Radiative transfer

http://folk.uio.no/matsc/iris4

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Radiative transfer tutorial

- Optically thick line formation
- Mg II h & k diagnostics
- C II, O I, Cl I diagnostics
- Visualization of synthetic observables

Optically thick line formation

- Thick vs thin
- Opacity, optical depth, source function
- Intensity, profile shape, central reversal
- Line width

Basic definitions

I_ν Intensity. $\mathrm{erg\ cm^{-2} s^{-1} sr^{-1} Hz^{-1}}$

η_ν Emissivity

χ_ν Opacity

Intensity gives the amount of energy per unit area perpendicular to the ray in a given direction per unit time per solid angle and per frequency bin. The intensity is constant with distance in the absence of emission and absorption/scattering processes.

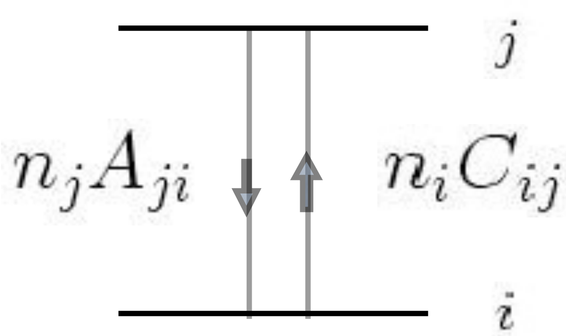
Emissivity gives the addition of energy to the ray through emission processes.

Opacity gives the removal of energy from the ray through absorption and scattering processes.

Transfer equation

$$dI_\nu = -\chi_\nu I_\nu ds + \eta_\nu ds$$

Coronal approximation: $\chi_\nu = 0$

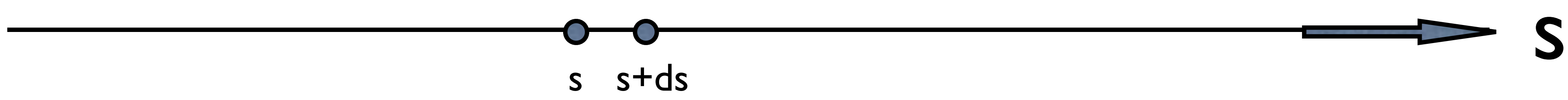


$$\eta_\nu = \frac{h\nu}{4\pi} n_j A_{ji}$$

$$n_j A_{ji} = n_i C_{ij}$$

Emissivity and rates set by local conditions.

Transfer equation



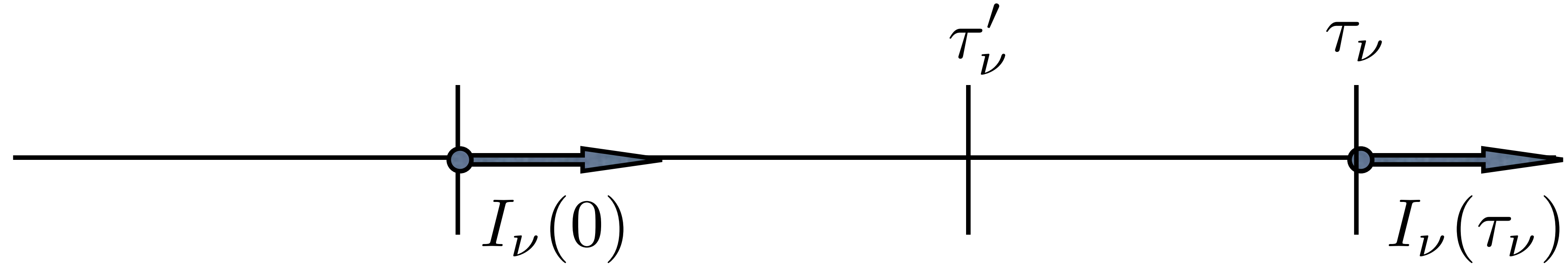
$$dI_\nu = -\chi_\nu I_\nu ds + \eta_\nu ds$$

Opacity non-zero case

$$d\tau_\nu \equiv \chi_\nu ds \quad S_\nu \equiv \frac{\eta_\nu}{\chi_\nu}$$

$$\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu$$

Transfer equation



$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(\tau'_\nu)e^{-(\tau_\nu - \tau'_\nu)}d\tau'_\nu$$

Homogeneous slab, small optical depth:

$$I_\nu(\tau_\nu) = I_\nu(0) + \tau_\nu(S_\nu - I_\nu(0))$$

Intensity (wavelength dependent!) is then an interpolation between the incoming intensity and the source function and thus always between the two. For I(0)>S we get an absorption line, for I(0)<S we get an emission line.

