambda_{w^*}} \psi \quad \text{and} \quad \frac{c_{w^*}}{\bar{w}} \geq \frac{\lambda_{w^*}}{\sigma + \lambda_{w_{\text{max}}}} \psi
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with \( \psi = \frac{\gamma}{1+\gamma} \left(1 - \frac{1}{Mm}\right) \).
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• For unemployed and marginal workers the net-of-search cost returns relative to average wages are then

\[
\rho - \frac{c_u}{\bar{w}} \in [-0.3, -0.5], \quad \text{and} \quad w^* - \frac{c_{w^*}}{\bar{w}} \in [-0.05, 0.3]
\]

...still negative!

• This assessment of the plausibility of the search costs is needed: calibrate, don’t estimate!
OJS with counteroffers (Postel-Vinay and Robin, 2002)

- When the worker meets a firm for the first time, the firm takes the whole surplus.

- When an employed worker gets an outside offer, the two firms Bertrand-compete: the worker takes the whole surplus and stays.

- Firms commit to wages: no renegotiation without outside offers.
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- For simplicity, suppose there is only one productivity level, $p$. This implies $w = p$ for workers who received counteroffers and

$$w^* = b - \frac{\lambda e}{r + \sigma} (p - b)$$

as a starting wage. Note $w^* < b$ and $Mm > 1/\rho$. 
A journey through the empirical search literature
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- Either accept implausible parameter estimates...
  - Postel-Vinay and Robin (ECA, 2002): for unskilled occupations, $r = 57\%$ per year
  - Flinn (ECA, 2006): for realistic values of $r$, $\rho = -4$
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• ...or need large unobserved heterogeneity
Cross-section vs. time-series

- **Shimer-Hall**: search model does not generate enough business-cycle volatility in vacancies and unemployment

- **Hagedorn-Manovskii**: we can save the standard search model as long as $\rho \approx 1$

- **Our paper**: if $\rho \approx 1$, no hope to get any frictional wage dispersion from search models

- Tension between time-series and cross-sectional implications of search model
Possible conclusions

1. Data still contain unobserved worker heterogeneity

   • Frictional wage dispersion is small in both data and model
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2. Data OK, but workers hate unemployment
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   - Plus: watch out for business-cycle implications!
Possible conclusions

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   - Frictional wage dispersion is small in both data and model

2. Data OK, but workers hate unemployment
   - Negative values for $\rho$ not plausible
   - Plus: watch out for business-cycle implications!

3. Data OK, and $\rho$ is moderately high
   - Basic search models fail
   - Latest OJS models (endogenous effort, counteroffers, wage-tenure contracts...) more promising