Homogeneous slab

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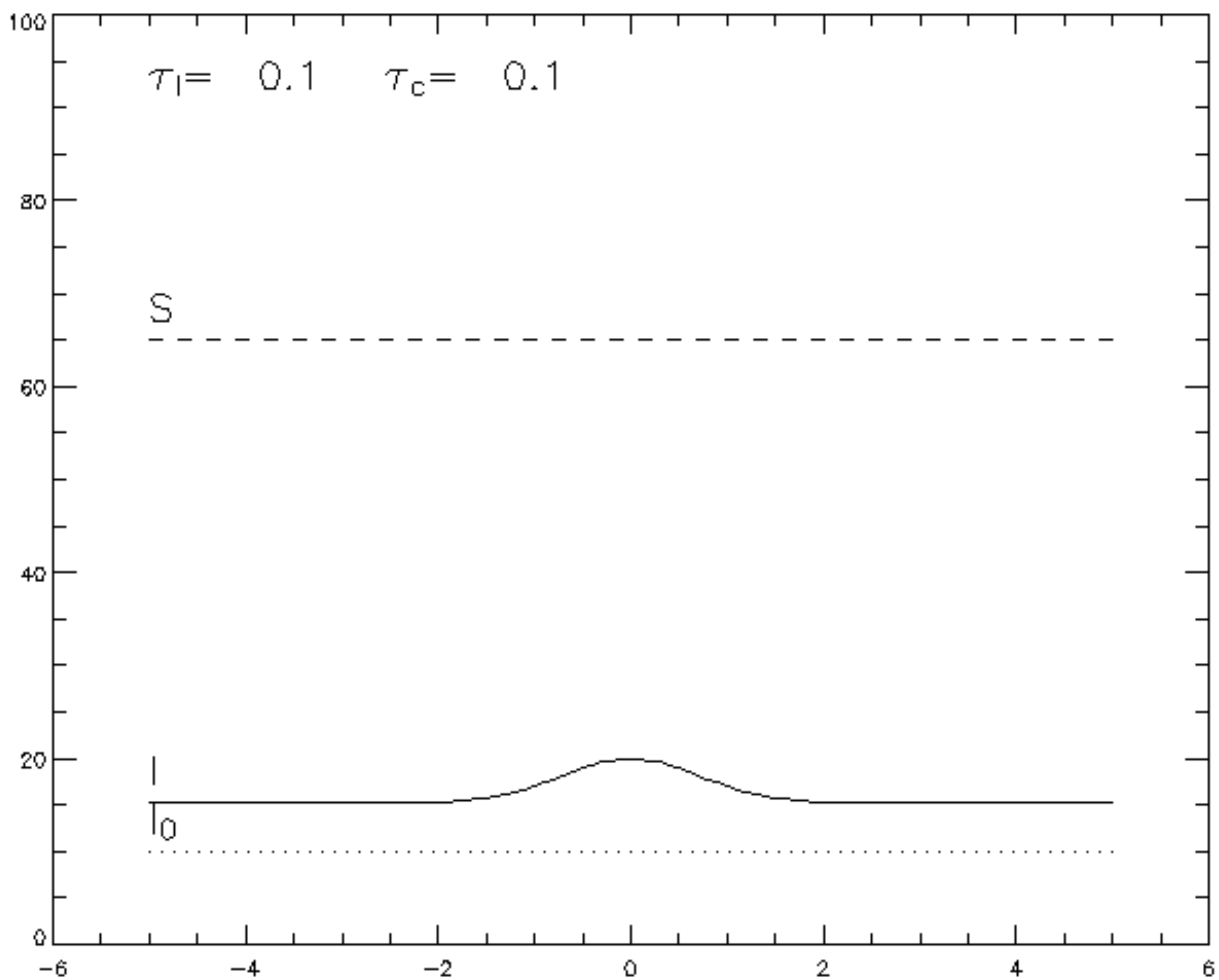
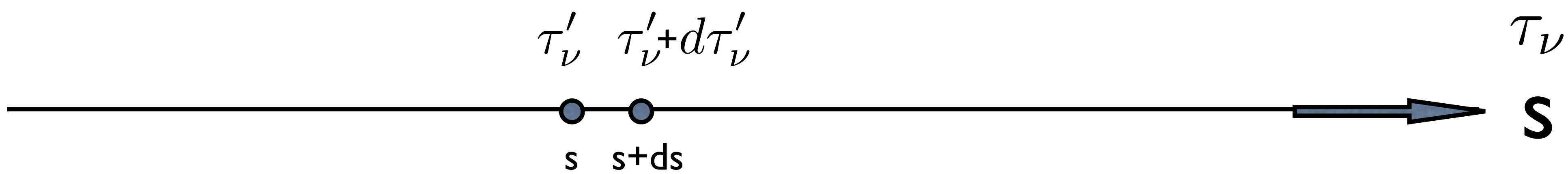


Transfer equation



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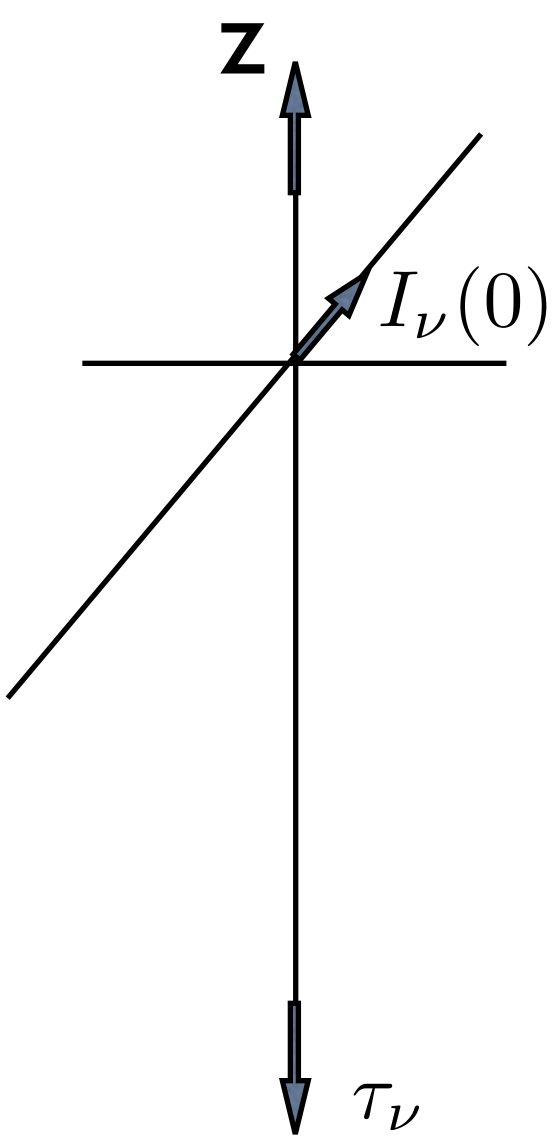


Semi-infinite atmosphere, 1D:

$$I_\nu(0) = \frac{1}{\mu}\int_0^{\infty} S_\nu(\tau'_\nu)e^{-\tau'_\nu/\mu}d\tau'_\nu$$

where mu is the cosine of the angle between the ray and the normal of the atmosphere

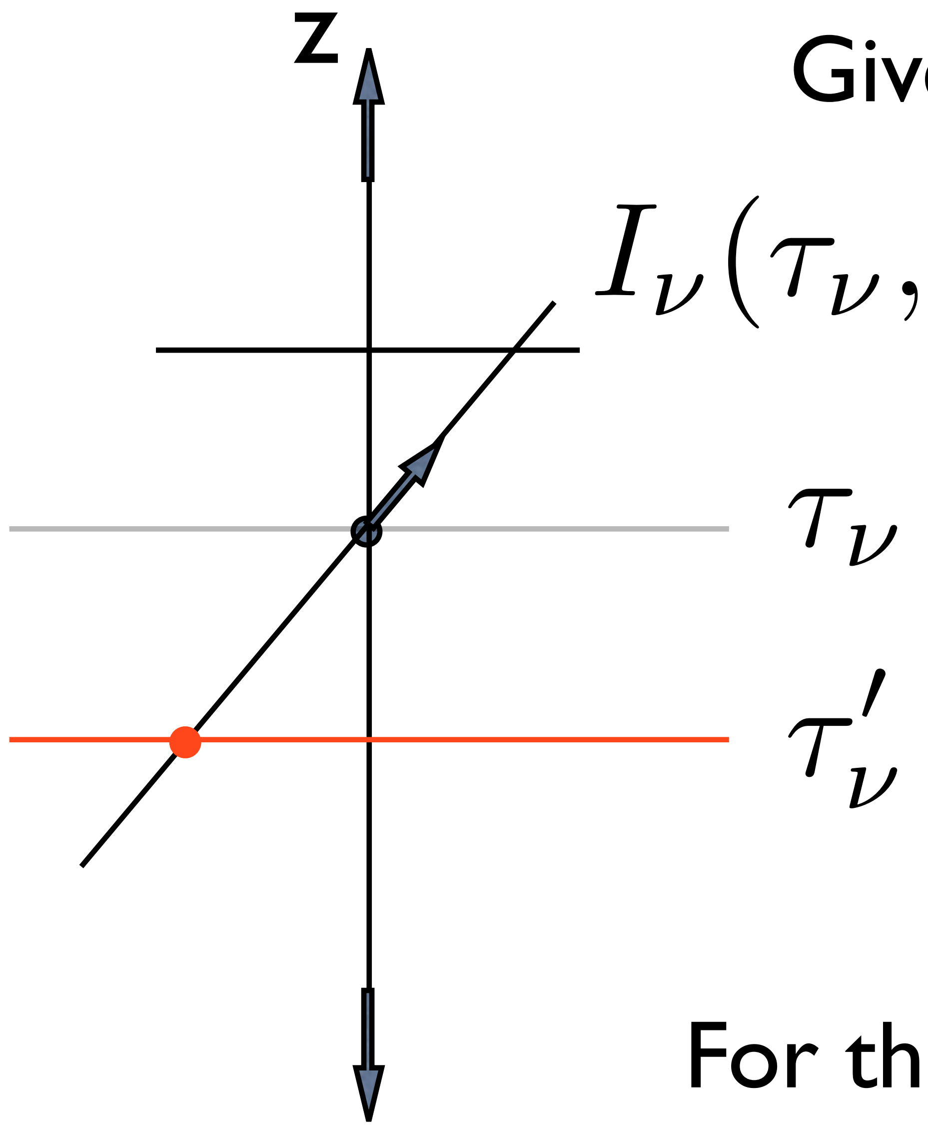
Eddington Barbier relation

Assuming a linear source function:

$$S_\nu(\tau'_\nu) = a + b\tau'_\nu$$

Gives a formal solution:

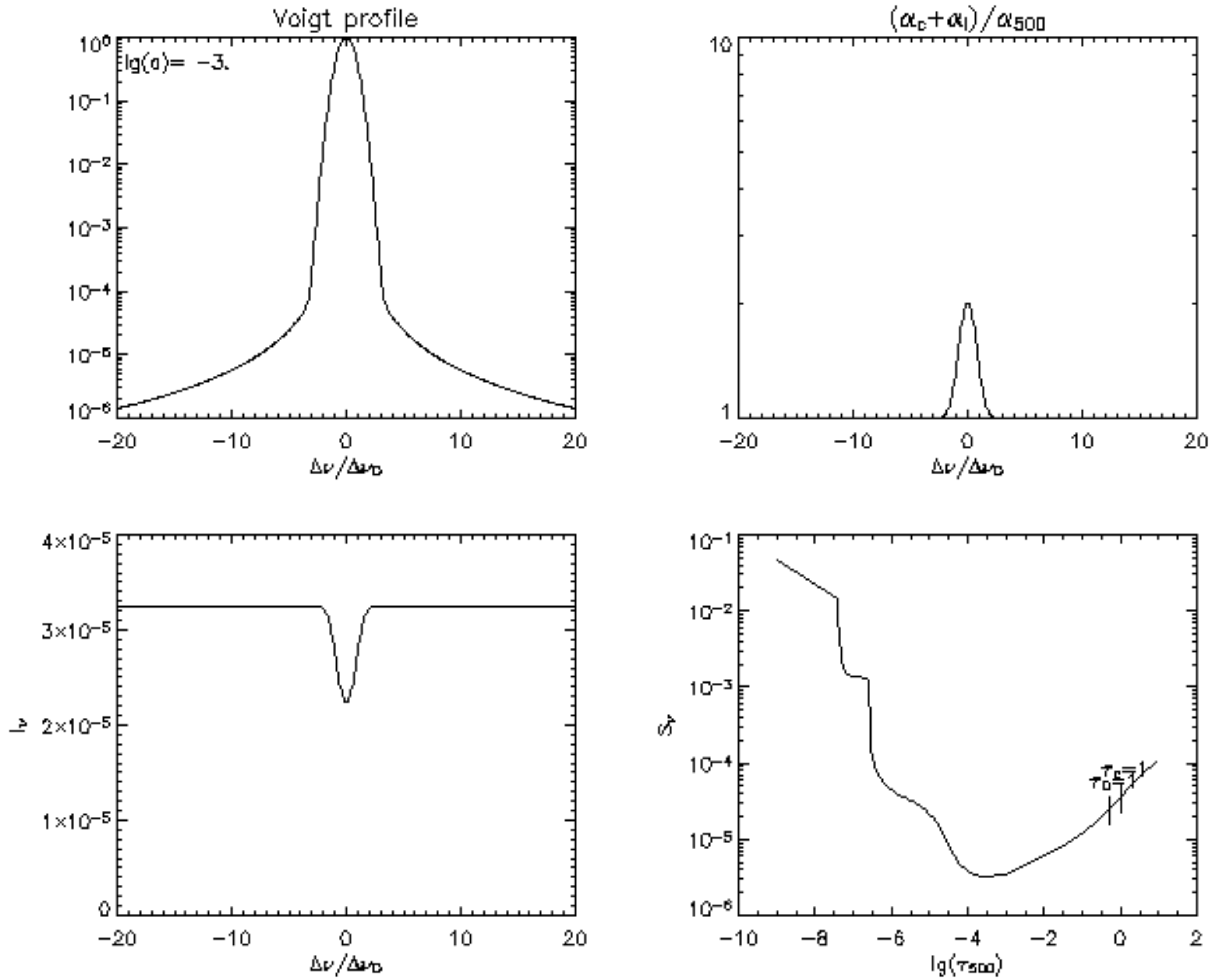
$$I_\nu(\tau_\nu, \mu) = S_\nu(\tau'_\nu = \tau_\nu + \mu)$$



For the emergent intensity:

$$I_\nu(0, \mu) = S_\nu(\tau_\nu = \mu) \qquad \text{EB}$$

Optically thick line formation



Where is optical depth unity?

$$\kappa_\nu^l = \frac{\pi e^2}{m_e c^2} n_l f_{lu} \phi_\nu \frac{1}{\rho}$$

$$\kappa_{\nu_0}^l = 0.02654 \frac{n_l}{N_{\mathrm{II}}} \frac{N_{\mathrm{II}}}{N_{\mathrm{el}}} \frac{N_{\mathrm{el}}}{N_{\mathrm{H}}} \frac{N_{\mathrm{H}}}{\rho} f_{lu} \frac{1}{\sqrt{\pi}\Delta\nu_D}$$

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$$\Delta\nu_D = \frac{1}{\lambda_0}\sqrt{\frac{2kT}{m} + \xi^2} = \frac{1}{\lambda_0}\Delta V_D$$

$\kappa_{\nu_0}^l$

$$\frac{\rm Mg\,II\,k}{\rm C\,II\,1335} = \frac{1}{4/6} \frac{N_{\rm II}}{N_{\rm el}} \frac{1}{6.8} \frac{5.2}{1} \frac{2800}{1335} \frac{1}{\Delta V_D} = 2.4(3.4) \frac{N_{\rm II}}{N_{\rm el}}$$

Contribution function to intensity

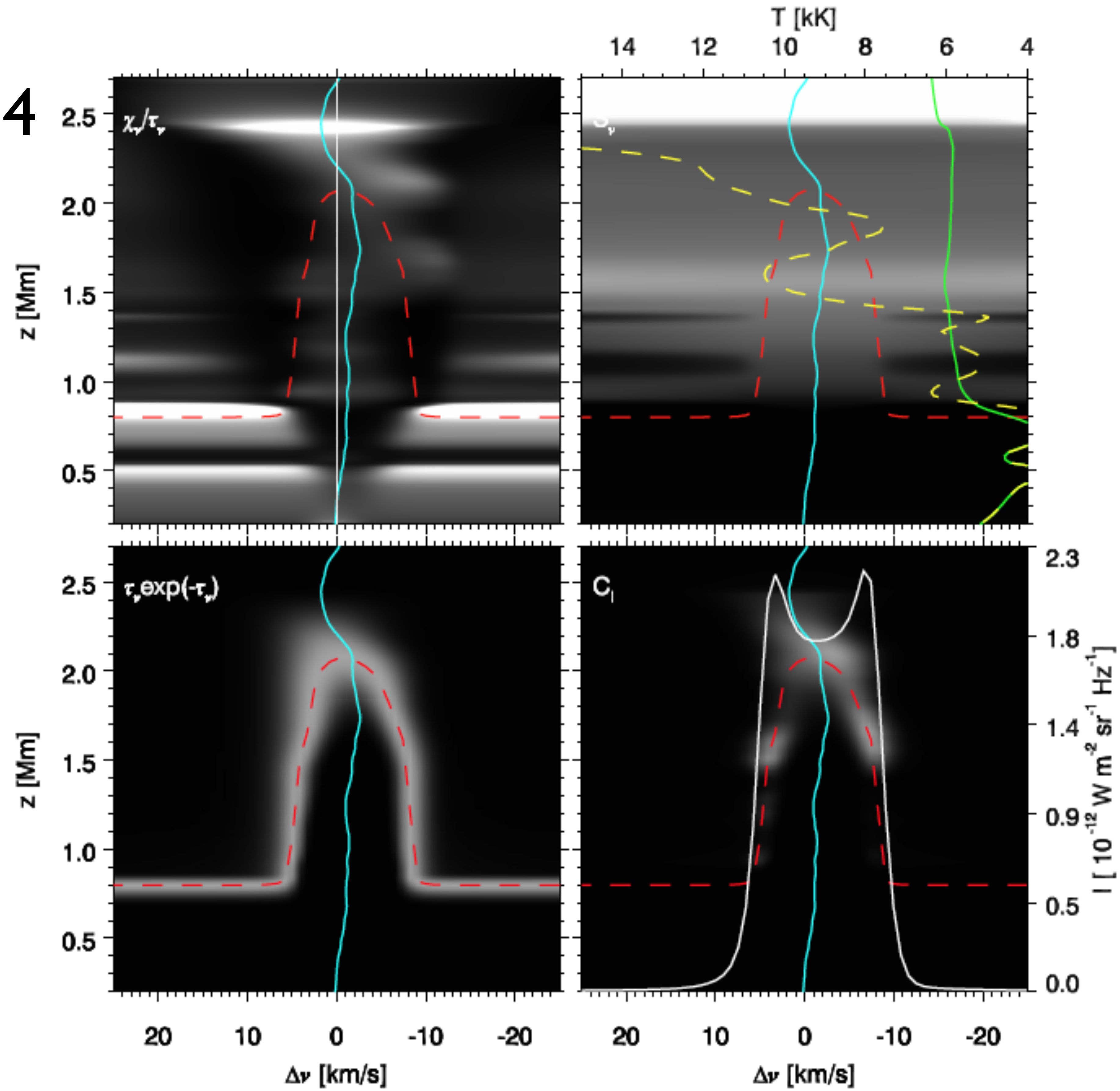
Disk center:

$$I_\nu = \int_0^\infty S_\nu e^{-\tau_\nu} \chi_\nu dz$$

$$C_{I_\nu}(z) = S_\nu e^{-\tau_\nu} \chi_\nu$$

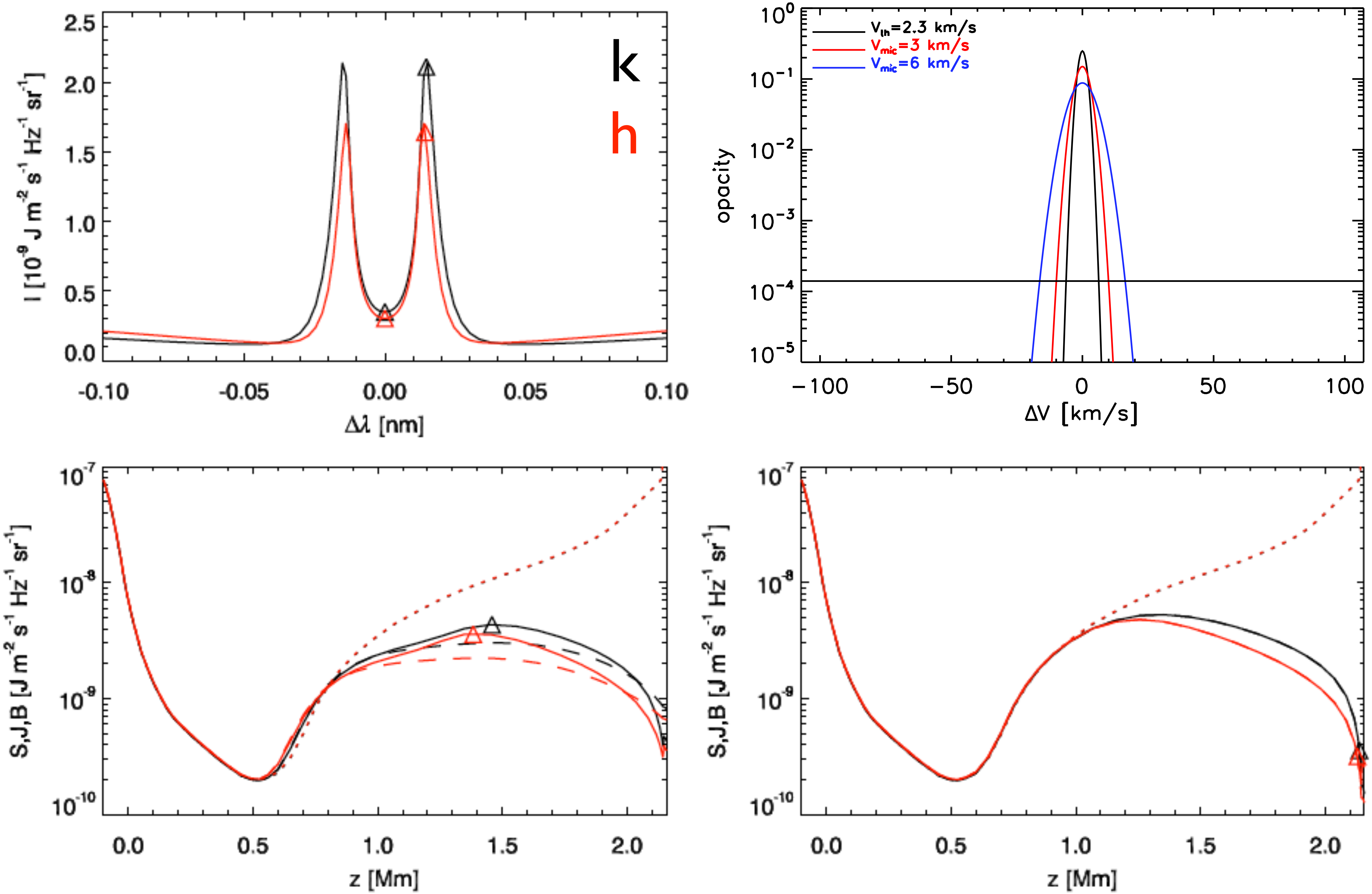
$$C_{I_\nu}(z) = S_\nu \tau_\nu e^{-\tau_\nu} \frac{\chi_\nu}{\tau_\nu}$$

C II 1334



T [kK]
14
12
10
8
6
4
2.5
2.0
1.5
1.0
0.5
z [Mm]
χ_ν/τ_ν
S_ν
$\tau_\nu \exp(-\tau_\nu)$
C_I
2.3
1.8
1.4
0.9
0.5
0.0
I [10^{-12} W m^{-2} sr^{-1} Hz^{-1}]
20
10
0
-10
-20
$\Delta\nu$ [km/s]

Line width and line ratio



Line width

$$\Phi_\nu = \frac{1}{\sqrt{\pi}\Delta\nu_D} e^{-\left(\frac{\Delta\nu}{\Delta\nu_D}\right)^2}$$

$$\Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m} + \xi^2}$$

Gaussian fit:

$$I_\nu = (I_0 - I_c)e^{-\frac{\Delta\nu^2}{2\sigma^2}} + I_c$$

$$\Delta\nu_D = \sqrt{2}\sigma$$

$$W_{\mathrm{FWHM}} = 2\sqrt{\ln 2}\Delta\nu_D$$

$$W_2 = \sigma.$$

$$d\tau_\nu = -\chi_\nu dz$$

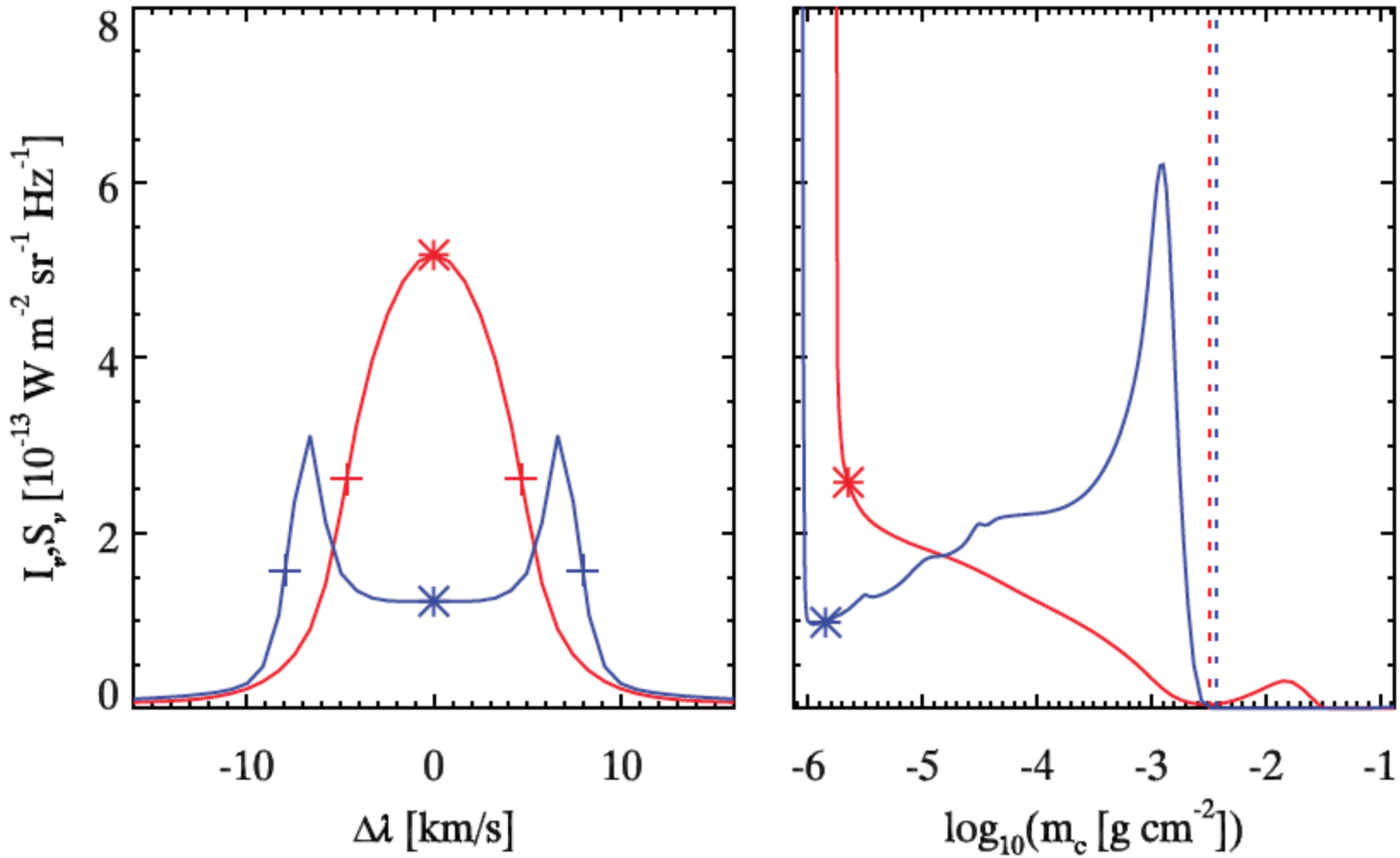
$$dm_c = -\rho dz$$

$$d\tau_\nu = \frac{\chi_\nu}{\rho} dm_c$$

$$1 = \int_0^{m_c(\Delta\nu)} \frac{\chi_\nu}{\rho} dm_c$$

$$\frac{\Delta\nu}{\Delta\nu_D} = \sqrt{\ln \frac{m_c(\Delta\nu)}{m_c(0)}}$$

Line width



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